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A review of functionally graded thick cylindrical and conical shells

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Abstract

Thick shells have attracted much attention in recent years particularly thick shells made from smart and graded material because of their unique properties. In this review paper, some critical issues and problems in the development of thick shells made from functionally graded piezoelectric material (FGPM) are discussed. This review has been conducted on various types of methods which are available for thick shell analysis and mainly focuses on elasticity theories, shear deformation theory, simplified theories and mixed theories since they were widely used in the modeling of FG thick shells. It is expected that this comprehensive study will be very beneficial to everyone involved or interested in the shell models.

Keywords: Thick-walled, Shell, Functionally graded Material (FGM), Piezoelectric.

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1. Introduction

A shell is a body that occupies a region in space lying between two surfaces called the faces of the shell. It may also have other bounding surfaces, called edges, transversal to the faces. The distance between the faces is assumed to be small in comparison with the other dimensions of the shell. That the thickness is small leads to several ways of approximating the equations of three-dimensional continuum mechanics [1]. Thick shells take many well-known shapes such as circular cylinders, cones, spheres, ellipsoids, paraboloids, and hyperboloids. They may be either solid or hollow. If they are hollow, they may be called shells of revolution. Shells of revolutions are considered as important structural elements, which are in various shapes as far as their applications are concerned in modern civil engineering, closed cylindrical shells and cone frustums in chemical industry, opened and closed cylindrical and conical shells with twist and tapered thickness in aerospace and turbomachinery industries [2-4]. Given that they are commonly used in most industrial equipment, it is necessary to gain insights into conditions of various loading.

Before the second world war the main interest in the investigation was concentrated on stationary thermal stresses. The unsteady thermal stresses can be divided into two groups, concerning quasi-static and dynamic thermal stresses respectively. In the case when the variation of temperature is slow in time then deformations as well as stresses may regard to be quasi-static, thus, the inertia terms appearing in the fundamental equations of the theory of thermal stresses can be neglected. The time variable occurring in the solutions is then regarded as a parameter, when the temperature change is rapid. Thermal stresses have attracted an increasing attention during the second war years. In this period of time numerous new papers on this subject have been published. The theory of thermal stresses, which previously, was regarded as a narrow branch of strength of materials and the theory of elasticity, became an independent subject connecting the theory of elasticity and thermodynamics. There appear many new synthetic works and monographs concerning the theory of thermal stresses. The general theory connecting the deformation field with the temperature field has been developed on the basis of irreversible thermodynamic processes. This theory is termed Thermo elasticity. The linear theory of therma1 stresses takes the assumptions of the classical, linear theory of elasticity and that the mechanical as well as thermal material properties are constant. These assumptions restrict the obtained solutions to definite ranges of temperature [5].

After creating the functionally graded materials (FGMs), researchers have focused on the analysis of the various structures made of these materials. FGMs have been created primarily by one Japanese group of material scientist (1980's). Properties of this group of materials varying continuously at the entire volume of the material. At the first years of decade 1990, researches on the thermal and vibration analysis of FGM have been started [6]. At the first years of decade 1990, researches on the thermal and vibration analysis of FGM have been started. As they are widely used in various industries, an attempt was made to modify the geometry and materials of shells so that their resistance against thermal and mechanical stresses increase and, where possible, to reduce the shell's weight. One of the important kinds of shells is the shells subjected to pressure. In today's advanced industry, special attention is paid to materials having low weight and high strength against damage done. FGMs have these properties, and these properties change smoothly and continuously along desired dimension and thus the stress distributions are smooth. A number of papers considering various aspects of FGM have been published in recent years [7-39].

With the development of smart materials and their application technology, piezoelectric ceramics and composites have been used not only as small components in electro-mechanical devices, but also as integrated structural elements just as shells. The same as FGM, mechanical and thermal properties of functionally graded piezoelectric material (FGPM) vary continuously in terms of their spatial coordinate system. FGPM have been introduced primarily to improve the structure of piezoelectric material and their application in various environments.

Therefore, studies on FGP thick shells caused a lot of researchers' concern, and many novel theories on coupled mechanics and relevant new and effective methodologies have been proposed in these decades [39-42]. This paper reviews many of the thick shell models associated with graded materials, with specific attention given to with FGMs, FGPMs.

2. Plane elasticity theory

The plane elasticity theories (PET) are derived from the 3D elasticity theory by making suitable assumptions on the kinematics of deformation or a tress state through the thickness of shells. These theories may account for both shear and normal deformation effects depending on the level of assumptions. The simplest equivalent-single-layer model is the classical plate theory, also known as Kirchoff theory [43], which ignores both shear and normal deformation effects. The classical plate model is based on the Kirchhoff-Love hypothesis that the straight lines remain straight and perpendicular to the midplane after deformation [44]. Many shell theories involve the Kirchhoff hypotheses. These require that fibers originally normal to the midsurface of the shell remain straight, normal and unchanged in length in any deformation. These assumptions imply the vanishing of the shear and normal strains, and consequently, neglecting the shear and normal deformation effects. The classical plate model is the simplest equivalent-single-layer model and it is only suitable for thin shells where the shear and normal deformation effects are inconsiderable. Using Kirchhoff-Love hypothesis, Lamé [45] presented an exact solution for thick-walled cylinders under inner and outer pressures. It has been supposed that the cylinder to be axisymmetric and isotropic. This solution is applicable for simple and quick solution of the pressure vessels and consequently mostly appears in the handbooks of pressure vessel.

Sherief et al. [46] considered the problem of an infinitely long annular cylinder whose inner and outer surfaces are subjected to known surrounding temperatures and are traction free. They used the Laplace transform with respect to time in the context of generalized thermo-elasticity theory with one relaxation time. Fukui and Yamanaka [47] studied the elastic problem of thick-walled tubes of a functionally graded material under internal pressure in the case of plane strain. The model components of the FGM are based on the combination of a matrix with three kinds of elastic moduli. The distribution profile is assumed by the expressions reported previously for a corundum/plaster FGM.

Horgan and Chan [48] investigated the effects of material inhomogeneity in another fundamental boundary-value problem of linear inhomogeneous isotropic elastostatics, namely the pressurized hollow cylinder (plane strain) or disk (plane stress) problem. The solution to the analogous problem for homogeneous isotropic materials (i.e., the classic Lamé problem) is discussed in virtually every textbook on linear elasticity. However, relatively little is known about the problem for inhomogeneous materials. Of course, in the classic books of Timoshenko and Goodier [49], special inhomogeneous configurations have been examined, for example, compound tubes composed of two different homogeneous isotropic materials. Such configurations involving shrink or force fits have been used in gun construction to alleviate stress build-up at the inner surface. Some aspects of the Lamé problem for inhomogeneous materials with continuously varying properties, have also been considered in many papers.

Tutuncu and Ozturk [50] presented the closed-form solutions for stresses and displacements in functionally graded cylindrical and spherical vessels subjected to internal pressure alone using the infinitesimal theory of elasticity. They assumed the material stiffness obeying a simple power law to vary through the wall thickness. Jabbari et al. [51] developed the general solution of steady-state on one-dimensional axisymmetric mechanical and thermal stresses for a hollow thick made of cylinder functionally graded porous material. They considered temperature as functions of the radial direction with general thermal and mechanical boundary-conditions on the inside and outside surfaces. It has been supposed the material properties are varying as a power function in terms of the radial coordinate system. With substitution of the derived temperature field in the Navier equation, the obtained differential equation has been analytically solved. Wu et al. [52] presented an analytical study for piezothermoelectric behavior of a circular cylindrical shell laminated with a functionally graded piezoelectric layer. For the case that the material properties obey an identical power law in the radial direction, exact solutions obtained through power series expansion method in conjunction with axisymmetric thermal or mechanical loading expanded as Fourier series. They considered both direct and inverse piezoelectric effects. Although this work considered the case in which the material constants are of power functions in the radial variable, the technique is applicable to other material inhomogeneity.

Galic and Horgan [53] developed an analytic solution to the axisymmetric problem of an infinitely long, radially polarized, radially orthotropic piezoelectric hollow circular cylinder rotating about its axis at constant angular velocity. The cylinder is subjected to uniform internal pressure, or a constant potential difference between its inner and outer surfaces, or both. They obtained an analytic solution to the governing equilibrium equations (a coupled system of second-order ordinary differential equations. Eraslan and Akis [54] obtained Analytical solutions for rotating solid shafts/disks by considering the nonlinear variation of the modulus of elasticity in radial direction. They used two different functions, one in exponential form, and the other in parabolic form, to describe the variation of the modulus of elasticity. Both forms are nonzero at the center and are sufficient to describe a reasonable variation of modulus of elasticity in the material. The use of the exponential profile in solid shaft/disk problems leads to an elastic equation of confluent hypergeometric type which assumed a closed form solution in terms of confluent hypergeometric functions. Dai et al. [55] presented an analytical study for magnetoelastic behavior of functionally graded material (FGM) cylindrical and spherical vessels placed in a uniform magnetic field, subjected to internal pressure. They determined the exact solutions for displacement, stress and perturbation of magnetic field vector in FGM cylindrical and spherical vessels by using the infinitesimal theory of magnetoelasticity.

Xiang et al. [56] proposed a very simple and convenient method to exactly analyze the N-layered elastic hollow cylinders submitted to the uniform pressures on the inner and outer surfaces by introducing two simple recursive algorithms. The extrusion stresses between two neighbor layers in the multi-layered cylinder are simply determined and then the exact solutions of the structure can be easily found based on Lame's solution. In this work, all the material parameters including the modulus of elasticity, Poisson's ratio, and the thickness of layers may be different in different layers. For the hollow cylinder with exponential graded properties and the linear graded properties, the exact solutions are found by solving a Whittaker equation or a hyper-geometric equation. Shi et al. [57] obtained by introducing two simple recursive algorithms, a very simple and convenient method to find the exact solutions of the N-layered elastic hollow cylinder submitted to uniform pressures on the inner and outer surfaces based on Lame's solution. For the hollow cylinder with continuously graded properties, it is found that the exact solutions can be obtained by solving a hyper-geometric equation. It demonstrated that the absolute value of the displacement in the radial direction in the layered cylinder will decrease with the increase of the number of layers, whether the cylinder is subjected to internal or external pressure. Tutuncu [58] obtained a tractable solution rather than numerical results to allow for further parametric studies. Stress and displacement solutions in the form of power series are presented in FGM thick-walled cylinders with exponentially-varying elastic modulus in the radial direction. A five-digit accuracy was obtained by taking twenty terms in the power series.

The effect of temperature is known as thermal effect while the effect of moisture absorption from the atmosphere is known as hygroscopic effect. The combined effect of temperature and moisture is known as hygrothermal effect. Heat gets conducted into the laminate when subjected to rise in the temperature. The laminate absorbs moisture when subjected to the wet conditions. Zenkour [59] described the hygrothermal responses in inhomogeneous piezoelectric hollow cylinders to present the interaction of electric potentials, electric displacement and elastic deformations.

The hybrid shell consists of a FGM host layer embedded with functionally graded piezoelectric material layers as sensors and/or actuators that are perfectly bonded to inner and outer surfaces of a shell. Saadatfar and Khafri [60] investigated the behavior of a rotating exponentially graded hybrid cylindrical shell subjected to an axisymmetric thermoelectromechanical loading and placed in a constant magnetic field. Although many earlier studies on rotating FG structures have considered different functions such as exponential or power for thickness and material distribution, the effect of thickness and material function type on the displacement, stresses and temperature distribution rarely researched. Jabbari et al. [61] presented a comparative study of thermoelastic analysis is given for material properties and disk thickness profiles using the Lame problem subjected to body force (e.g. angular velocity), internal pressure (e.g. shrink fit pressure) and thermal loads (e.g. temperature gradient).

3. Shear deformation theory

In shear deformation theory (SDT), the straight lines perpendicular to the central axis of the shell of revolution do not necessarily remain unchanged after loading and deformation, suggesting that the deformations are axially axisymmetric and change along the longitudinal direction. In other words, the elements have rotation, and the shear strain is not zero. The SDT developed by Reissner [62] which accounts for the shear deformation effect. Mindlin [63] accounts for the shear deformation effect by the way of a linear variation of the in-plane displacements through the thickness. Mirsky and Hermann [64] applied the first order shear deformation theory (FSDT) for the analysis of an isotropic cylinder.

A high order theory to examine the electromechanical behavior of piezoelectric generic shells with graded material properties in the thickness direction presented by Wu et al. [65]. They derived different types of charge equations, depending upon whether the driving signal of piezoelectrics is free charge or electric voltage. The obtained equations can be readily reduced to typical structures, such as beams, plates and circular cylindrical shells. The high order theory can be used to study the sensing and actuating behavior of a simply supported inhomogeneous piezoelectric circular cylindrical shell and, for comparison and validation purposes, a homogeneous shell. Eipakchi et. al. [66] investigated the governing equations of homogeneous cylinders with variable thickness using FSDT and represents the solution of the equations using perturbation theory. The further extended their previous work by considering homogenous and isotropic conical shells with variable thickness using FSDT and second-order shear deformation theory (SSDT) and solved the conducted equations by perturbation theory [67]. Bayat et al. [68] developed new linear and non-linear equilibrium equations for an FG axisymmetric rotating disk with bending and thermal loading. The material properties of FGM disks are assumed to vary continuously through the thickness of the disk graded according to a power-law distribution of the volume fraction of the constituents. First-order plate theory and von-Karman theory are used and both small and large deflections are considered. In case of small deflection, an exact closed form solution for displacement field is given. Power series solutions are employed to solve for displacement field under large deflection.

Ghannad et al. [69] presented the analytical solution of a thick homogenous and isotropic conical shell, making use of the FSDT. In line with the energy principle and the FSDT, the equilibrium equations have been derived. They analytically solved using the matched asymptotic method (MAM) of the perturbation theory, the system of differential equations which are ordinary and have variable coefficients. Eipakchi [70] improved the work of Ghannad et al. by calculated the stresses and displacements of a thick conical shell with varying thickness under nonuniform internal pressure using third-order shear deformation theory (TSDT). The obtained equations were solved the matched asymptotic expansion method, which are a system of differential equations with variable coefficients. This method does not require knowledge of the inner and outer profiles of the shell and loading distribution before formulation, and can explain the behavior of the shell successfully even near the boundaries. The method involved solving a system of algebraic equations and two systems of differential equations with constant coefficients. These systems of equations have closed-form solutions. The material properties are assumed to vary with a power law function along the thickness of cylinder. Arefi and Rahimi [71] proposed an analytical method for two-dimensional analysis of a clamped-clamped FG cylinder. Hamilton principle and FSDT are employed for derivation of the principle differential equations. Solution of these differential equations includes the particular and homogenous solutions. Homogenous solution can be obtained by recognizing the imaginary and real roots of characteristic equation. With imposing the natural boundary condition at the end of cylinder and the symmetric condition at the middle of cylinder, the homogenous solution can be analytically obtained. Rahimi et al. [72] employed SDT for electro elastic analysis of a functionally graded piezoelectric cylinder as a physical sensor for estimation and controlling the internal pressure. Ghannad et al. [73] performed an elastic analysis for axisymmetric clamped-clamped pressurized thick truncated conical shells made of FGMs based on the FSDT, and the virtual work principle.

In modern time, it has become increasingly popular to transport various types of liquids by sea and the storage tanks required for this purpose are often made from expensive materials. It is therefore desirable to make the tanks with as little material as possible. One way of decreasing the structural weight is to vary the wall thickness. Short transition sections of shells of revolution with variable wall thickness form part of the shell structures. In ref. [74] a theoretical analysis is provided for the axisymmetric deformation of varying thickness cylinder subjected to a varying lateral pressure. In this work, Ghannad et al. presented an analytical solution for deformations and stresses of axisymmetric clamped–clamped thick cylindrical shells with variable thickness made of FGMs subjected to internal pressure, using FSDT and MAM. Nejad et al. [75-77] derived an elastic solution for the purpose of determining displacements and stresses in thick shells under uniform pressure where multilayered method (MLM) has been used for the solution. They obtained that MLM is very suitable for the purpose of calculation of radial stress, circumferential stress, shear stress, and radial displacement, but it is not useful for axial stress and not useful at all for axial displacement. Also, they used this method for the analysis of a rotating truncated conical shell.

Viola et al. [78] extended the generalized TSDT with the normal and shear stress recovery to various types of FG truncated conical panels and shells using the generalized differential quadrature method (GDQM). By considering this formulation it is possible to apply a general loading condition with the satisfaction of all the boundary conditions. It is shown how the mechanical response for graded truncated conical panels or shells changes with the variation of the assumed theory. It should be noticed that the procedure introduced in this paper can also be extended to other types of graded panels or shells. Making use of FSDT and MLM, Nejad et al. [79, 80] performed a semi-analytical solution for the purpose of elastic analysis of rotating thick shells with variable thickness made of axially FGM under non-uniform pressure. Using the generalized plane strain one-dimensional equilibrium equation of thermo-elasticity, Alsubari et al. [81] presented a hygro-thermo-elastic analysis of anisotropic cylindrical shells under hygrothermal and mechanical loading. Based on the realistic variation of displacements from the elasticity approach, they proposed a new 13-term HSDT for the analysis of an anisotropic cylindrical shell strip. They concluded that higher-order terms in the HSDT play an important role in achieving accurate results. Kar and Panda [82] analyzed the linear and nonlinear flexural behavior of different geometries of FG shell panels under the thermo-mechanical load. This is the first time a general nonlinear mathematical model of FG shell panel is developed based on the HSDT kinematics and Green–Lagrange nonlinearity. The governing differential equation is derived using the variational principle and discretized with the help of suitable isoparametric FE steps. The desired responses are computed numerically using a direct iterative method. Lai et al. [83] obtained based on FSDT considering the effect of transverse shear, the displacement-based differential equation set for general problems of moderately thick cylindrical shells; the problem is simplified into a solvable eighth-order ordinary differential equation and a second-order ordinary differential equation by introducing four displacement functions.

Sabik and Kreja [84] presented an effectiveness study of the proposed FEM model for multilayered composite plates and shells under the temperature influence. The multilayered shell body was considered as an Equivalent Single Layer with the average resultant stiffness of the multilayered cross-section, whereas the First Order Shear Deformation theory kinematic assumptions were taken into account.

Zhang et al. [85] analyzed based on the classical shell theory, the transient thermal stresses of a thin FG cylindrical shell subjected to thermal shock load on the inner surface. Firstly, the transient temperature field assumed in the form power series; is obtained by using Laplace transform in combination. Then, GDQM is introduced to obtain the numerical solutions of the transient thermal stresses. Using TSDT and MLM, Jabbari et al. [86, 87] carried out a semi-analytical solution for the purpose of a thermo-elastic analysis of axially FG rotating thick shells with variable thickness subjected to the temperature gradient and pressure loading.

Nejad et al. [88] developed a general formulation for thermo-elastic analysis of a functionally graded thick shell of revolution with arbitrary curvature and variable thickness subjected to thermo-mechanical loading by using HSDT and MLM. They derived the final relations in general state for every arbitrary structure and material property distributions.

4. Shell theories

The general theory of thin isotropic elastic shells (where the effects of transverse normal stress, transverse hear deformation, and rotatory inertia are also discussed), a set of stress displacement relations is deduced which is entire consistent with the assumptions for the stresses and displacements in a thin shell. A thick shell model incorporating the effect of rotary inertia and the shear deformation was used. To describe the shell geometry a cylindrical polar coordinate system. These results were obtained by means of a variational theorem due to Reissner. Naghdi and Cooper [89] considered two systems of equations of motion for elastic cylindrical shells using the basic equations of Reissner. This theory which include the effects of both transverse shear deformation and rotatory inertia**.**

Irie et al. [90] presented an analysis for the free vibration of a truncated conical shell with variable thickness by use of the transfer matrix approach. They assumed the applicability of the classical thin shell theory and the governing equations of vibration of a conical shell are written as a coupled set of first order differential equations by using the transfer matrix of the shell. Takahashi et al. [91] discussed a free vibrations investigation of a truncated conical shell by means of the improved theory of shells. They considered equations of vibration and the boundary conditions in general forms developed from stationary conditions of Lagrangian of a conical shell. These equations solved in the conical shell with linearly varied thickness along axis add the effects of boundary conditions, number of nodal generators, thickness and semi-vertex angle on frequencie.

Cui et al. [92] introduced new variable transformation formulas to solve the basic governing differential equations for conical shells. They transformed the basic governing differential equations for conical shells into a second-order differential equation with complex constant coefficients, by performing magnitude order analysis and neglecting the quantities with h/R magnitude order.

Wu et al. [52] investigated the elastic stability of an FG cylinder. They employed the shell Donnell's theory to derive the strain deformation relations. Stress-strain relations have been obtained by regarding the effect of thermal strain in Hooke's low. Three nonlinear equations of equilibrium according to Donnell's theory were applied. With imposing the condition of pre-buckling and a function for the radial displacement, the results have been defined by minimizing the critical load with respect to the parameter of the problem. Pelletier and Vel [93] investigated an exact solution for the steady state thermo-elastic response of FG orthotropic cylindrical shells using Flugge and Donnell shell theories. Sofiyev et al. [94] developed analytical formulations and solutions for the stability analysis of heterogeneous orthotropic conical shell with mixed boundary conditions using Donnell shell theory. The basic equations of heterogeneous orthotropic truncated conical shells are derived and solved applying the Galerkin's method for the two cases of mixed boundary conditions. Wu et al. [95] presented on the basis of the classical elasticity theory, the exact analysis of the transversely isotropic axisymmetric electro-magneto-thermo-elastic circular cylinder using the general solution and Lur'e symbolic method. Since the present theory is derived without requirement of any ad hoc assumptions concerning the deformation or the stress state, the results based on them are of high accuracy, appeal to application, and help to describe problems in an incisive way.

5. Generalized elasticity theory

Based on the theory of elasticity, generalized governing equations of hollow cylinders in cylindrical coordinate can be derived. Based on the 3D theory of elasticity and assuming the material properties to be two-dimensionally dependent, which means the material properties vary according to described functions continuously in two directions. Conical shells are the shells of revolution, however, when we applied their variable transformation formulas to conical shells, it was found that these formulas are singular. In order to overcome this problem, Witt [96] derived the differential equation of a conical shell subjected to asymmetrical temperature distributions. In order to obtain a particular solution to the differential equation, he assumed the expression for the temperature distributions to be the sum of hyperbolic and cubic functions. Hausenbauer and Lee [97] determined stresses for the radial, tangential and axial stresses in a thick-walled cone subject to internal and/or external pressures. They considered to be valid for long shells with small tapering angles where the effect of shear is small. For short cones with large tapering angles, stresses are no longer negligible and the results of this investigation do not apply.

Panferov [98] used the method of perturbing the shape of the boundary to determine the stress state of thick-walled conical and biconical isotropic shells, under the assumption that the shells are closed and that their shape deviates but little from a spherical one. They used the method of successive approximations to obtain the solution of the problem of thermal loading of an elastic, transversely isotropic conical pipe (generally speaking, truncated) of constant thickness. Tarn [99] studied the thermomechanical states in a class of functionally graded cylinders under extension, torsion, shearing, pressuring, and temperature change. Referred to the cylindrical coordinates, the material is cylindrically anisotropic. The only material symmetry considered reflectional symmetry with respect to the cylindrical surfaces r=constant.

Jabbari et al. [100] developed the analytical solution for the non-axisymmetric thermal and mechanical stresses in a thick hollow cylinder made of functionally graded material. The method of solution is based on the direct method and uses power series, rather than the potential function method. The advantage of this method over the potential function method is its generality and mathematical power to handle any type of the mechanical and thermal boundary conditions. Jane and Wu [101] analyzed the thermoelastic transient response of the laminated circular conical shell. They discussed the thermoelastic problem of circular conical shell composed of multilayer of different materials. The finite difference and the Laplace transform methods were employed to obtain the numerical results. Application of the presented method to laminated circular conical shells reveals that the previous method is rapidly convergent. There is no limit to the number of layers in such a circular conical shell.

Shao [102] investigated the thermo elastic analysis of a thick-walled cylinder under the mechanical and thermal loads. The cylinder has been divided into many annular sub cylinders in the radial direction. Based on this division, properties of every sub cylinder may be assumed to be isotropic. In the following, the thermal and the equilibrium equation have been individually employed for every sub cylinder. After solution of the thermal and the equilibrium equation in every sub cylinder, compatibility equations for the thermal and mechanical components within every two layers are imposed. By performing this procedure for the complete cylinder, distribution of temperature and displacement have been obtained.

Kang [103] employed the three-dimensional formulation of elasticity for modeling a thick shell of revolution with variable thickness and curvature. The achieved formulation in that study was valid for an isotropic material. Nejad, Rahimi, and Ghannad [104] could derive a set of field equations for a functionally graded shell of revolution. The tensor-based formulation was employed for the purpose of analyzing a functionally graded piezoelectric shell of revolution by Arefi and Rahimi [105]. Dehghan et al. [106] obtained the general form of equations in coupled electro-thermo-elastic analysis of shells of revolution with variable thickness and material properties that are graded in three directions. In this paper, primarily, the basic equations in a curvilinear system are introduced and base vectors, in the form of covariant and contra-variant, are developed. This type of equations can be easily transformed to cylindrical, spherical or any existing ordinary orthogonal curvilinear coordinates systems for coupled electro-thermo-elastic behavior of material. They used the energy method and Hamilton's principle in order to obtain energy functional and derive the equations of motion. Jabbari et al. [107] studied an attempt the problem of general solution for the thermal and mechanical stresses in a thick FGPM hollow cylinder due to the two-dimensional non-axissymmmetric steady-state loads. The method of solution is based on the direct method and uses power series, rather than the potential function method. The advantage of this method is its mathematical power to handle both simple and complicated mathematical function for the thermal and mechanical stresses boundary conditions. The potential function method is capable of handling complicated mathematical functions as boundary condition.

Alibeigloo [108] presented an analytical solution for infinitesimal deformations of a functionally graded hollow cylindrical shell with the inner and the outer surfaces perfectly bonded to piezoelectric layers and excited by thermo-electro-mechanical loads. The analysis was carried out by using a Navier type solution that identically satisfies boundary conditions at the simply-supported and electrically grounded left and the right end faces of the hybrid cylinder. The resulting ordinary differential equations are solved by the state space method. There is one-way coupling for the thermal effects in the sense that the temperature change affects the mechanical and the electric deformations but is not affected by them. Fesharaki et al. [109] presented the analytical solution for the two-dimensional electro mechanical behavior of a hollow cylinder made of FGPM. Zozulya and Zhang [110] presented a high order theory for FG axisymmetric cylindrical shells based on the expansion of the axisymmetric equations of elasticity for FGMs into Fourier series in terms of Legendre's polynomials. Starting from the axisymmetric equations of elasticity, the stress and strain tensors, the displacement, traction and body force vectors are expanded into Fourier series in terms of Legendre's polynomials in the thickness coordinate. In the same way, the material parameters that describe the functionally graded material properties are also expanded into Fourier series. All equations of the linear elasticity including Hooke's law are transformed into the corresponding equations for the Fourier series expansion coefficients.

Alashti et al. [111] carried out asymmetric deformation and stress analysis of a functionally graded hollow cylindrical shell under the effect of thermo-mechanical loads using the differential quadrature method. Without losing the generality, material properties of the cylindrical shell are assumed to be graded in the radial direction obeying a power law, while the Poisson ratio is assumed to be constant. The governing partial differential equations are expressed in terms of displacement and thermal fields in series forms with the help of two versions of differential quadrature methods, namely the polynomial and Fourier quadrature methods.

6. Finite element models

A numerical study of the dynamic thermoelastic response of FGMs was performed by Reddy and Chin [112]. Two problems, viz., a one-dimensional, axisymmetric functionally graded cylinder, with temperature and spatially dependent material properties and subjected to two different thermal loading conditions are studied. Since these problems involved a high thermal loading, the study is aimed at examining the effect of thermomechanical coupling in each case. It is found that there is no significant difference in temperature distribution for the uncoupled and coupled formulations, partly due to the static boundary conditions imposed on the cylinder.

Raju et al. [113] obtained a simple finite element which fits the above configuration is obviously a conical shell finite element. They derived the stiffness matrix for a conical shell finite element is derived using Novozhilov's strain-displacement relations for a conical shell. Over the past years, some researchers have managed to analyze thick shells of revolution for isotropic and functionally graded materials. Semi-analytical finite element formulation, using first-order shear deformation theory, was used by Ganesan and Kadoli [114] to analyze the shell of revolution in curvilinear coordinate. Sladek et al. [115] applied a meshless local Petrov-Galerkin method to orthotropic shallow shells under a thermal load by the Reissner-Mindlin theory, which takes the shear deformation into account. Material properties are considered continuously varying along the shell thickness. They used the Laplace-transform technique to eliminate the time variable in the considered diffusion equation. Hosseini et al. [116] developed the meshless local Petrov–Galerkin (MLPG) method to analyze the coupled thermoelasticity problem based on the Green–Naghdi theory (without energy dissipation) in functionally graded thick hollow cylinders. The governing equations in the time-domain form are written in cylindrical coordinate system. The local integral equations are derived from the weak form of the governing equations with considering Heaviside step function as the test functions. The spatial variation of the temperature and radial displacement are approximated using an interpolation based on multiquadric radial basis function. After substitution of spatial approximations into the local integral equations a system of ordinary differential equations (ODE) is obtained. The system of ODE is solved by the Newmark finite difference method. Asemi et al. [117] described the elastic analysis of FG truncated thick hollow cone with finite length using the finite element method, and Rayleigh–Ritz energy formulation.

7. Conclusions

FGPMs are multiphase materials in which the material parameters change continuously along the thickness or radial direction, so as to achieve the purpose of optimizing the structure. FGPMs are widely used in aerospace, nuclear reactors, internal combustion engines and other fields. A large number of researchers have devoted themselves to this scientific area, and great achievements have been made. To this purpose, this review is presented on researches performed in the area of coupled mechanics on FGPM thick shells. Various theories for the modeling and analysis of coupling problems of FGPM thick shells, and main coupling problems of FGPM thick shells have been comprehensively reviewed and thoroughly discussed in this paper. The main aim of this review article is to collate the research performed in the area of coupled mechanics on FGPM tick shell, thereby giving a broad perspective of the state of art in this field.

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