



## Non-rigid star pattern recognition for preparation of IOD's observations

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### ABSTRACT

The invention of electro-optical devices at the beginning of the 21st century was really a rebirth in the geodetic astronomy. Today, the digital cameras with relatively high geometric and radiometric accuracy have opened a new insight in the satellite attitude determination and the study of the Earth's surface geometry and physics of its interior, i.e., the computation of astronomical coordinates and the vertical deflection components. In the automatic star detection, high precision and reliability in extraction of the star's centers from the captured images and corresponding them with the astronomical coordinates is the most important point. In this article, the probabilistic method has been applied for the star matching. The registration is treated as a Maximum Likelihood estimation problem with the motion constraint over the velocity field such that the catalogue coordinates set moves coherently to align with the pixels coordinates set. The motion coherence has been constrained to the matching problem and derives a solution of regularized ML estimation through the vibrational approach, which leads to an elegant kernel form. In this way, the EM algorithm has been applied for the penalized ML optimization with deterministic annealing. This method finds correspondence between stars coordinates in catalogue and image without making any prior assumption of the transformation model except the motion coherence. This method can estimate the gnomonic transformations between the catalogue and the image and is shown to be accurate and robust in the presence of image noise and outliers. The result of evaluation by proposed algorithm on the image taken by the TZK2-D camera, indicated that the point matching is achieved by standard deviation less than 0.001 pixel.

### KEYWORDS

Star matching  
Initial orbit determination  
Coherent motion theory

### 1. Introduction

The celestial positioning has been used for navigation purposes for many years. The stars as the extra-terrestrial benchmarks provide unique opportunity in absolute point positioning. However, astronomical field data acquisition and data processing of the collected data is very time-consuming. The advent of the Global Positioning System (GPS) nearly made the celestial positioning system obsolete. The new satellite-based positioning system has been very popular since it is very efficient and convenient for many daily life applications. Nevertheless, the celestial positioning method is never replaced by satellite-based positioning in absolute point positioning sense. From the ancient times, determination and prediction of orbit was a challenge for the scientists and it has a historical

background in celestial mechanics. Today, satellite orbit determination is known as a primary part of each space mission. In recent years, the majority of launched satellites have been in the lower limits of the earth's atmosphere—these satellites are called low earth orbit satellites. The orbit determination of these satellites has been addressed by organizations, universities and related international institutions ([http://www.aiub.unibe.ch/index\\_eng.html](http://www.aiub.unibe.ch/index_eng.html)). Besides scientific satellites, spy and military satellites lie in this layer too. Exploring and predicting the orbit of such satellites have a vital role in the field of passive defense. In other words, the direction and height of rise and decent of these satellites in a certain local horizon are among the most sensitive parameters required for the electronic and defense warfare (Montenbruck & Gill, 2000).

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Additional observations and appropriate initial value are used for exact orbit determination. So, in any mission that a low number of observations from intended object's orbit is available, the initial orbit determination with high accuracy is considered as a critical matter. According to different types of observations such as range, range rate, angle and angle rate, different methods have been used to determine the initial orbit. For example, Gauss and Laplace methods are among the classical algorithms that are used to determine the initial orbit. So far, various techniques have been presented for satellite tracking, among which, three common methods are: the laser ranging technology, the radar tracking technology and the optical tracking technology (Amann et al., 2001). According to the researches, if the ideal condition is available, the accuracy of optical tracking systems (surveying the sky using optical telescopes) is better than the range-based methods. In addition, due to the simple structure and inexpensive equipment of optical systems, they have a high flexibility in environmental conditions (Hejduk et al, 2004). Also, the satellites which are in high earth orbits do not access to range observations and we have to use angular observations. Thus, the optical tracking is preferred to the other above-mentioned methods. Since the optical tracking has more optimality than the radar and laser techniques and because of the availability of digital cameras with high geometric and radiometric accuracy, the visual-based geodetic astronomy has been used for different applications including satellite positioning, astronomical coordinates, and vertical deviation components (Li et al., 2014). At the beginning of the 21st century, some fundamental changes

were occurred in geodetic astronomy using the electro-optical method (Hirt & Burki, 2006). This method is an automatic and efficient method with the ability to real-time data analysis and the base of that is astronomical imaging by CCDs (Hirt & Flury, 2008). The images captured by this method have a lot of information about the stars, galaxy, satellites' streak, etc. (Stoveken & Schildknecht, 2005). In these systems, CCD provides an image from stars around the vertical axis and then the extracted stars from the image must be matched with the available stars in the catalogue to obtain the celestial coordinates of stars. Photometry by CCD is a novel method that nowadays is used in different sciences such as astronomical measurements. By using CCD and an appropriate telescope, we can create a method with high visibility to record stars with the low brightness which are not visible even by ordinary telescopes. Figure 1 shows the principals of the determination of astronomical coordinates. In all the mentioned applications, the most important step is the identification of the available stars in the image. After building a CCD-based star-tracker by Salmin (1976) (Salomon & Goss, 1976), scientists such as JPL, Strkwerd, Turner, and Junkins (Junkins & Strikwerda, 1979; Junkins & White, 1977) attempted to perform some researches for real-time identification of stars after image capturing. The main goal of their research is the presentation of a matching method to identify stars in the astronomical digital images. Figure 2a shows an image from the sky; after extracting the stars' centers (bright spots) by image processing techniques, they must be identified in the catalogue of Figure 2b.

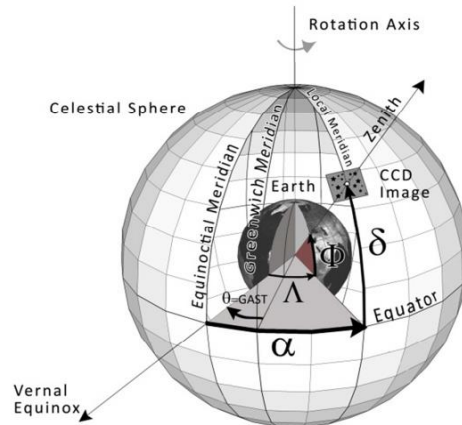


Figure 1. The basics principals of determining astronomical coordinates

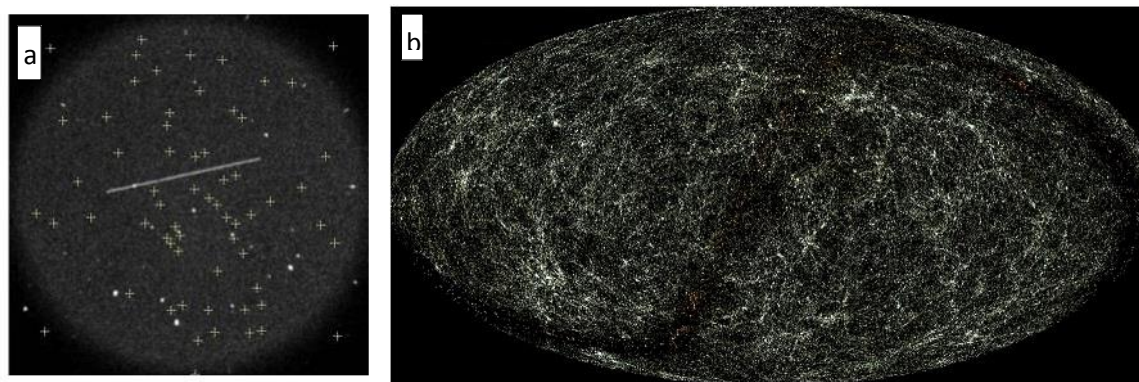


Figure 2. (a) Captured image from the sky along with extracted stars, (b) sample star catalogue that includes more than 1 milliard stars

One of the factors that make problem in star identification procedure is the presence of outlier points and noise (due to CCD error) in data. In other words, all the bright points in the image aren't star, so the identification algorithms must be able to detect obtained stars between lots of stars in the catalogue. During the last years, different methods have been proposed to resolve this problem (Clouse & Padgett, 2000; Chen et al., 2004; Jiancheng et al., 2005). Generally, these methods can be divided into three categories: a) shape-based methods; b) network-based methods and c) intelligent methods. Gottlieb (1978) presented the shape-based methods; the main idea of this method is the selection of some stars in camera's field of view to form different geometric shapes such as triangle, quadrangle, etc. and the angular distance between the points is used as matching the index. Triangulation among points is an optimal and common method that has been used widely. In this method, other geometric shapes like spherical triangle have been used to reduce the adaptation (match) error of geometric shapes (Hejduk et al., 2004). In the network-based method the star identification problem was first introduced by Padgett and Kreutz-Delgado as a pattern recognition problem (Padgett & Kreutz-Delgado, 1997). In other words, a pattern is defined for each star based on its adjacent stars. So each star has its own pattern that its corresponding pattern must be found in the formed patterns of the catalogue. In intelligent methods, neural networks have been used to pattern recognition. The neural network designed by Li and Zhang (2003) includes sub-networks that are parallel with error propagation algorithm (Li et al., 2000). In this method, first a plane mapped matrix is created as neural network input, then the classification determines the selected sub-network with the error propagation algorithm and finally, the network is trained by the input data. Paladugu (2003) has used Cohen network to identify the stars (Hirt & Flury, 2008). Paladugu (2004) has used the genetic algorithm to identify the corresponding pattern of stars in the catalogue (Paladugu et al., 2004). The network-based method is more reliable than the shape-based method to match stars and is suitable for cameras with the medium field of view. Although this method has less computation than the shape-based method, it will be

time-consuming and inefficient in cases that the points in a system have been rescaled or the camera's field of view is small. The intelligent methods have resolved the mistake matching problem and complicated calculations of the previous methods. However, it is time-consuming still (Spratling & Mortari, 2009). In this article, the probabilistic method has been applied for non-rigid star matching. The registration is treated as a Maximum Likelihood (ML) estimation problem with the motion coherence constraint over the velocity field such that the catalogue coordinates set moves coherently to align with the pixels coordinates set. In this way, the Gaussian Mixture Model GMM has been fitted to the star catalogue coordinates set, whose Gaussian centroids are initialized from the star image coordinates set and the process of adapting the Gaussian centroids have been considered from their initial positions to their final positions as a temporal motion process, and imposes a motion coherence constraint over the velocity field. This motion coherence constraint penalizes the derivatives of all orders of the underlying velocity field.

## 2. Method

In the following, we discuss our proposed approach which is divided into four steps described as follows.

### 2.1 Map projection in imaging stars

The imaging of stars is based on gnomonic map projection (Grafarend & Krumm, 2006). Gnomonic map projection is an azimuthal map projection that can be considered as conformal map projection around the point that the camera's axis is realized. According to that fact, the points of both systems are almost without the distortion—caused by map projection—around their centers so that the distortion is increased by getting away from the image's center (Figure 3). Therefore, the mentioned methods lose their capability by getting away from the image's center because in all methods, the realization of the conformality condition between the extracted stars from the image and the catalogue according to the medium Field Of View (FOV) has been used. Figure 4 shows the distortion in angle, area, and distance of the gnomonic map projection, respectively

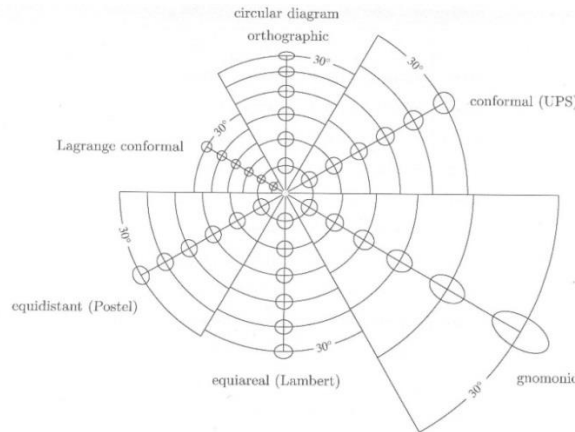


Figure 3. Pie chart comparison of six map projection based on distance from center

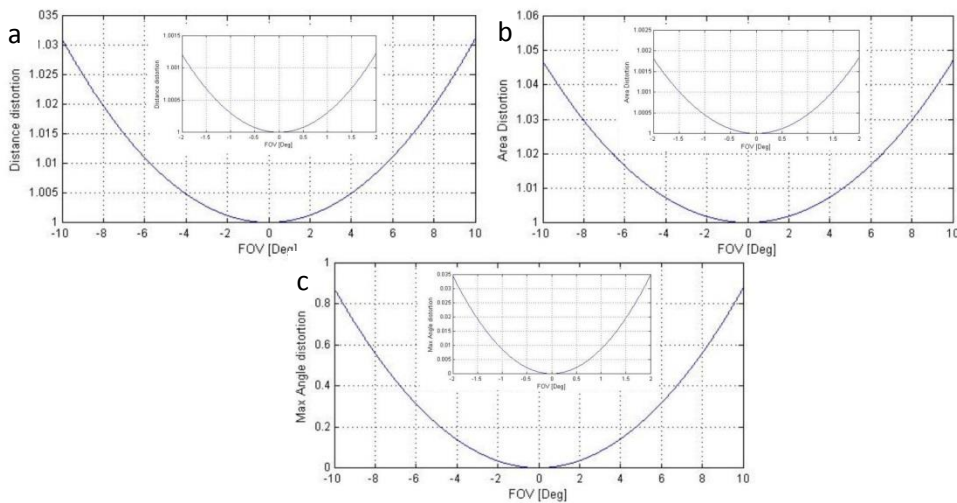


Figure 4. (a) Distance, (b) area, and (c) max angle distortion in gnomonic map projection

In all the existing applications including the star tracker, one of the most important factors in identifying the stars is the relationship between FOV and the mentioned distortions. So the problem is the use of a method that its pattern recognition is independent of the camera's FOV. In this paper, first, we discover the corresponding points using the probability theory of Bayesian and then remove the mistake points by the M-estimator Sample and Consensus (MSAC) algorithm. As we will see in the following sections, the proposed algorithm is more effective to reduce the consumed time and computations, compared to other previous methods. Figure 4 shows the flowchart of the proposed algorithm. In addition, due to the use of MSAC algorithm, the probability of errors is eliminated in the matching process.

## 2.2 Extracting astronomical coordinates

As it is clear according to the proposed algorithm (Figure 5), the first step for matching between the pixel and celestial coordinates of stars is the extraction of star celestial coordinates from the catalogue based on the captured position. Star catalogues have different dimensions and can include from thousands to hundreds of millions of stars, and it is used as an important database in different applications. Star catalogues' information is collected by

special satellites or Ground-based telescopes and usually contains some of stars' positions along with position-based errors. A star catalogue contains information such as apparent and absolute magnitude in different bands, triangle properties, photometry data, information about the special movement of the star, spectrum information, etc. (Hoffleit & Jaschek, 1982). One of the important features of star catalogue is the high accuracy of star position and completeness of their number. There are many agencies and observatories in the world that have tried to make different kinds of star catalogues with a high number of stars by improving the methods and measurement equipment, each of which has its own special applications and features. One of the available catalogues is the Hipparcos catalogue. This catalogue is the primary result of the observations and data reductions taken by the satellite that includes 118218 data with the astronomical accuracy of (about) 1 arc-millisecond and certain result for twofold and multifold systems. The mean magnitude accuracy of Hipparcos catalogue for fixed stars in magnitude range 0.0004-0.007 (mag) and in the interval of (2-12) mag with corresponding transmission errors is about 0.003-0.05 (mag). The normal accuracies around magnitude 8 is about 0.0015 (mag) on the middle and about 0.011 (mag) on the individual errors. The positional accuracy of Hipparcos catalogue in 1991/25 time-interval was about 1-3 milliseconds which was a great



accuracy at that time. However, the normal positional errors increased to about 15 arc-milliseconds in 2005 time-interval. The accuracy of specific movements is also about 1-2 arc milliseconds per year. This range for brighter stars is about 1-2 (mag). Finally, two-color photometry was published from Tayko test which concluded that this range can be better than 0.1 mags. With the international agreement, the Hipparcos catalogue is a standard reference catalogue for photometry that presents the positions in celestial inertial reference system at optic wavelengths. However, in 1999 a deal was agreed upon, and based on that, near 1800 stars and double stars were kept out of the catalogue. In addition, Hipparcos includes wide band of visual photometric data that contains information related to the variable stars. It should be noted that information of star positions in the catalogue are prepared on 1 January 2000. As a result, the position information of the catalogue must be updated to take advantage of catalogue's information. In this regard, special movement corrections and time shift of star coordinates are used. Thus, the star celestial coordinates are calculated according to the intended epoch (Perryman et al., 1997).

**2.3 Accurate extraction of stars' centers**

After capturing images from the stars, it is necessary to extract stars' centers with high accuracy. In other words, the aim of this step is the extraction of stars' centers at pixel coordinates. For this propose, SIFT algorithm has been used in this study (Lowe, 1999).

**2.4 Matching (identifying) stars**

Before describing the proposed algorithm, it is necessary to explain some of the concepts used in this study as follows:

**Gaussian mixture model:** Mixture models are a group of density functions that are formed by some (normally

Gaussian) distribution functions which are combined to form a multi-mode distribution function. In a Gaussian mixture model, it is assumed that each  $x_i$  is created independently by a mixture with the following density:

$$f(x_i | \theta) = \sum_{k=1}^K p_k h(x_i | \mu_k, \Sigma_k) \tag{1}$$

$p_k$  is mixing ratio of (by assumption  $0 < p_k < 1$  for each  $k$  and  $(p_1 + \dots + p_K = 1)$   $k^{\text{th}}$  component and  $h(x_i | \mu_k, \Sigma_k)$  is  $d$ -dimensional Gaussian distribution function that its average is  $\mu_k$  and its covariance matrix is  $\Sigma_k$ . Also,  $\theta$  is a vector of mixture parameters where  $\theta = (p_1 + \dots + p_K, \mu_1 + \dots + \mu_K, \Sigma_1 + \dots + \Sigma_K)$  (Bilmes, 1998).

**Bayes estimator:** Bayesian estimator  $\tau(\theta)$  which is showed by  $T_{l,G} = t_{l,G}(X_1, \dots, X_n)$ , is an estimator that has minimal Bayes risk towards the loss function  $L$  and previous cumulative distribution  $G$  (Langley et al., 1992).

**Convergent motion theory:** In convergence theory, if two sets of corresponding points are considered, the velocity field which is obtained using differences in the points' positions of the first system based on the location differences of the points is a converge and smooth field (Figure 6). So, the velocity field of corresponding points can be achieved by applying the convergence condition (Yuille & Grzywacz, 1989).

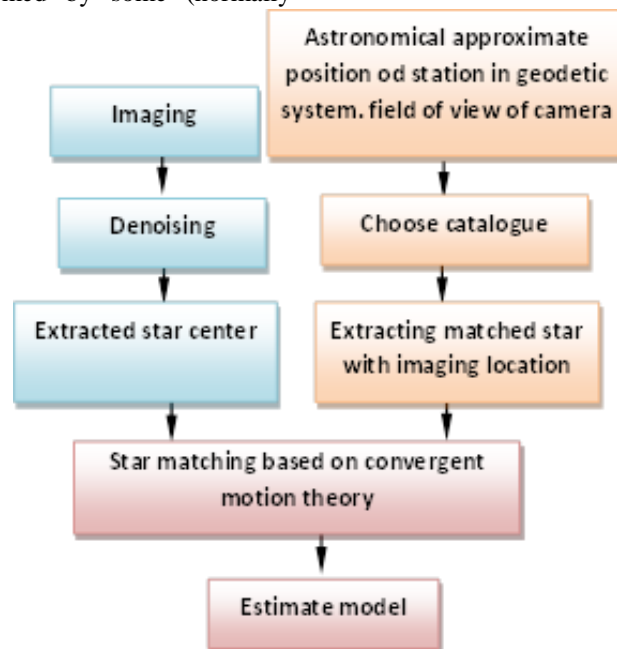


Figure 5. Proposed algorithm

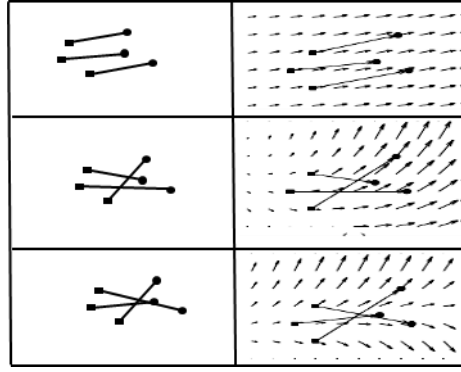


Figure 6. Different matching of 2 point sets cause to create velocity field with different values of smoothness as first image points are matched correctly (Yuille & Grzywacz, 1988)

In the proposed algorithm for matching between the extracted celestial and pixel coordinates, matching is mentioned as a maximum expectation estimation problem with convergence condition on the velocity field; as one set of points has a convergent motion for matching on the second set. The base of this method is matching a Gaussian mixture model on the first set of points, as their initial centers are points' positions in the first coordinate system. The procedure of matching the Gaussian model centers to their final position (the points at the second coordinate system) is considered as a temporary motion with convergence condition.

The celestial coordinates of stars are considered as reference points' set. We indicate them and their pixel coordinates by  $X_{N \times 2}$   $Y_{M \times 2}$  respectively. The aim of this algorithm is detecting  $K$  corresponding stars from  $M$  stars in the pixel coordinate system and  $N$  stars in the celestial coordinate system, so  $K$  value is between  $M$  and  $N$  values. The image points are considered as initial centers of a Gaussian mixture model ( $Y_0$ ) and the continues function of velocity ( $v$ ) of these points is defined so that the star position at any moment is equal to:

$$Y = v(Y_0) + Y_0 \quad (2)$$

Gaussian mixture density function is defined as follows:

$$p(x) = \sum_{m=1}^M \frac{1}{M} p(X | m) \quad (3)$$

where,  $X | m \sim N(y_m, \sigma^2 I_2)$  and  $Y$  are the two-dimensional centers of Gaussian function with equal covariance. In order to impose the smooth motion condition, the following constraint is applied:

$$p(Y | \lambda) \propto \exp\left(-\frac{\lambda}{2} \phi(Y)\right) \quad (4)$$

$p(Y | \lambda)$  is the initial probability function of the image points,  $\lambda$  is weight constant and  $\phi(Y)$  is the function that controls the smoothness of motion. By using Bayesian theory, the aim is finding  $Y$  through maximization of the secondary probability or equivalently through minimization of the following energy function:

$$E(Y) = -\sum_{n=1}^N \log \sum_{m=1}^M e^{-\frac{1}{2} \left\| \frac{x_n - y_m}{\sigma} \right\|^2} + \frac{\lambda}{2} \phi(Y) \quad (5)$$

We assume the points are independent and have identical distributions and ignore the independent terms of  $Y$ . Function  $\phi$  which should be smooth finally shows the initial information about motion. In this research, our aim is smoothing the produced velocity field by movement of the image points. As we know, the oscillatory behavior of a function specifies the level of smoothing (Girosi et al., 1995). In differentiable functions class, one function is smoother than the other one if it has less oscillation; in other words, this function has less energy in high frequency. The high frequencies of a function can be achieved by using a high pass filter and accordingly the power of obtained frequencies can be calculated. So, function  $\phi$  can be defined as follows:

$$\phi(s) = \int_{\mathbb{R}^2} \frac{|\tilde{v}(s)|^2}{\tilde{G}(s)} ds \quad (6)$$

$\tilde{v}$  indicates the Fourier transform of velocity and  $\tilde{G}$  is a symmetric low pass filter. By rewriting the Eq. (6), we will have:

$$E(\tilde{s}) = -\sum_{n=1}^N \log \sum_{m=1}^M e^{-\frac{1}{2} \left\| \frac{x_n - y_m}{\sigma} \right\|^2} + \frac{\lambda}{2} \int_{\mathbb{R}^2} \frac{|\tilde{v}(s)|^2}{\tilde{G}(s)} ds \quad (7)$$

It can be proved that the function which minimizes the above energy function is in the form of a radial basic function.

$$v(z) = \sum_{m=1}^M W_m G(z - y_{0m}) \quad (8)$$

We consider  $G$  as a Gaussian function. This choice has various reasons that are (Chui & Rangarajan, 2003):

1- It satisfies our expected features (for example, it is symmetric, positive definition and the limit of its Fourier transform is zero when the norm of  $S$  limits to zero).

2- The Gaussian form is without oscillation in time and frequency domains.

3- This choice has caused our regularization parameter to be similar to the regularization parameter in the convergent motion theory.

By substituting the above results in Eq. (7), we will obtain:

$$E(W) = -\sum_{n=1}^N \log \sum_{m=1}^M e^{-\frac{1}{2} \left\| \frac{x_n - y_{0m} - \sum_{k=1}^M w_k G(y_{0k} - y_{0m})}{\sigma} \right\|^2} + \frac{\lambda}{2} \text{tr}(W^T G W) \quad (9)$$

$G_{M \times M}$  is gram matrix whose elements are  $g_{ij} = e^{-\frac{1}{2} \left\| \frac{y_{0i} - y_{0j}}{\beta} \right\|^2}$  and  $W_{M \times 2}$  is Gaussian weight kernel.

Now, by using maximum expectation algorithm, we can determine the upper bound of the Eq. (9):

$$Q(W) = \sum_{n=1}^N \sum_{m=1}^M P^{old}(m | X_n) \frac{\|x_n - y_{0m} - G(m, \cdot)W\|^2}{2\sigma^2} + \frac{\lambda}{2} \text{tr}(W^T G W) \quad (10)$$

$P^{old}$  Indicates the secondary possibilities and  $G(m, \cdot)$  is  $m^{\text{th}}$  row in matrix  $G$ . The minimization of an upper bound of  $Q$  will cause the energy function of Eq. (9) to minimize. If we differentiate Eq. (10) with respect to  $W$  and rewrite the

equation in a matrix form, we will have (maximization step):

$$\frac{\partial Q}{\partial W} = \frac{1}{\sigma^2} G(\text{diag}(P1))(Y_0 + G W) - P X + \lambda G W = 0 \quad (11)$$

$P$  is the matrix of secondary possibilities with the following elements:

$$P_{mn} = \frac{e^{-\frac{1}{2} \left\| \frac{y_m^{old} - x_n}{\sigma} \right\|^2}}{\sum_{m=1}^M e^{-\frac{1}{2} \left\| \frac{y_m^{old} - x_n}{\sigma} \right\|^2}} \quad (12)$$

Also,  $1$  is a columnar matrix whose elements are one. By multiplying Eq. (11) by  $\sigma^2 G^{-1}$ , we find a linear equation system.

$$(\text{diag}(P1))G + I \sigma^2 W = P X - \text{diag}(P1)Y_0 \quad (13)$$

The output of the proposed algorithm will be matrix  $P$  whose  $i^{\text{th}}$  column indicates the matching probability of  $i^{\text{th}}$  star with all of the stars in the catalogue. Unfortunately, this algorithm is unstable versus noise and mistake points; in other words, there will be some mistake points among the matched points that aren't are not matched. So, we attempt to apply MSAC algorithm to solve this problem and discover the points that are incorrectly matched (Torr & Zisserman, 2000).

#### Finding algorithm of corresponding points

- 1- Determine values for  $\beta, \lambda, \sigma_0$  parameters
- 2- Create gram matrix with initial values:  $Y = Y_0$
- 3- Deterministic annealing algorithm
  - 3-1- Maximum expectation algorithm
    - 3-1-1 Calculate correspondence matrix  $P$
    - 3-1-2 Calculate matrix  $W$
    - 3-1-3 Determine new values of  $Y$ :  $Y = Y_0 + G W$
  - 3-2- Sigma reduction :  $\sigma = r \sigma_0$
- 4- Calculate field of velocity :  $v(z) = G(z, 0)W$

Figure 7. Matching algorithm

### 3. Results

The proposed method is implemented on the two images that have been used by CCD's of Ixon Ultra 888 for experiments and analyses. That machine has been equipped with back-illuminated technology and possesses an electron magnifying set up to record the single photons that achieve to the CCD's surface. Table 1 and Figure 8 presents the key specifications of the machine. As it can be observed the machine has 1024\*1024 square pixels with the pixel size of 13 microns. This sensor has high quantum efficiency and attracts and saves more than 90% of the electrons that achieve to the surface of the measurer. The high reading rate of this sensor provides an appropriate capability for consecutive imaging of the celestial bodies. The central processor of this measurer digitalizes the attracted electrons in the form of 14-bit and provides them to users in the form of files with fits format. Figure 9 shows the sample images

used in this study. As it was presented in the flowchart of the proposed algorithm, denoising is done after the imaging to prepare the image for matching algorithm's matters. In this study, we use image denoising algorithm that uses heat equation and its numerical solution (Perona & Malik, 1990). One of the features of this method is that in denoising process, it prevents the softening of existing edges in the denoised image. It is obvious that the temperatures of different points are changed toward a balance temperature. Now, if we match the gray levels of the image with the temperatures of different points, then pursuant to solve the heat equation, the gray levels of the Image approach each other and this leads to denoising. It should be noted that the diffusion coefficient of edges is selected large in order to preserve the edges. The results of this section are shown in Figure 9.



Figure 8. CCD of iXON Ultra 888

Table1. Key specification of iXON Ultra 888

<b>Active pixel (H x V)</b>	1024×1024
<b>Pixel size (W x H; μm )</b>	13×13 μm
<b>Image area (mm)</b>	13.3×13.3
<b>Active Area Pixel Well Depth (e- )</b>	80e-
<b>Max Readout Rate (MHz)</b>	30MHZ
<b>Frame rates (fps)</b>	26(full frame)-9690
<b>Read noise (e-)</b>	<1 with EM gain
<b>QE Max</b>	>90%

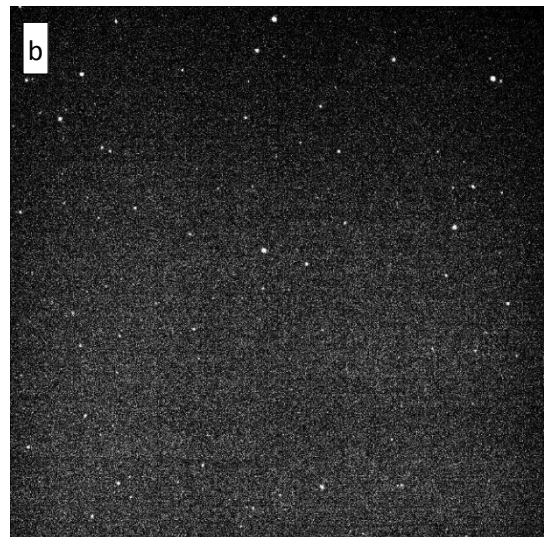
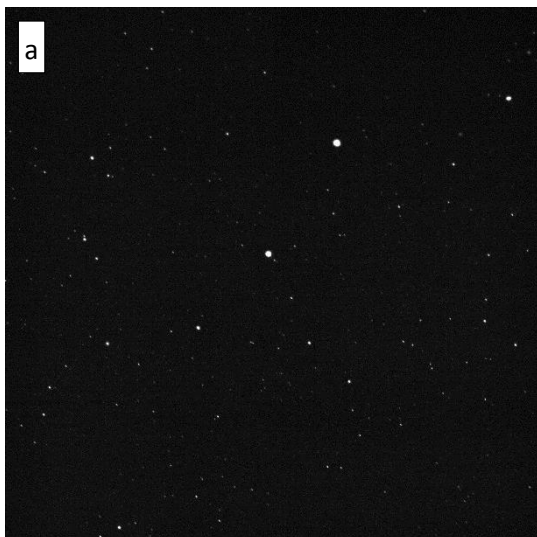


Figure 9. (a) The first sample, and (b) the second sample image form sky at known station by CCDs of Ixon Ultra 888



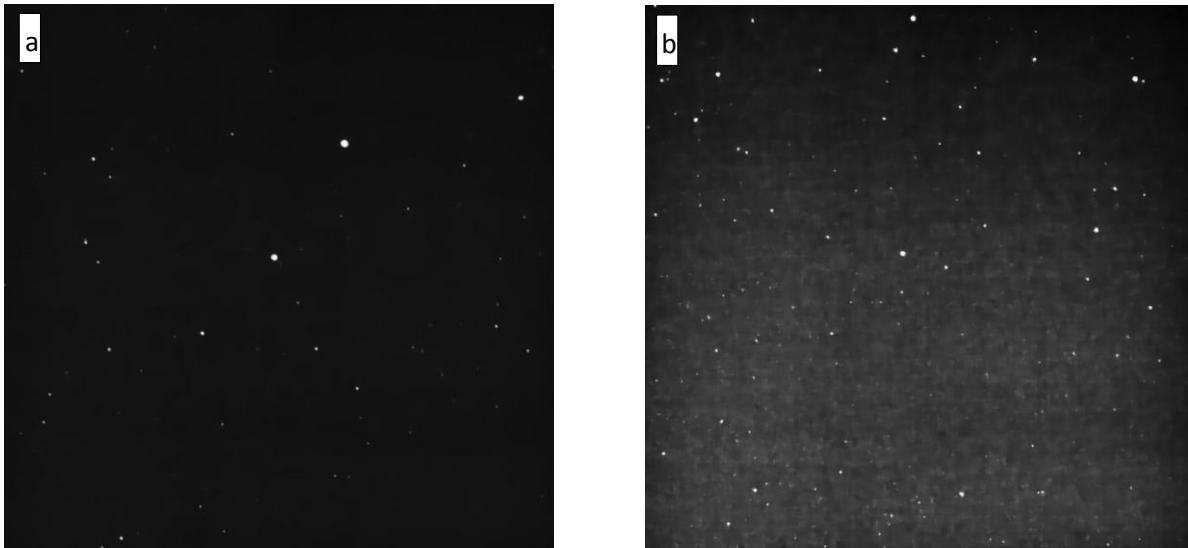


Figure 10. (a) Result of denoising in first sample image, and (b) Result of denoising in second sample image by diffusion equation

Table 2. SIFT parameters

SIFT Parameters	
<b>Scl</b>	2.1
<b>Threshold</b>	10
<b>Radius1</b>	4
<b>Radius2</b>	4
<b>Radius3</b>	4
<b>Octaves</b>	5
<b>Sigma</b>	1.5
<b>Edge Ratio</b>	5

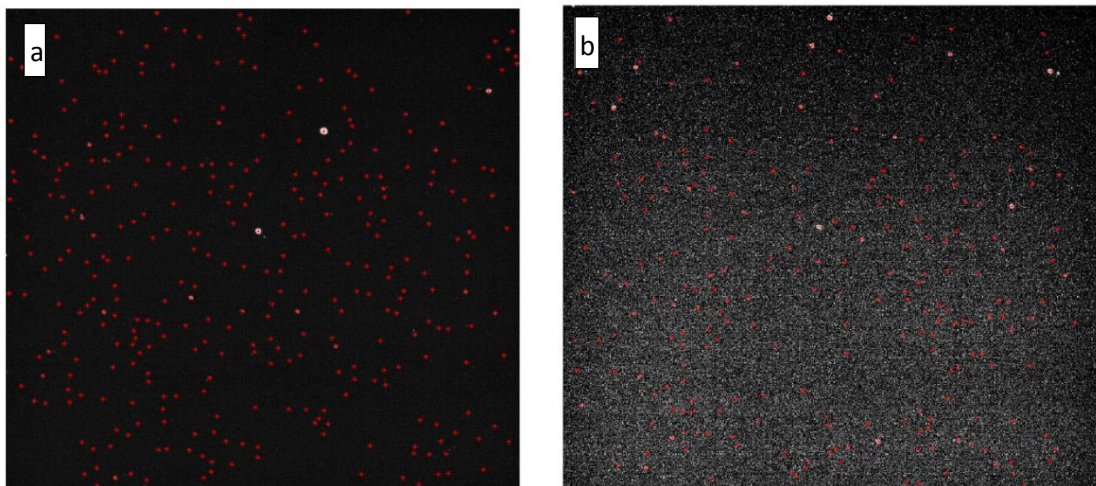


Figure 11. (a) Detected star in first sample images, and (b) detected star in second sample images

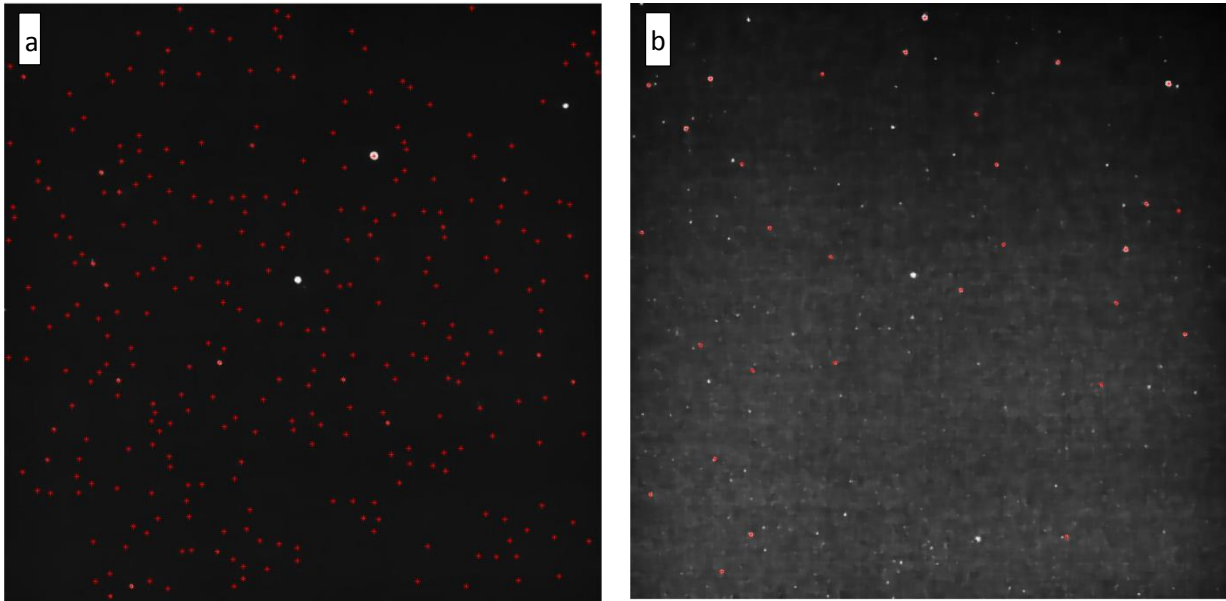


Figure 12. Result of star matching process at (a) first sample image, and (b) second sample image

The algorithm of extracting stars was applied to the image that its noises had been eliminated (Figure 10). Therefore, by taking advantage of SIFT parameters presented in Table 2, the centers of stars are highlighted as red points in Figure 11 using the proposed algorithm by Sharifi, Samadzadegan, and Farzaneh (2008). It should be mentioned that with having geodetic coordinates of capturing station that is recorded with GPS and FOV of the camera, we can extract the corresponding range of the sky from the catalogue in observations epoch. We apply the stars' coordinates of both systems (pixel and celestial) to matching algorithm based on the matching parameters of Table 3. The output of this step is probability matrix of matching. Where  $(i, j)$  element represents the matching probability between  $i^{\text{th}}$  extracted star from the image and  $j^{\text{th}}$  star of the catalogue. Figure 12 shows the position of matched stars using the proposed algorithm.

#### 4. Conclusion

The invention of electro-optical devices at the beginning of the 21<sup>st</sup> century was really a rebirth in the geodetic astronomy. Today, the digital cameras with relatively high geometric and radiometric accuracy have opened a new insight in satellite attitude determination and the study of the Earth's surface geometry and physics of its interior, i.e., computation of the astronomical coordinates and the vertical deflection components. In the automatic star detection, high precision and reliable extraction of the star's centers from the captured images, and corresponding them with the astronomical coordinates, is the most important point. In this article, the star's centers are extracted by the advanced image processing technique with sub-pixel precision. Using the theory of coherent motion, the corresponding stars have been detected.

In the proposed algorithm for matching between the extracted celestial and pixel coordinates, matching is mentioned as a maximum expectation estimation problem with convergence condition on the velocity field; as one set

of points has a convergent motion for matching on the second set. The base of this method is matching a Gaussian mixture model on the first set of points, as their initial centers are points' positions in the first coordinate system. The procedure of matching the Gaussian model centers to their final position (the points at the second coordinate system) is considered as a temporary motion with convergence condition.

It should be noted that this algorithm will be a more efficient method than the previous methods in term of time and calculation volume.

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