JCAMECH

Vol. 48, No. 2, December 2017, pp 319-330

DOI: 10.22059/jcamech.2017.239995.178

On the Analysis of Laminar Flow of Viscous Fluid through a Porous Channel with Suction/Injection at Slowly Expanding or Contracting Walls

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Received: 18 Aug. 2017, Accepted: 27 Aug. 2017

Abstract

The vast biological and industrial applications of laminar flow of viscous fluid through a porous channel with contracting or expanding permeable wall have attest to the importance of studying the flow process. In this paper, two-dimensional flow of viscous fluid in a porous channel through slowly expanding or contracting walls with injection or suction is analyzed using variation parameter method. From the parametric studies using the developed approximate analytical solutions, it is shown that increase in the Reynolds number of the flow process leads to decrease in the axial velocity at the center of the channel during the expansion. The axial velocity increases slightly near the surface of the channel when the wall contracts at the same rate. Also, as the wall expansion ratio increases, the velocity at the center decreases but it increases near the wall. The results of the approximate analytical solution are verified by numerical solution using shooting method coupled with Runge-Kutta method. The results of the variation parameter method are in excellent agreement with the results obtained using numerical method.

Keyword: Viscous; Porous Channel; Expanding or Contracting walls; Variation parameter method; Biological applications.

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1. Introduction

The study of laminar flow of viscous fluid through a porous channel or pipe with contracting or expanding permeable walls have attracted good number of research efforts in the past few decades. This is because of its biological applications such as transport of biological fluids through contracting or expanding vessels, filtration in kidneys and lungs, flow inside lymphatics, the synchronous pulsation of porous diaphragms, modeling of air circulation in respiratory system. Also, the wide range of attracted research attentions of the laminar flow process is due to its industrial applications as it is evident in the model of regression of burning surface in solid rocket motors, binary gas diffusion, chromatography, ion exchange, ground water movement, transpiration cooling and the separation of ^{235}U from ^{238}U by gaseous diffusion and also flow in multichannel filtration systems such as the wall flow monolith filter used to reduce emissions from diesel engines [1–8]. In all these applications, it is established that the equations governing the flow process are generally nonlinear. In order to predict and determine the actual flow behavior, different analytical, approximate analytical and numerical methods have been employed to solve the governing nonlinear equations. In the past studies of laminar flow through a porous channel, Majdalani [5] and Majdalani and Roh [6] adopted asymptotic formulations using Wentzel-Krammers-Brillouin (WKB) and multiple-scale techniques to study the oscillatory channel flow with wall injection. Jankowski and Majdalani [9] also applied the multiple-scale techniques to analyze oscillatory channel flow with arbitrary suction. Jankowski and Majdalani [10] developed an analytical solution by means of the Liouville-Green transformation for laminar flow in a porous channel with large wall suction and a weakly oscillatory pressure while Zhou and Majdalani [11] used finite difference method and asymptotic technique (variation of parameters and small parameter perturbations) to investigate the mean flow for slab rocket motors with regressing walls. The results from the two methods were compared for different Reynolds numbers *Re* and the wall regression rate *a*, and it was observed that accuracy of the analytical solution deteriorates for small *Re* and large *a*. A good agreement between the solutions was observed for large values of *Re*. A similar analysis was done by Majdalani and Zhou [12] for moderate-to-large injection and suction driven channel flow with expanding or contracting walls. Multiple solutions associated with this problem have been reported by Robinson [13], Zarturska et al. [14] and Si et al. [15, 16]. Majdalani et al. [17] applied regular perturbation method to study two-dimensional viscous flow between slowly expanding or contracting walls with weak permeability. In a recent study, Dinarvand et al. [18] adopted homotopy analysis and homotopy perturbation methods to solve Berman's model of two-dimensional viscous flow in porous channels with wall suction or injection. They concluded that the HPM solution is not valid for large Reynolds numbers, a weakness earlier observed in the case of other perturbation techniques. Using the homotopy analysis method (HAM), Xu et al. [19] developed highly accurate series approximations for two-dimensional viscous flow between two moving porous walls and obtained multiple solutions associated with this problem. Also, the same method was adopted by Dinarvand and Rashidi [20] to analyze two dimensional viscous flow in a rectangular domain bounded by two moving porous walls. Although, the homotopy analysis method is a reliable and efficient semi-analytical technique, it suffers from a number of limiting assumptions such as the requirements that the solution ought to conform to the so-called rule of solution expression and the rule of coefficient ergodicity. Moreover, the solution comes with large number of terms. In practice, analytical solutions with large number of terms and conditional statements for the solutions are not convenient for use by designers and engineers [21].

Variation parameter method (VPM) is an approximate analytical method which has been applied to solve linear and nonlinear differential equations [22-26]. It is different from variational iteration method (VIM) in many aspects. In VIM, the multiplier used called Lagrange Multiplier can be of different forms of exact, semi-exact and approximate multipliers. However, in VPM, there is no concept of exact and approximate multipliers. The multiplier used in VPM is calculated using Wronskian technique. Also, VIM takes into account the complete equation for the solution purposes while VPM gives the solution of the problem without taking highest order term into consideration [27]. Its main advantages is in its ability to solve nonlinear integral and differential equations without linearization, discretization, closure, restrictive assumptions, perturbation, approximations, round-off error and discretization that could result in massive numerical computations. It provides excellent approximations to the solution of non-linear equations with high accuracy. Moreover, the need for small perturbation parameter as required in traditional perturbation methods, the difficulty in determining the Adomian polynomials, the rigour of the derivations of differential transformations or recursive relation as carried out in DTM, the restrictions of HPM to weakly nonlinear problems as established in literatures, the lack of rigorous theories or proper guidance for choosing initial approximation, auxiliary linear operators, auxiliary functions, auxiliary parameters, and the requirements of conformity of the solution to the rule of coefficient ergodicity as done in HAM, the search Langrange multiplier as carried in VIM, and the challenges associated with proper construction of the approximating functions for arbitrary domains or geometry of interest as in Galerkin weighted residual method (GWRM), least square method (LSM) and collocation method (CM) are some of the difficulties that VPM overcomes. The results of VPM are completely reliable and physically realistic. Therefore, in this present work, variation parameter method is utilized to analyze unsteady two-dimensional flow of viscous fluid through a porous channel with expanding/contracting walls with injection or suction. Also, the developed approximate analytical solutions are used to study the effects of the flow parameters in the expanding or contracting porous channel. In order to support and verify the approximate analytical solution by VPM, a numerical solution is also obtained using fourth-order Runge-Kutta method coupled with shooing techniques. From the analysis, the results obtained by VPM are in excellent agreements with the results of the numerical method.

2. Problem Formulation

Consider a fully developed unsteady, laminar, isothermal, and incompressible flow in a two-dimensional porous channel bounded by two permeable surfaces or walls that enable the viscous fluid to enter or exit during successive expansions or contractions as shown in Fig. 1. One end of the channel is closed by a compliant solid membrane. Both walls are assumed to have equal permeability and to expand uniformly at a time dependent rate, $\dot{a}(t)$. A coordinate system is chosen with the origin at the center of the channel as shown in the figure.

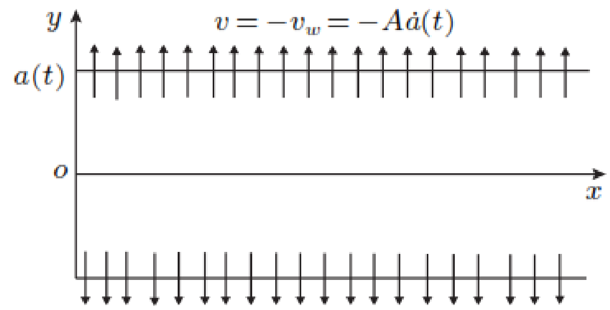


Fig. 1. The model of the porous channel with expanding or contracting walls.

Following the assumptions, the equations for continuity and motion are

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0 \tag{1}$$

$$\rho\left(\frac{\partial \bar{u}}{\partial t}+\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right)=-\frac{\partial \bar{p}}{\partial \bar{x}}+\mu\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)-\sigma B_o^2\bar{u} \tag{2}$$

$$\rho\left(\frac{\partial \bar{v}}{\partial t}+\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}}+v\frac{\partial \bar{v}}{\partial \bar{y}}\right)=-\frac{\partial \bar{p}}{\partial \bar{y}}+\mu\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) \tag{3}$$

Assuming no slip condition, the appropriate boundary conditions are given as

$$\bar{y}=a(t), \quad \bar{u}=0, \quad \bar{v}=-V_w=-\frac{a}{c} \tag{4}$$

$$\bar{y}=0, \quad \frac{\partial \bar{u}}{\partial \bar{y}}=0, \quad \bar{v}=0 \tag{5}$$

$$\bar{x}=0, \quad \bar{u}=0 \tag{6}$$

where $c=\dfrac{\dot{a}}{V_w}$ is the wall presence or injection/suction coefficient i.e. which is the measure of permeability

On introducing the following stream functions and the mean flow vorticity

$$\bar{u}=\frac{\partial \bar{\psi}}{\partial \bar{y}}, \quad \bar{v}=\frac{\partial \bar{\psi}}{\partial \bar{x}} \tag{7}$$

The pressure term in Eqs. (2) and (3) can be eliminated and the vorticity transport equation is obtained as

$$\rho\left(\frac{\partial \bar{\xi}}{\partial t}+\bar{u}\frac{\partial \bar{\xi}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{\xi}}{\partial \bar{y}}\right)=\mu\left(\frac{\partial^2 \bar{\xi}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{\xi}}{\partial \bar{y}^2}\right)+\sigma B_o^2\bar{u}\frac{\partial \bar{u}}{\partial \bar{y}} \tag{8}$$

where

$$\bar{\xi}=\frac{\partial \bar{v}}{\partial \bar{x}}-\frac{\partial \bar{u}}{\partial \bar{y}} \tag{9}$$

Also, the above partial differential equation can be converted to ordinary differential equation using the following similarity variables

$$\bar{u} = \frac{\mu x}{\rho H^2}\bar{f}'(\eta,t), \quad \bar{v} = \frac{-\mu}{\rho H}\bar{f}(\eta,t) \tag{10}$$

where

$$\eta = \frac{\bar{y}}{H}, \quad \bar{f}'(\eta,t) = \frac{\partial \bar{f}(\eta,t)}{\partial \eta}$$

Substituting Eq. (9) and (10) into Eq. (8), we arrived at a fourth order ordinary differential equation

$$\bar{f}_{\eta\eta\eta\eta} + \alpha(t)\left(\eta \bar{f}_{\eta\eta\eta} + 3 f_{\eta\eta}\right) + \overline{ff}_{\eta\eta\eta} - \bar{f}_{\eta}\bar{f}_{\eta\eta} - \frac{\rho a^2}{\mu}\bar{f}_{\eta\eta t} - Ha^2 f_{\eta\eta} = 0 \tag{11}$$

And the boundary conditions becomes

$$\eta = 0, \quad \bar{f} = 0, \quad \bar{f}_{\eta\eta} = 0$$

$$\eta = 1, \quad \bar{f} = Re, \quad \bar{f}_{\eta} = 0 \tag{12}$$

where $\alpha(t) = \frac{\rho a \dot{a}(t)}{\mu}$ is the non-dimensional wall dilation rate which is positive for expansion and negative for contraction.

And $Re = \frac{\rho a V_w}{\mu}$ is the permeation Reynolds number, which is positive for injection and negative for suction. Using the following variables,

$$\psi = \frac{\bar{\psi}}{a\dot{a}}, \quad u = \frac{\bar{u}}{\dot{a}}, \quad v = \frac{\bar{v}}{\dot{a}}, \quad f = \frac{\bar{f}}{Re} \quad \Rightarrow \quad \psi = \frac{xf}{c}, \quad u = \frac{xf'}{c}, \quad v = \frac{-f}{c}, \quad c = \frac{\alpha}{Re} \tag{13}$$

Eqs. (11) and (12) are normalized as

$$f_{\eta\eta\eta\eta} + \alpha(t)\left(\eta f_{\eta\eta\eta} + 3 f_{\eta\eta}\right) + Re\left(ff_{\eta\eta\eta} - f_{\eta} f_{\eta\eta}\right) - \frac{\rho a^2}{\mu} f_{\eta\eta t} - Ha^2 f_{\eta\eta} = 0 \tag{14}$$

with the boundary conditions

$$\eta = 0, \quad f = 0, \quad f_{\eta\eta} = 0$$
$$\eta = 1, \quad f = 1, \quad f_{\eta} = 0 \tag{15}$$

Assuming $\alpha(t) = \frac{\rho a \dot{a}(t)}{\mu}$ remains constant during the flow process, and $f = f(\eta)$, then $f_{\eta\eta t} = 0$ and Eq. (14) reduces to

$$\begin{aligned} &f_{\eta\eta\eta\eta} + \alpha\left(\eta f_{\eta\eta\eta} + 3 f_{\eta\eta}\right) \\ &+ Re\left(ff_{\eta\eta\eta} - f_{\eta} f_{\eta\eta}\right) - Ha^2 f_{\eta\eta} = 0 \end{aligned} \tag{16}$$

The boundary conditions still remain as Eq. (15).

If α=*0* in the above Eq. (16), the Berman's model [1] for channels with stationary walls is recovered.

3. The Procedure of Variation Parameter Method

The basic concept of VPM for solving differential equations is as follows: The general nonlinear equation is in the operator form

$$Lf(\eta) + Rf(\eta) + Nf(\eta) = g \tag{17}$$

The linear terms are decomposed into $L + R$, with L taken as the highest order derivative which is easily invertible and R as the remainder of the linear operator of order less than L. where g is the system input or the source term and u is the system output, Nu represents the nonlinear terms.

The VPM provides the general iterative scheme for Eq. (17) as:

$$f_{n+1}(\eta)= f_0(\eta)+\int_0^{\eta}\lambda(\eta,\xi)\left(-Rf_n(\xi)-Nf_n(\xi)-g(\xi)\right)d\xi \tag{18}$$

where the initial approximation $f_0(\eta)$ is given by

$$f_0(\eta)=\sum_{i=0}^{m}\frac{k_i f^i(0)}{i!} \tag{19}$$

m is the order of the given differential equation, k_i s are the unknown constants that can be determined by initial/boundary conditions and $\lambda(\eta,\xi)$ is the multiplier that reduces the order of the integration and can be determined with the help of Wronskian technique.

$$\lambda(\eta,\xi)=\sum_{i}^{m}\frac{(-1)^{i-1}\xi^{i-1}\eta^{m-1}}{(i-1)!(m-i)!}=\frac{(\eta-\xi)^{m-1}}{(m-1)!} \tag{20}$$

From the above, one can easily obtain the expressions of the multiplier for $Lf(\eta)=f^n(\eta)$

$$n=1,\quad \lambda(\eta,\xi)=1$$

$$n=2,\quad \lambda(\eta,\xi)=\eta-\xi$$

$$n=3,\quad \lambda(\eta,\xi)=\frac{\eta^2}{2!}-\eta\xi+\frac{\xi^2}{2!}$$

$$n=4,\quad \lambda(\eta,\xi)=\frac{\eta^3}{3!}-\frac{\eta^2\xi}{2!}+\frac{\eta\xi^2}{2!}-\frac{\xi^3}{3!}$$

$$n=5,\quad \lambda(\eta,\xi)=\frac{\eta^4}{4!}-\frac{\eta^3\xi}{3!}+\frac{\eta^2\xi^2}{2\cdot 2!}-\frac{\eta\xi^3}{3!}+\frac{\xi^4}{4!}$$

$$n=6,\quad \lambda(\eta,\xi)=\frac{\eta^5}{5!}-\frac{\eta^4\xi}{4!}+\frac{\eta^3\xi^2}{2\cdot 3!}-\frac{\eta^2\xi^3}{2\cdot 3!}+\frac{\eta\xi^4}{4!}-\frac{\xi^5}{5!}$$

$$n=7,\quad \lambda(\eta,\xi)=\frac{\eta^6}{6!}-\frac{\eta^5\xi}{5!}+\frac{\eta^4\xi^2}{2\cdot 4!}-\frac{\eta^3\xi^3}{6\cdot 3!}+\frac{15\eta^2\xi^4}{2\cdot 4!}-\frac{\eta\xi^5}{5!}+\frac{\xi^6}{6!}$$

$$n=8,\quad \lambda(\eta,\xi)=\frac{\eta^7}{7!}-\frac{\eta^6\xi}{6!}+\frac{\eta^5\xi^2}{2\cdot 5!}-\frac{\eta^4\xi^3}{6\cdot 4!}+\frac{\eta^3\xi^4}{6\cdot 4!}-\frac{\eta^2\xi^5}{2\cdot 5!}+\frac{\eta\xi^6}{6!}-\frac{\xi^7}{7!}$$

$$n=9,\quad \lambda(\eta,\xi)=\frac{\eta^8}{8!}-\frac{\eta^7\xi}{7!}+\frac{\eta^6\xi^2}{2\cdot 6!}-\frac{\eta^5\xi^3}{6!}+\frac{\eta^4\xi^4}{24\cdot 4!}-\frac{\eta^3\xi^5}{6!}+\frac{\eta^2\xi^6}{2\cdot 6!}-\frac{\eta\xi^7}{7!}+\frac{\xi^8}{8!}$$

$$n=10,\quad \lambda(\eta,\xi)=\frac{\eta^9}{9!}-\frac{\eta^8\xi}{8!}+\frac{\eta^7\xi^2}{2\cdot 7!}-\frac{\eta^6\xi^3}{36\cdot 5!}+\frac{\eta^5\xi^4}{24\cdot 5!}-\frac{\eta^4\xi^5}{24\cdot 5!}+\frac{\eta^3\xi^6}{36\cdot 5!}-\frac{\eta^2\xi^7}{2\cdot 7!}+\frac{\eta\xi^8}{8!}-\frac{\xi^9}{9!}$$

Consequently, an exact solution can be obtained when n approaches infinity.

Using the standard procedure of VPM as stated above, one can write the solution of Eq. (16) for the case of negligible magnetic field as

$$\begin{aligned} f_{n+1}(\eta)&=k_1+k_2\eta+k_3\frac{\eta^2}{2}+k_4\frac{\eta^3}{6}\\ &-\int_0^{\eta}\left(\frac{\eta^3}{3!}+\frac{\eta^2\xi}{2!}+\frac{\eta\xi^2}{2!}+\frac{\xi^3}{3!}\right)\left[\alpha\left(\eta\left(f_{\eta\eta\eta}\right)_n+3\left(f_{\eta\eta}\right)_n\right)+Re\left(\left(ff_{\eta\eta\eta}\right)_n-\left(f_{\eta}f_{\eta\eta}\right)_n\right)\right]d\xi \end{aligned} \tag{21}$$

Here, k_1, k_2, k_3, *and* k_4 are constants obtained by taking the highest order linear term of Eq. (16) and integrating it four times to get the final form of the scheme.

The above equation can also be written as

$$f_{n+1}(\eta) = f(0) + f'(0)\eta + f''(0)\frac{\eta^2}{2} + f'''(0)\frac{\eta^3}{6}$$
$$-\int_0^{\eta}\left(\frac{\eta^3}{3!}+\frac{\eta^2\xi}{2!}+\frac{\eta\xi^2}{2!}+\frac{\xi^3}{3!}\right)\left[\alpha\left(\eta\left(f_{\eta\eta\eta}\right)_n+3\left(f_{\eta\eta}\right)_n\right)+Re\left(\left(ff_{\eta\eta\eta}\right)_n-\left(f_{\eta}f_{\eta\eta}\right)_n\right)\right]d\xi \tag{22}$$

From the boundary conditions in Eq. (15)
$f(0)=0, \quad f''(0)=0$

Using the above statement and inserting the boundary conditions of Eq. (15) into Eq. (22), we have

$$f_{n+1}(\eta) = k_1\eta + \frac{k_2\eta^3}{6}$$
$$-\int_0^{\eta}\left(\frac{\eta^3}{3!}+\frac{\eta^2\xi}{2!}+\frac{\eta\xi^2}{2!}+\frac{\xi^3}{3!}\right)\left[\alpha\left(\eta\left(f_{\eta\eta\eta}\right)_n+3\left(f_{\eta\eta}\right)_n\right)+Re\left(\left(ff_{\eta\eta\eta}\right)_n-\left(f_{\eta}f_{\eta\eta}\right)_n\right)\right]d\xi \tag{23}$$

For the sake of conformity to variation parameter analysis, Eq. (28) will be written in form of

$$f_{n+1}(\eta) = k_1\eta + \frac{k_2\eta^3}{6} - \int_0^{\eta}\left(\frac{\eta^3}{3!}+\frac{\eta^2\xi}{2!}+\frac{\eta\xi^2}{2!}+\frac{\xi^3}{3!}\right)\left[\alpha\left(\eta f_n'''+3f_n''\right)+Re\left(ff_n'''-f_n'f_n''\right)\right]d\xi \tag{24}$$

From the iterative scheme, it can easily be shown that the series solution is given as

$$f_0(\eta) = k_1\eta + \frac{k_2\eta^3}{6}$$
$$f_1(\eta) = k_1\eta + \frac{k_2\eta^3}{6} - \frac{\alpha k_2\eta^5}{30} + \frac{Re k_2^2\eta^7}{2520} \tag{25}$$

$$f_2(\eta) = k_1\eta + \frac{k_2\eta^3}{6} - \frac{\alpha k_2\eta^5}{30} + \frac{Re k_2^2\eta^7}{2520} + \frac{\alpha^2 k_2\eta^7}{210} + \frac{\alpha k_2 k_1\eta^7}{630} - \frac{\alpha Re k_2^2\eta^9}{113480}$$
$$-\frac{\mathrm{Re}\, k_2^2 k_1\eta^9}{45360} - \frac{Re^2 k_2^3\eta^{11}}{2494800} + \frac{\alpha^2 Re k_2^2\eta^{11}}{178200} - \frac{\alpha Re^2 k_2^3\eta^{13}}{16216200} + \frac{Re^3 k_2^4\eta^{15}}{247665600}$$

Similarly, the other iterations $f_3(\eta), \quad f_4(\eta), \quad f_5(\eta), \quad f_6(\eta), \quad f_7(\eta), \quad \ldots f_{15}(\eta)$ are obtained

$$f(\eta) = k_1\eta + \frac{k_2\eta^3}{6} - \frac{\alpha k_2\eta^5}{30} + \frac{Re k_2^2\eta^7}{2520} + \frac{\alpha^2 k_2\eta^7}{210} + \frac{\alpha k_2 k_1\eta^7}{630} - \frac{\alpha Re k_2^2\eta^9}{113480}$$
$$-\frac{\mathrm{Re}\, k_2^2 k_1\eta^9}{45360} - \frac{Re^2 k_2^3\eta^{11}}{2494800} + \frac{\alpha^2 Re k_2^2\eta^{11}}{178200} - \frac{\alpha Re^2 k_2^3\eta^{13}}{16216200} + \frac{Re^3 k_2^4\eta^{15}}{247665600} + \ldots \tag{26}$$

Where the constants k_1 and k_2 are determined using the boundary conditions in Eq. (15) i.e. $f(1)=1, \quad f'(1)=0$

Also, the first-order derivatives of *f(1)* is developed as

$$f'(\eta) = k_1 + \frac{k_2\eta^2}{2} - \frac{\alpha k_2\eta^4}{6} + \frac{Re k_2^2\eta^6}{360} + \frac{\alpha^2 k_2\eta^6}{30} + \frac{\alpha k_2 k_1\eta^6}{90} - \frac{9\alpha Re k_2^2\eta^8}{113480}$$
$$-\frac{\mathrm{Re}\, k_2^2 k_1\eta^8}{5040} - \frac{Re^2 k_2^3\eta^{10}}{226800} + \frac{\alpha^2 Re k_2^2\eta^{10}}{16200} - \frac{\alpha Re^2 k_2^3\eta^{12}}{1247400} + \frac{Re^3 k_2^4\eta^{14}}{16511040} + \ldots \tag{27}$$

4. Results and Discussion

In order to support and verify the approximate analytical solution using VPM, a numerical solution is also obtained using fourth-order Runge-Kutta method coupled with shooing techniques. The comparisons of the results through the two different methods are shown in Table 1 and Table 2. From the analysis, the results obtained by VPM are in excellent agreements with the results obtained by the numerical method. This shows that the results of VPM are completely reliable and physically realistic as the efficiency and accuracy of VPM is demonstrated.

Also, the developed solutions are used to study the effects of the flow parameters in the expanding or contracting porous channel with injection and suction.

Figs. 1 show the effects of the permeation Reynolds number and non-dimensional wall dilation rate on the dimensionless flow velocities. Fig. 1a-d, 2a-2d, 3a and 3b show the effects of Reynolds number, Re, on the velocity at constant non-dimensional wall dilation rate on the dimensionless axial velocity. Increase in the Reynolds number decreases the axial velocity at the center of the channel during the expansion while the axial velocity increases slightly near the surface of the channel when the wall contracts at the same rate. The behavior of axial velocity for different permeation Reynolds number, over a range of non-dimensional wall dilation rate were plotted in Figs. 2, 2a-2d. The figures depict that, for every level of injection or suction, the velocity is maximum at the center of the channel and near the point, the velocity is increased when the channel is expanding and decrease when the channel contracts. That is because the flow toward the center becomes greater to make up for the space caused by the expansion of the wall and as a result the axial velocity also becomes greater near the center. As the wall expansion ratio increases, the velocity at the center decreases and increases near the wall. Similarly, for the case of contracting wall as shown in Fig. 2a-d and 3a, increasing contraction ratio leads to lower axial velocity near the center and the higher near the wall because the flow toward the wall becomes greater and as a result the axial velocity near the wall becomes greater. So, both the expansion and suction through the wall reinforce the flow through the channel and similarly does the wall contraction and injection through the surface. The results of the present study show that for every level of injection or suction, in the case of expanding wall, increasing expansion ratio leads to higher axial velocity near the center and the lower axial velocity near the wall.

Table 1: Comparison of results of flow for large Reynolds number and suction

$f(\eta)$	**Re = 5, α=0.5**	
η	NM	VPM
0.0	0.000000	0.000000
0.1	0.152874	0.152874
0.2	0.301551	0.301551
0.3	0.442606	0.442606
0.4	0.573196	0.573196
0.5	0.690876	0.690876
0.6	0.793373	0.793373
0.7	0.878373	0.878373
0.8	0.943297	0.943297
0.9	0.985090	0.985090
1.0	1.000000	1.000000

Table 2: Comparison of results of flow for large Reynolds number, injection and suction

$f'(\eta)$	**Re = 5, α=0.5**	
η	NM	VPM
0.0	1.536154	1.536154
0.1	1.151411	1.151411
0.2	1.453855	1.453855
0.3	1.362554	1.362554
0.4	1.245207	1.245207
0.5	1.104631	1.104631
0.6	0.941489	0.941489
0.7	0.754210	0.754210
0.8	0.539188	0.539188
0.9	0.290458	0.290458
1.0	0.000000	0.000000

5. Conclusion

In this work, variation parameter method has been applied to analyze two-dimensional unsteady flow of viscous fluid in a porous channel through expanding/contracting walls with large injection or suction. From the results, it was established that increase in the Reynolds number decreases the axial velocity at the center of the channel during the expansion while the axial velocity increases slightly near the surface of the channel when the wall contracts at the same rate. Also, as the wall expansion ratio increases, the velocity at the center decreases and increases near the wall. For every level of injection or suction, in the case of expanding wall, increasing expansion ratio leads to higher axial velocity near the center and the lower axial velocity near the wall. The approximate analytical solution was verified by numerical solution using shooting method coupled with Runge-Kutta method. The results of the variation parameter method are in excellent agreement with the results obtained using numerical method.

Nomenclature

$\dot{a}$ time-dependent rate
B_0 electromagnetic induction
Ha Hartmann number
$\bar{p}$ Pressure
Re permeation Reynolds number
t time
$\bar{u}$ velocity component in x-direction
$\bar{v}$ velocity component in y-direction
V_w fluid inflow velocity at the wall
$\bar{x}$ coordinate axis parallel to the channel walls
$\bar{y}$ coordinate axis perpendicular to the channel walls
ρ_{nf} density of the nanofluid
ρ_f density of the fluid
μ_{nf} dynamic viscosity of the nanofluid
ρ_s density of the nanoparticles
ϕ fraction of nanoparticles in the nanofluid
σ electrical conductivity
α dimensionaless wall dilation rate

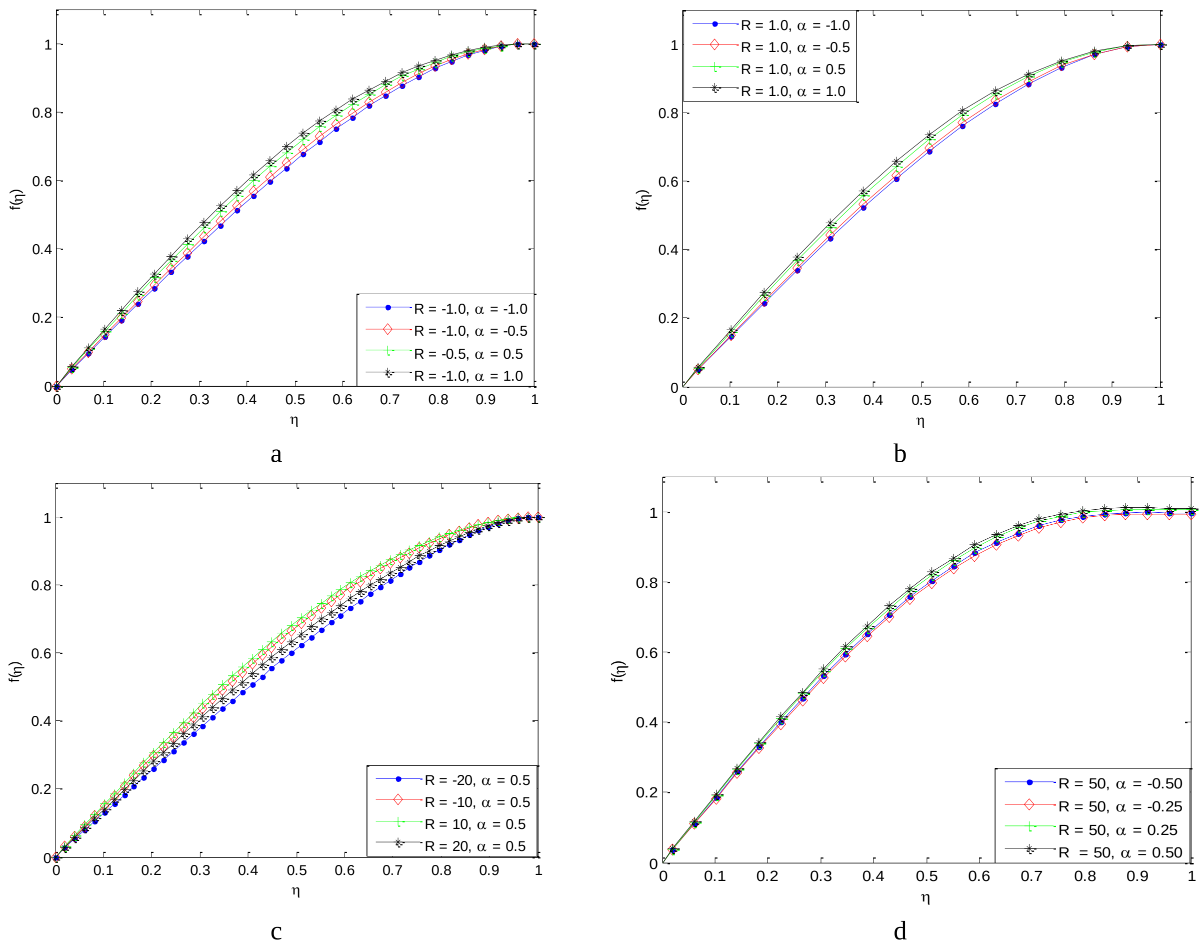


Fig. 1 Variation of f (η) for different expansion and contraction ratio, α and different small values of Re

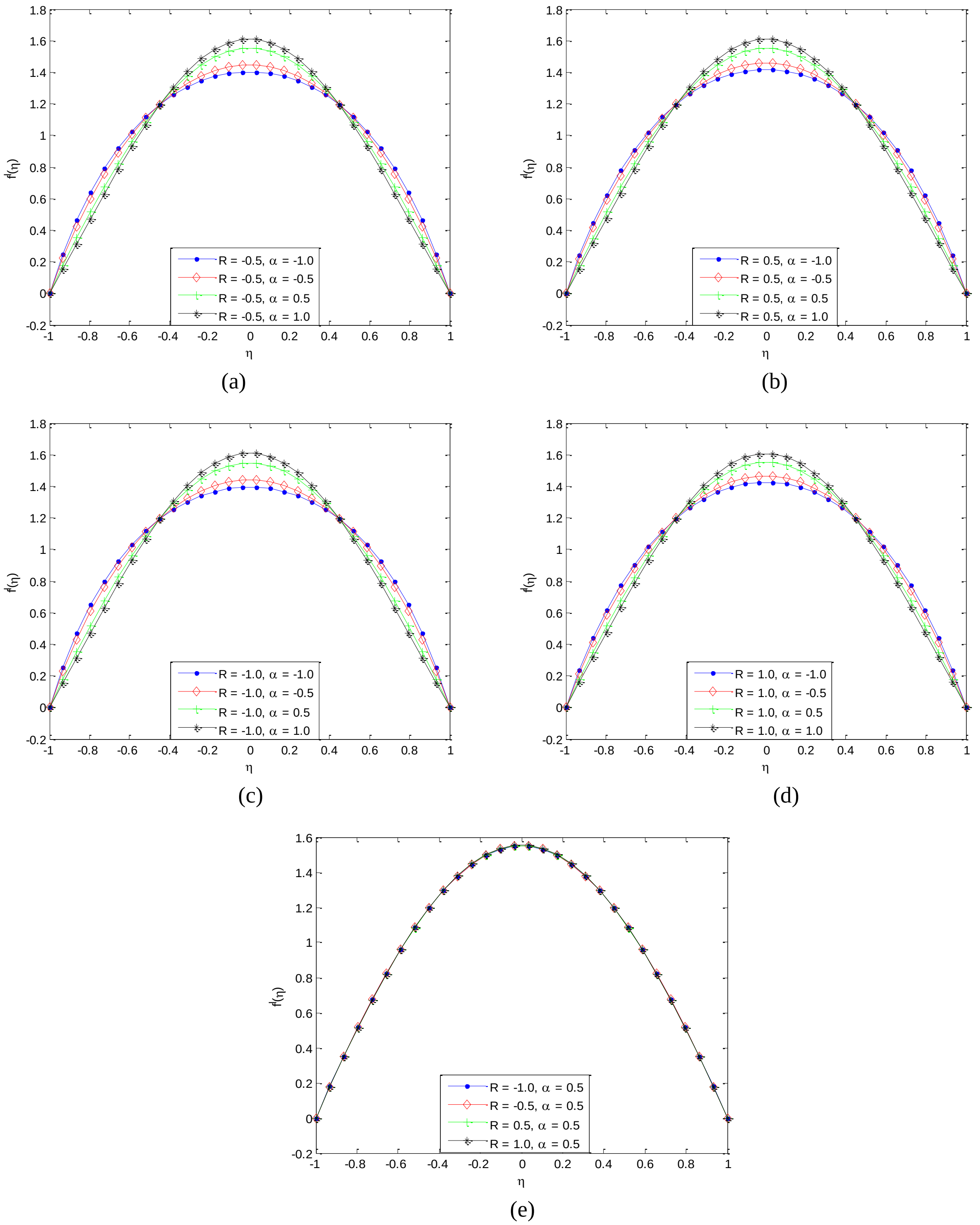
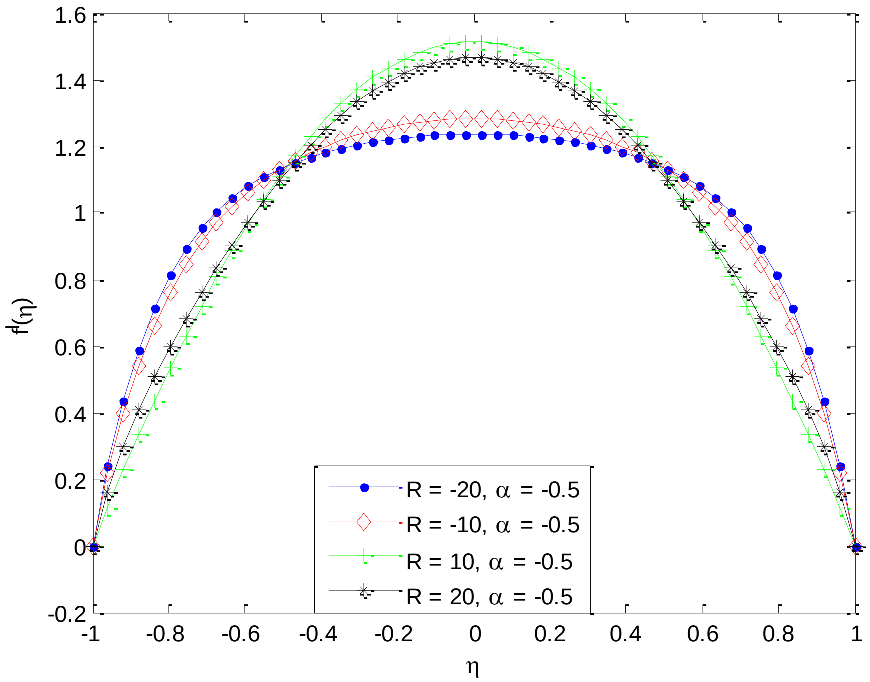
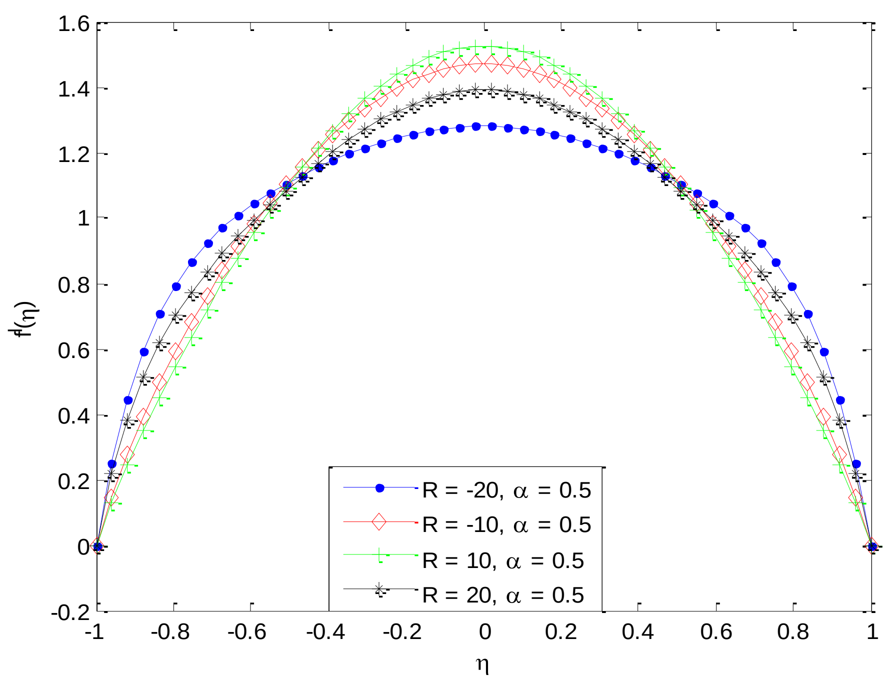


Fig. 2 Variation of f' (η) for different expansion and contraction ratio, α and different small values of Re



(a)



(b)

Fig. 3 Variation of f' (η) for different expansion and contraction ratio, α and different values of Re

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