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Evaluating the Effectiveness of Integrated Benders Decomposition Algorithm and Epsilon Constraint Method for Multi-Objective Facility Location Problem under Demand Uncertainty

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Abstract

One of the most challenging issues in multi-objective problems is finding Pareto optimal points. This paper describes an algorithm based on Benders Decomposition Algorithm (BDA) which tries to find Pareto solutions. For this aim, a multi-objective facility location allocation model is proposed. In this case, an integrated BDA and epsilon constraint method are proposed and it is shown that how Pareto points in multi-objective facility location model can be found. Results are compared with the classic form of BDA and the weighted sum method for demand uncertainty and deterministic demands. To do this, Monte Carlo method with uniform function is used, then the stability of the proposed method towards demand uncertainty is shown. In order to evaluate the proposed algorithm, some performance metrics including the number of Pareto points, mean ideal points, and maximum spread are used, then the t-test analysis is done which points out that there is a significant difference between aforementioned algorithms.

Keywords

Multi-objective optimisation, Benders decomposition algorithm, Demand uncertainty.

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Introduction

Facility location problem is an important issue in many strategic programs and has a crucial role in many transportation and communication problems. Facility location problem has an impressive effect on efficiency and cost, and has an application in transhipment, switching, and sorting points as well as supply chain (Melo et al., 2009; Rahimi et al., 2016; Tang et al., 2016). The capacitated facility location problem extends this model by selecting from many possible facilities for locating utilities which minimises the cost including the sum of transportation costs and facilities opening cost. Based on the model of Daskin et al. (2005), one of the core forms of capacitated facility location is:

$$Min \sum_{i}\sum_{j} c_{ij} z_{ij} \quad (1)$$

subject to

$$\sum_{j} Z_{ij} = h_i \quad i \in I, \quad (2)$$

$$\sum_{i \in I} Z_{ij} \le b_j X_{ij} \quad j \in J, \quad (3)$$

$$Z_{ij} \ge 0 \quad i \in I; j \in J, \quad (4)$$

where I is the set of plants and J is the set of customers; Parameters include c_{ij} The fixed cost of transporting between plant i and customer j, h_i The capacity of plant i, b_j the demand of customer j; *and* the decision variables include z_{il} the amount of product transported between plant i and customer j; X_{ij} is equal to 1 if customer j is served by plant i, 0 otherwise.

The objective Function (1) minimises the total cost (fixed facility cost plus shipment). Equation (2) limits total products that should be transported between facilities and customers for every customer. Constraint (3) limits total products that should be transported to each facility. Equation (4) is a simple non-negative equation.

However, several issues in facility location should be considered such as cost, environment, social factors, efficiency, and so on.

Performance measurement of facility location has been one of the important keys of decision-making policies recently. Regarding this, data envelopment analysis has been studied by some scholars (Klimberg & Ratick, 2008; Moheb-Alizadeh et al., 2011). Data Envelopment Analysis (DEA) is one of the mathematical programmings having been used widely for measuring the performance of decision-making units (DMUs) which uses the same inputs and outputs. The DEA model was first introduced by Charnes et al. (1978). One of the advantages of DEA is no need to convert input or output measures to some common metrics, meaning that they remain in their natural form. The fraction of the sum of weighted output to the sum of weighted inputs is defined as the DEA score (Mirghafoori et al., 2014; Danesh Asgari & Haeri, 2017; Torabi & Mahlooji, 2017).

The most important issue in multi-objective optimisation is first how to find Pareto optimal. Dabia et al. (2013) showed that a multi-objective optimization problem is a non-deterministic polynomial NP-hard problem. In Wang, Lai, and Shi's study (2011), Pareto optimal solutions are defined as the non-dominated solution for Multi-Objective Optimization (MOO) problem. In this issue, several approaches including metaheuristic and exact algorithms have been developed to deal with multi-objective optimization problems. Metaheuristic algorithms are approximate and usually non-deterministic; Also, these methods are not problem-specific. Metaheuristics are based on a systematic progression of random evolution, and it has no mathematical proof, and its convergence cannot be proved (Pishvaee et al., 2014). It has been proven that some metaheuristics need some modifications to guarantee finding a local optimum. Other main disadvantages of metaheuristics, when used for optimisation, are mainly related to the apparent difficulties to control diversity (Coello et al., 2007). Moreover, the role of the parameters of the metaheuristic algorithms in its convergence and its loss of diversity has been scarcely studied. It is valuable to notice that there is no guarantee of the convergence of approximate methods. In contrast, some mathematical programming by exhaustive search looks into all available space and tries to find the optimal solution, so it is guaranteed to find the optimal solution. These approaches by exhaustive search in all available search space, try to find the best guaranteed optimal solutions among other solutions. One of these approaches for solving these multi-objectives optimisation and generating Pareto optimal is aggregating objectives using numerical scalar weights. Using each weight results in the corresponding Pareto solution and a varying number of scalar weights could produce the corresponding set of Pareto solution. Weakness of the most current methods is the lack of the well-distributed set of Pareto solutions (Wang et al., 2011; Ismail-Yahaya & Messac, 2002).

Messac et al. (2003) applied normalised normal constraint method for solving the multi-objective NP-hard problem and generating Pareto frontier. Wang et al. (2011) applied normalised normal constraint method used in Das and Dennis's study (1998) for solving their multi-objective NP-hard problem. They have used posterior preference articulation approach and could produce special Pareto solutions for DMs (decision makers). They have used their methods in the four-echelon supply chain with six nodes including suppliers, plants, warehouses and customers. However, they did not consider uncertainty, different transportation modes in their multi-objectives NP-hard problems.

Bender decomposition algorithm (BDA) is one of the techniques in mathematical programming with guaranteed convergence. There are two stages in Benders decomposition algorithm. The variables of the original problem are split into two subsets so that a first-stage master problem (MP) is solved with the first set of variables, and then the solution of the first stage is used for the second stage so that the second set of variables in the second stage is determined by a given first-stage solution. If the sub-problem shows that master problem is infeasible or is not optimal yet, then feasibility and optimality Benders cuts are generated and added to the master problem, the algorithm is repeated until no cuts can be generated (Benders, 1962).

Also, Benders decomposition provides a rich framework for designing metaheuristics (Boschetti & Maniezzo, 2009). Boschetti and

Maniezzo (2009) solved capacitated facility location problem as an NP-hard problem with Benders decomposition metaheuristics. There are some advantages of using BDA, for example, it is guaranteed that this algorithm always converges and achieves an optimal solution; moreover, it is based on algebra concepts (Pishvaee et al., 2014). In this regard, Pishvaee et al. (2014) designed a multi-objective possibility programming model for a real medical needle and syringe supply chain network design problem under epistemic uncertainty of input data. The resulted model is strongly NP-hard; a BDA is devised to solve the model efficiently.

Abdolmohammadi and Kazemi (2013) developed BDA for economic dispatch problem, also their algorithm provides a useful framework for the non-convex region. Üster & Agrahari (2011) have used strengthened cut for the convergence of lower and upper bounds for their network design problem related to strategic decisions such as location and capacity decisions. Osman and Demirli (2010) proposed bilinear goal programming and BDA for their strategic problem. The most contribution of their work is to reduce time in reaching an optimal solution. In Yang and Lee’s (2012) work, reducing computational time and the number of iteration are two main issues which have been focused. They have proposed their approach to a multiproduct batch plant scheduling problem. de Sá et al. (2013) addressed BDA for single allocation hub location problem.

Recently, Abdolmohammadi and Kazemi (2013), and Charwand et al. (2014) used Normal Boundary Intersection (NBI) and BDA for their multi-objective framework; however, one drawback of the combined NBI algorithm is that Pareto optimality solution is not guaranteed (Das & Dennis, 1998; Ghane-Kanafi & Khorram, 2015).

Moreover, an uncertain parameter in supply chain network design models is another important characteristic. The uncertainties can be classified into two groups. They are random or stochastic, and non-random or strategic uncertainties.

Aghezzaf (2005), Chan et al. (2001), Longinidis and Georgiadis (2011), and Baghalian et al. (2013) are some authors who have worked on uncertain demands. In this paper, demand uncertainty with

Monte Carlo method would be discussed. Monte Carlo methods are based on computing algorithm using repeated random number to compute results (Rahimi et al., 2016). Table 1 reviews some works which have applied BDA.

Table 1. A review of BDA

Author	Description
Abdolmohammadi & Kazemi (2013)	Heat and power economic dispatch
Üster & Agrahari (2011)	Distribution network design
Montemanni (2006)	Robust spanning tree problem
Kagan & Adams (1993)	Multi-objective distribution planning problem
Osman & Demirli (2010)	Integrating goal programming and BDA for supplier selection
Oliveira et al. (2014)	Petroleum product supply chain
de Sá et al. (2013)	Hub location problem
Fortz & Poss (2009)	Multi-layer network design
Çakır (2009)	Multi commodity multi-mode distribution planning
de Camargo et al. (2008)	Multi allocation hub location problem
Esmaili et al. (2013)	Hybrid power market
Chu & You (2013)	Scheduling and dynamic optimisation
Al-Agtash & Yamin (2004)	Electricity market
Charwand et al. (2015)	Multi-objective electricity market

In the next section, the model is explained, then the classic form of BDA, as well as the proposed method are applied to the model to find the optimal Pareto solution. This model is an extension of Klimberg and Ratick’s (2008); they have considered capacitated facility location combined with data envelopment analysis as a bi-objective and indeterministic model, then a new framework solution procedure has been proposed for solving the proposed multi-objective facility location problem under deterministic and uncertainty cases. For demand uncertainty, Monte Carlo method with uniform function has been proposed. Then, results of the proposed algorithm are compared with the results found by the classic form of BDA, and the weighted sum method and some statistical tests are examined to evaluate the proposed solution approach.

Methodology

The problem above based on Klimberg and Ratick’s (2008) model is as below:

$$M\text{ax} \quad \sum_{k=1}^{k}\sum_{l=1}^{l} 1 - d_{kl} \tag{5}$$

$$Min \sum_{k=1}^{k}\sum_{l=1}^{l} c_{kl} b_{kl} + \sum_{k=1} F_k y_k \tag{6}$$

$st.$

$$\sum_{k=1}^{k} x_{kl} = 1 \ \ \forall l \tag{7}$$

$$x_{kl} \le y_k \ \ \forall k,l \tag{8}$$

$$\sum_{k=1}^{k} b_{kl} = dem_l \quad \forall l, \tag{9}$$

$$b_{kl} \le \min[dem_l, O_{mkl}] y_k \quad \forall k,l, \tag{10}$$

$$\sum_{i=1}^{I} v_{ikl} I_{ikl} = x_{kl} \ \ \forall k,l \tag{11}$$

$$\sum_{j=1}^{J} u_{jkl} o_{jkl} + d_{kl} = x_{kl} \ \ \forall k,l \tag{12}$$

$$\sum_{j=1}^{J} u_{jkl} o_{jrs} - \sum_{i=1}^{I} v_{ikl} I_{irs} \le 0, \forall k,l,r,s (k \ne r, l \ne s) \tag{13}$$

$$\mathrm{u}_{klj}\, o_{klj} \le x_{kl} \ , \forall k,l,j \tag{14}$$

$$b_{kl} \ge x_{kl} \quad \forall k,l, \tag{15}$$

$$x_{k,l}, y_k = 0,1 \tag{16}$$

$$u_{k,l,j}, v_{k,l,j}, b_{kl} \ge 0 \tag{17}$$

where k is the index of facility locations, l is the index of demand locations, t is the set of inputs, f is the set of output.

i= 1. . . *I:* Inputs used at DMU

j= 1. . . *J:* Outputs produced at DMU

k= 1. . . *r*, . . ., *K*DMUs

dem(l) is the demands of the customer; Cap _k is the capacity of facility *k;* c_{kl} is the cost of transportation from facility *k* to demand point *l;* x_{kl} is equal to 1 if facility *k* serves demand *l*, 0 otherwise; y_k is equal to 1 if facility *k* is open; 0 otherwise; F_k is the fixed cost of facility k; d_{kl} is the inefficiency of facility (DMU) k that serves demand point l; b_{kl} are products which are transported between facility k and demand point l; O_{jkl} is the amount of the jth output for the kth facility that serves demand point l; I_{ikl} is the amount of the ith input for the kth facility which serves demand point l; uj is the weight assigned to the jth output; vi is the weight assigned to the ith input.

Equation (5) maximises the total efficiency of all decision-making units (DMUs), Equation (6) minimises the total cost including transportation and facility location of DMUs. Constraint (7) makes sure every demand must be satisfied by at least one facility. Constraint (8) guarantees if one demand is served by one facility, that facility must be open before. Equation (9) shows the demand of every customer is equal to the total products that are transported from facilities. Constraint (10) depicts maximum products which their transport is equal to demand or capacity of the facility. Equation (11) shows if facility k is open to serve demand l, then the sum of weighted input of facility must be equal to 1; otherwise, 0. The variable d_{kl} is defined as the inefficiency of DMU k. Therefore, constraint (12) depicts efficiency of facility k that is open. Constraint (13) shows that the sum of weighted output is less than the sum of corresponding weighted inputs. Constraint (14) indicates that weight of output of facility k which is open is less than 1. Equation (15) depicts that at least one product is transported for demand l if facility k serves it. Constraint (16) and (17) show that variables $x_{k,l}$, y_k are binary, and variables, $u_{k,l,j}$, $v_{k,l,j}$, b_{kl} are continuous. The problem mentioned above is the capacitated formulation of combined DEA and facility location model which simultaneously solves spatial efficiency and facility efficiency. The first issue is measured by optimising cost while the latter one is measured by DEA linear programming. The formulation of this combination in a multi-objective framework has been able to provide the decision makers with a way for measuring the interaction between facility location pattern and those open sites' performance. These formulations obtain a trade-off between facility location and facility efficiency, and may provide a strong approach for multi-objective location analysis.

The specific problem which is formulated in Equations (5) to (17) is known as mixed integer programming optimization problem and is defined as a combination of facility location problem and data envelopment analysis. In the literature, several articles mentioned that facility location problem is a complex issue and exact solution of this kind of problem is NP-hard, and also an approximation of this problem with a small error is known NP-hard (Fowler et al., 1981; Megiddo & Tamir, 1982; Gonzalez, 1985).

The Proposed BDA

First of all, two objectives must be converted to a single one, in this regard, epsilon constraint would be used. In this approach, the number of objective functions is reduced, and the number of constraints is increased. In minimization problem, an upper bound (epsilon value) is considered for one of the objective functions as a new constraint added to the model, then the problem is solved for the single objective function (Laumanns et al., 2006; Aghaei et al., 2011; Khalili-Damghani & Amiri, 2012).

First, for this purpose, initializing the value of epsilon, that can change for constrained objective to find Pareto front, is used. Then, the above problem is split into two parts; the master problem and sub-problem that can be solved iteratively by using their solution (Benders, 1962). The sub-problem includes continuous variables and their related constraints, while the master problem includes integer variables and a continuous one which relates the master and the sub-problem together. The optimal solution for the master problem provides lower bound for the objective function. By using this solution that has been gained from the master problem and fixed integer variables as inputs for the dual of the sub-problem, it is solved resulting in upper bound for the objective function. Also, by solving dual of primal sub-problem, one Benders cut is produced including continuous variables that are added to the master problem. In the next iteration, this cut is added to the master problem and then updates the lower bound for the objective function that it is guaranteed which current solution is not worse than previous lower bound. So, the master and the sub-problem are solved iteratively until the end. The requirement for finishing this loop is to reduce the distance between lower and upper bound. After finding one optimal point, value of epsilon should be changed to obtain a new optimal point. BDA is dependent to decompose one mixed integer programming into a master problem and one sub-problem that iteratively would solve using the other solution. The master problem includes only integer variables and one auxiliary variable. The primal master problem and the sub-problem are introduced as follow:

Primal master problem

$$Min \sum_{k=1}^{K} F_k . y_k \tag{18}$$

subject to

Equations (7), (8). (19)

where Equation (18) is the binary section of Equation (6).

Benders sub-problem

Benders sub-problem (*BSP* (*u*, *v*|*x*, *y*)) is a maximisation problem that finds the optimal value of continuous variable *x*, *y* for fixed integer variables $\bar{x}_{kl,}\overline{y_k}$. The primal sub-problem is written as below:

$$Min \ \sum_{k=1}^{k} \sum_{l=1}^{l} c_{kl} b_{kl} \tag{20}$$

subject to

$$\sum_{k=1}^{k} \sum_{l=1}^{l} 1 - d_{kl} \geq \varepsilon \tag{21}$$

$$\sum_{i=1}^{I} v_{ikl} I_{ikl} = \bar{x}_{kl} \quad \forall k, l \tag{22}$$

$$\sum_{j=1}^{J} u_{jkl} o_{jkl} + d_{kl} = \bar{x}_{kl} \quad \forall k, l \tag{23}$$

$$\sum_{j=1}^{J} u_{jkl} o_{jrs} - \sum_{i=1}^{I} v_{ikl} I_{irs} \leq 0, \forall k, l, r, s (k \neq r, l \neq s) \tag{24}$$

$$\sum_{k=1}^{k} b_{kl} = dem_l \quad \forall l, \tag{25}$$

$$b_{kl} \le \min[dem_l, O_{mkl}]\overline{y}_k \quad \forall k,l, \tag{26}$$

$$\mathrm{u}_{klj}\, o_{klj} \le \overline{x}_{kl} \ , \forall k,l,j \tag{27}$$

$$b_{kl} \ge \overline{x}_{kl} \quad \forall k,l, \tag{28}$$

Notice that here integer variables are fixed that already are found in the previous step by solving the master problem.

For producing Benders cut for the master problem, dual of $BSP(u,v|x,y)$ would be used ($\pi_{kl}^1, \pi_{kl}^2, \pi_{kl}^3, \pi_{klj}^4, \pi_{kl}^5, \pi_{kl}^6, \pi_{klj}^7, \pi_{kl}^8$). For using dual variables $\pi_{kl}^1, \pi_{kl}^2, \pi_{kl}^3, \pi_{klj}^4, \pi_{kl}^5, \pi_{kl}^6, \pi_{klj}^7, \pi_{kl}^8$ would be employed for every Constraint (21)-(28). It is assumed that Equations (22), (23), and (25) are considered as an inequality. By using these variables, dual problem, $DBSP(\pi_{kl}^1, \pi_{kl}^2, \pi_{kl}^3, \pi_{klj}^4, \pi_{kl}^5, \pi_{kl}^6, \pi_{klj}^7, \pi_{kl}^8 | \overline{x}, \overline{y})$ is as follow:

$$Min \ \ \pi_{kl}^1.\overline{x_{kl}} + \pi_{kl}^2.\overline{x_{kl}} - \pi_{kl}^4.\overline{\mathrm{x}_{kl}} + \pi_{kl}^5.\mathrm{dem}_l - \pi_{kl}^6.\min[\mathrm{dem}_l].\overline{y_k} + \pi_{kl}^7.\overline{\mathrm{x}_{kl}} + \pi_{kl}^8.\varepsilon \tag{29}$$

$$s.t \ \ \pi_{kl}^2 \ge 1 \quad \forall k,l \tag{30}$$

$$\pi_{kl}^1, \pi_{kl}^2 \ \ free \tag{31}$$

$$\pi_{kl}^4, \pi_{kl}^5, \pi_{kl}^6, \pi_{kl}^7, \pi_{kl}^8 \ge 0 \tag{32}$$

Benders' master problem

Benders' master problem is as follows:

$$\max \ \ z \tag{33}$$

subject to

$$z \ge \sum_{k=1}^{k}\sum_{l=1}^{l} 1 - d_{kl} + \overline{\pi_{kl}^1}.x_{kl} + \overline{\pi_{kl}^2}.x_{kl} - \overline{\pi_{kl}^4}.\mathrm{x}_{kl} + \overline{\pi_{kl}^5}.\mathrm{dem}_l - \overline{\pi_{kl}^6}.\min[\mathrm{dem}_l].y_k + \overline{\pi_{kl}^7}.\mathrm{x}_{kl} + \pi_{kl}^8.\varepsilon \ \ \forall k,l \tag{34}$$

$$\overline{\pi_{kl}^1}.x_{kl} + \overline{\pi_{kl}^2}.x_{kl} - \overline{\pi_{kl}^4}.\mathrm{x}_{kl} + \overline{\pi_{kl}^5}.\mathrm{dem}_l - \overline{\pi_{kl}^6}.\min[\mathrm{dem}_l].y_k + \overline{\pi_{kl}^7}.\mathrm{x}_{kl} + \pi_{kl}^8.\varepsilon \le 0 \tag{35}$$

Equations (7) and (8).

In this model, Equation (33) presents the objective function of the master problem. Equation (34) is the optimality cuts added to the master problem. Also, in these equation dual parameters are from the sub-problem; the solutions used are fixed in this step. Equation (35) is the feasibility cut which in case the sub-problem is infeasible. It is added to the master problem. Figure 1 shows the procedure of the proposed method.

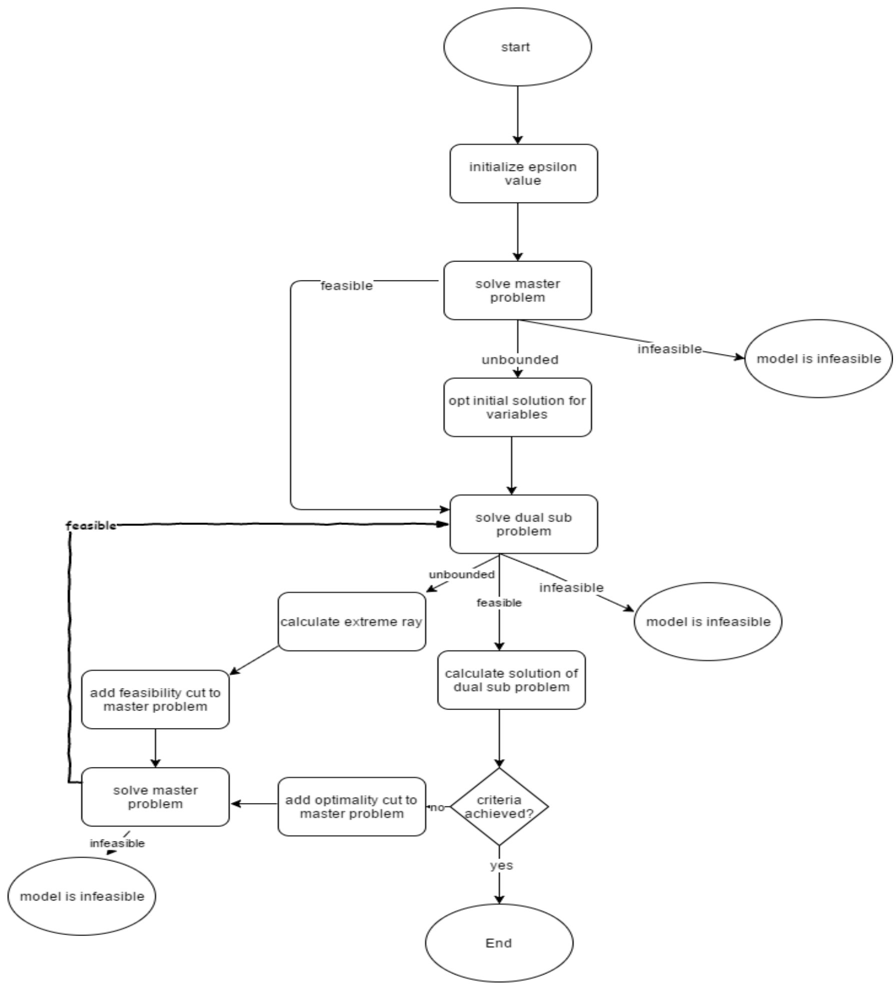

Fig. 1. Flowchart of the proposed method

where Equation (36) is as below:

$$\sum_{k=1}^{k}\sum_{l=1}^{l}1-d_{kl}+\overline{\pi_{kl}^{1}}.x_{kl}+\overline{\pi_{kl}^{2}}.x_{kl}-\overline{\pi_{kl}^{4}}.\mathrm{x}_{kl}+\overline{\pi_{kl}^{5}}.\mathrm{dem}_{l}-\overline{\pi_{kl}^{6}}.\min[\mathrm{dem}_{l}].y_{k}+\overline{\pi_{kl}^{7}}.\mathrm{x}_{kl}+\pi_{kl}^{8}.\varepsilon \quad \forall k,l \qquad (36)$$

Results

In this section, numerical analysis would be tested and compared with classic BDA. Both cases, that are deterministic and uncertainty under Monte Carlo method with the uniform function will be shown to verify the stability of the proposed method towards different conditions. GAMS software is especially designed for the above model. To illustrate the proposed method, numerical analysis from Klimberg and Ratick (2008) would be used. The proposed model and solution approach are applied to a simulation data based on the real case study. In this regard, the value of efficiency is based on DEA efficiency and cost is based on Malaysia's currency (Ringgit). There are four facilities which tend to be located in 8 potential location sites and 18 customer locations that their demand must be satisfied. It is tried to use standard data ranged in real, so we simulated real data from the website www.investpenang.gov, based on a starting business in Malaysia. However, in this approach, classical BDA is applied for finding Pareto points. This cost includes facility location and transportation cost. When DEA objective is optimised, another one (cost objective) increases significantly since the model tries to locate the minimum number of facilities and assign demands to the minimum available warehouses to maximise DEA efficiency score. The model is formulated for multi-products plants, for this matter, Table 2 shows demands of customers based on kilogram per unit. It has been considered 8 products which should be served by 18 customer zones.

Table 2. The demand of customers

product	Demand(kg)																	
	i1	**i2**	**i3**	**i4**	**i5**	**i6**	**i7**	**i8**	**i9**	**i10**	**i11**	**i12**	**i13**	**i14**	**i15**	**i16**	**i17**	**i18**
product1	37	61	96	45	43	45	43	55	65	34	54	33	54	23	44	65	66	76
product2	64	53	54	78	29	27	35	52	27	44	28	31	59	60	67	30	60	76
product3	24	73	21	33	26	27	53	59	47	68	55	43	61	49	22	72	21	68
product4	26	65	40	26	56	79	71	79	74	37	68	50	26	68	38	45	74	56
product5	69	26	68	60	68	41	20	66	48	63	66	61	66	21	56	42	37	44
product6	52	33	76	77	68	76	74	66	72	39	63	24	51	35	55	29	72	50
product7	53	36	54	41	62	68	23	33	47	50	52	27	33	53	37	56	51	75
product8	46	45	53	38	66	22	55	59	56	58	27	44	52	71	76	63	57	59

To facilitate sites in potential location, various plants with different size and capacity are considered. Table 3 depicts fixed cost of opening plants. The size of products, weights, distance, and type of vehicles are factors which are considered to measure transportation cost. For this issue, Table 4 implies transportation cost between plants and customer zones.

Table 3. The fixed cost of opening plants in different candidate locations (*100 RM)

No	j1	j2	j3	j4	j5	j6	j7	j8
p1	350	630	240	420	380	430	450	530
p2	520	420	360	450	430	510	740	510
p3	420	520	280	670	360	310	640	340
p4	360	480	650	720	780	680	630	520

Table 4. The transportation cost between facilities and customers (RM)

	i1	i2	i3	i4	i5	i6	i7	i8	i9	i10	i11	i12	i13	i14	i15	i16	i17	i18
j1	805	850	598	850	990	931	869	528	823	962	500	564	697	761	681	969	874	900
j2	792	924	612	623	958	986	554	595	549	861	539	927	662	936	802	934	955	896
j3	966	898	960	614	676	924	963	677	778	673	899	639	548	636	588	699	922	975
j4	765	709	793	694	795	570	611	506	558	869	923	714	500	745	655	959	717	814
j5	829	972	881	721	846	676	518	752	585	761	957	833	508	986	940	633	709	548
j6	850	896	809	605	833	654	519	834	769	632	867	565	763	782	728	628	512	565
j7	942	550	958	804	610	818	834	819	546	858	817	812	771	768	994	988	939	659
j8	734	746	611	615	604	971	910	526	535	575	575	554	628	696	932	937	881	1000

As it is evident from the comparison of Figure 2 with Figure 3, the proposed method is more able to capture more Pareto points, rather than classic Benders decomposition under uncertainty condition. For uncertainty, demands are assumed to be distributed by Monte Carlo method and uniform function. The classic form of BDA under uncertainty is fragile, but the integration of epsilon constraint and BDA catches more suitable Pareto solution. Classic Benders decomposition approach can find some Pareto points in deterministic case. However, there is a drawback in finding true Pareto optimality while the proposed method is more capable of finding more optimality points in better quality and discipline. In other words, the proposed algorithm benefits from epsilon constraint solution that could cover non-convex points while the classic form of BDA and the weighted sum method possesses the weakness of not being able to find Pareto points of the non-convex optimisation problem.

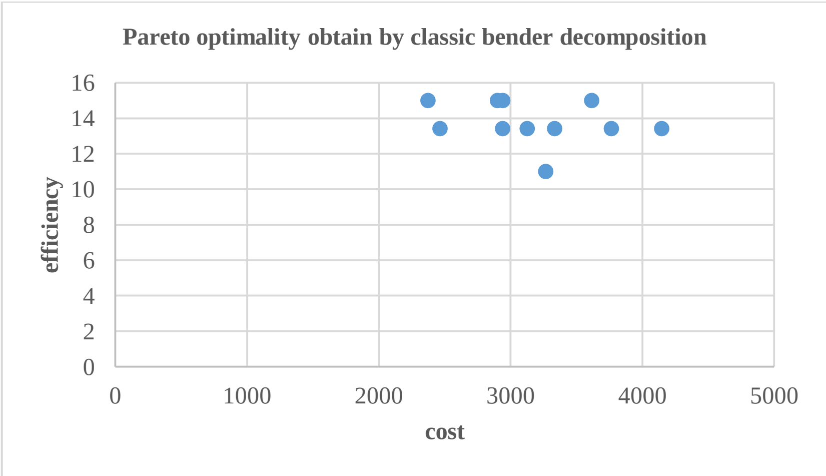

Fig. 2. Pareto front for uncertainty by classic BDA

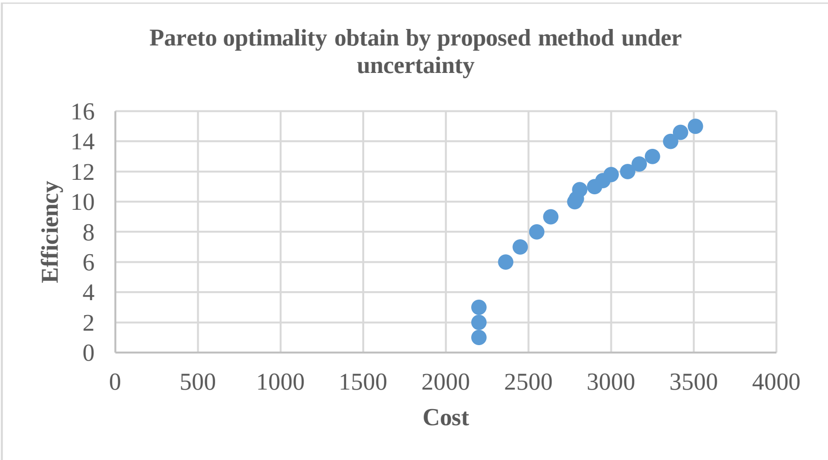

Fig. 3. Pareto front for uncertainty case by the proposed method

Quality is the most important characteristic that is found in applying the proposed BDA against the classic form of BDA under uncertainty (Figs. 2 and 3). For uncertainty Monte Carlo method, which relies on repeating, random sampling with uniform function is used. As it is apparent from Figure 2, there is no quality in Pareto optimality when classic BDA is used while the proposed method is stable against uncertainty (Figure 3).

In the next section, for further evaluating these algorithms, some performance metrics including some Pareto points, mean ideal points, and maximum spread are used which are described as below.

Number of Pareto Solution

To check the reflect of change and variation under different conditions, design of experiment (DOE) is proposed, thus, Taguchi method is suggested (Taguchi, 1986; Behmanesh & Rahimi, 2012). It consists of five parameters including the fixed cost, the transportation cost, the inputs, the outputs, and the demand as factors for DOE and three levels including low level (number 1), medium level (number 2), and high level (number 3) resulting in 27 experiments. Table 5 shows the parameter set for the design of experiments that should be done. The numbers allocated in Table 5 indicate the level of each parameter.

Table 5. Design of experiments

Factor A	Factor B	Factor C	Factor D	Factor E
1	1	1	1	1
1	1	1	1	2
1	1	1	1	3
1	2	2	2	1
1	2	2	2	2
1	2	2	2	3
1	3	3	3	1
1	3	3	3	2
1	3	3	3	3
2	1	2	3	1
2	1	2	3	2
2	1	2	3	3
2	2	3	1	1
2	2	3	1	2
2	2	3	1	3
2	3	1	2	1
2	3	1	2	2
2	3	1	2	3
3	1	3	2	1
3	1	3	2	2
3	1	3	2	3
3	2	1	3	1
3	2	1	3	2
3	2	1	3	3
3	3	2	1	1
3	3	2	1	2
3	3	2	1	3

The first metric used to measure is the number of Pareto solution that is gained with two algorithms. Based on experiments which are designed and depicted in Table 5, 27 tests are run, and results have been presented in Table 6.

Table 6. The number of Pareto solution for the weighted sum method and the proposed algorithm

Experiment run	The weighted sum method	The proposed algorithm
1	3	13
2	3	14
3	3	14
4	3	15
5	4	13
6	4	14
7	4	14
8	5	14
9	5	13
10	5	14
11	4	15
12	4	14
13	5	13
14	3	14
15	4	14
16	5	13
17	4	14
18	4	15
19	5	14
20	3	14
21	4	14
22	4	14
23	3	14
24	4	14
25	4	14
26	4	14
27	4	14

From Table 6, it is observed that the proposed algorithm has more Pareto points than the weighted sum method which is more desirable.

Mean Ideal Point (MID)

Another factor for measuring performance is mean ideal point which means the closeness of Pareto point solutions and ideal points (Arjmand & Najafi, 2015). This metric is formulated as below.

$$MID = \frac{\sum_{i=1}^{n} \sqrt{f_{1j} + f_{2j}}}{n} \quad (37)$$

which n shows the number of Pareto solutions, and f_{1j}, f_{2j} implies the first and the second objective values of the ith objective function.

With regard to the above, MID measurement for these two algorithms is calculated and presented in Table 7.

Table 7. The mean ideal point for the sum weighted method and the proposed algorithm

Experiment run	The weighted sum method	The proposed algorithm
1	50.285	16.26
2	50.86	16.28
3	50.48	15.26
4	48.14	15.32
5	47.45	15.66
6	46.65	15.85
7	48.85	15.65
8	49.95	14.98
9	50.15	15.62
10	50.23	14.35
11	49.32	15.62
12	48.65	14.95
13	50.62	15.84
14	48.61	14.63
15	48.23	14.29
16	50.31	14.36
17	49.32	14.63
18	48.96	15.13
19	50.31	14.91
20	48.65	14.35
21	49.42	14.62
22	48.54	15.61
23	48.61	15.48
24	48.56	15.87
25	49.62	14.51
26	49.63	14.62
27	48.16	14

Table 7 presents that the proposed algorithm possesses is less mean ideal points than the weighted sum method and these values are preferable for us.

Maximum Spread (MS)

The last metric used here is maximum spread (Zitzler, 1999), and it is calculated as below.

$$D = \sqrt{\sum_{m=1}^{M} (\max_{i=1:|Q|} f_m^i - \min_{i=1:|Q|} f_m^i)^2} \qquad (37)$$

Those M and Q are the number of objective functions and the number of Pareto solutions, respectively. A higher value of MS is more desirable; the desirable values are the bigger ones. Table 8 presents the details.

Table 8. The maximum spread for the sum weighted method and the proposed algorithm

Experiment run	The sum weighted method	The proposed algorithm
1	81	360
2	80	352
3	78	356
4	83	364
5	65	362
6	69	378
7	58	365
8	85	345
9	75	390
10	91	393
11	69	410
12	73	423
13	56	351
14	74	362
15	81	453
16	82	342
17	86	385
18	92	345
19	63	362
20	62	312
21	85	432
22	69	423
23	73	329
24	82	356
25	86	375
26	92	346
27	91	395

By using the t-test, Table 9 shows that there is a significant difference between the means of experiments which are used here.

In the next section (Tables 10 and 11), same experiments are designed to compare the effectiveness of the proposed algorithm and the classic form of Benders decomposition.

Table 9. The t-test results for the sum weighted method and the proposed algorithm

The performance metrics	The t-test		
	The p-value	Result	Final result
NPS	0	H0 is rejected	proposed algorithm
MID	0	H0 is rejected	proposed algorithm
MS	0	H0 is rejected	proposed algorithm

Table 10. The results of NPS, MID, and MS for the classic BDA

Run	NPS	MID	MS
1	4	40	65
2	5	39	62
3	5	42	36
4	5	45	59
5	4	44	75
6	4	39	74
7	4	45	76
8	5	38	72
9	3	37	71
10	3	46	70
11	4	51	65
12	3	36	68
13	4	44	67
14	5	44	76
15	5	38	73
16	5	37	71
17	4	42	72
18	4	38	70
19	4	37	64
20	4	44	65
21	5	33	63
22	5	41	62
23	4	43	60
24	4	49	69
25	3	39	73
26	3	38	74
27	3	52	75

Table 11. The t-test results for the comparison between the classic BDA and the proposed algorithm

The performance metrics	The t-test		
	The p-value	Result	Final result
NPS	0	H0 is rejected	Proposed algorithm
MID	0.003	H0 is rejected	Proposed algorithm
MS	0	H0 is rejected	Proposed algorithm

Regarding to Tables above and t-test results from the comparison between the proposed algorithm, the classic BDA, and the sum weighted method. It is observed that there is a significant difference between these algorithms.

Conclusion

In this paper, integrated BDA and epsilon constraint have been applied for multi-objective facility location model; the primary goal of this model is to find the optimal number of potential customers, the minimum cost and high efficiency to serve. In this regard, for the uncertainty of demand, Monte Carlo approach is applied. Firstly, by using epsilon constraint the proposed model converts into the single one, then BDA is used, in this case, the original model is divided into two segments; the master problem and the sub-problem. In this approach, the master problem includes facility location objective and strategic decision and the sub problem possesses DEA objective. This model is an extension of Klimberg and Ratick (2008). The capacitated facility is located in conjunction with the analysis of data coverage in a deterministic approach, while the proposed model is considered with the demand uncertainty. The studied approach is compared with the classic BDA under deterministic and uncertainty cases. Results show that the proposed approach is more stable in comparison with the classic form of BDA and the weighted sum method. The proposed algorithm has been applied for the particular problem which was introduced by Klimberg and Ratick (2008) and has been compared with the weighted sum method that Klimberg and Ratick (2008) utilized to solve their problem. The statistical analysis and results show that there exist significant improvements in the proposed algorithm against the weighted sum method and the classic BDA.

As a future work, it is recommended to apply the proposed solution approach for other multi-objective models. Also, it is interesting to use some improvements regarding BDA /epsilon approach to accelerate/ improve the proposed algorithm. Regarding uncertainty, this paper used Monte Carlo method, it is suggested to use other uncertainty cases.

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