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Expansion of Location Theories of Firms and Products' Consistency Using Triangular Distribution Approach

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Abstract

One of the main criticisms of location models is simplistic assumptions concerning the consumers' distribution on the street or city. The location models usually make use of uniform distribution of consumers while it is not true in reality, and mostly the consumers' accumulation is more in the city centers rather than suburb areas. This study deals with selection of optimal location of firms using Lijesen and Reggiani (2013) spoke model and changing the consumers' distribution from uniform to triangular in a two-step game. In this game, the firms select their location, at the first stage, and enter the price competition at the second stage. Results indicate that the increased number of streets and transportation costs leads to price increase and this indicates that the farther away are the firms from each other, their competition in the market would decrease and the price would increase. If both firms are located in the same distance from city center, they would gain the same market share and more inclined toward being closer to city center or having minimum distance. Moreover, when in the consistency issue include the triangular distribution, Nash equilibrium price is less compared with Rohlfs model (1971) and a greater range of consumers purchase the product.

Keywords: Location, Triangular Distribution, Consistency, Nash Equilibrium.

JEL Classification: D43, E61, C62.

1. Introduction

The location dimension is one of the most important effective factors on individuals and firms' decision making. Finding appropriate

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location, which its selection results in decreasing competition between firms, increasing product differentiation and minimizing distribution cost of goods and services to customers, plays a significant role in increasing market share and firms' profit. Since making industries' establishment policies without knowledge and awareness, leads to decrease or disappearance of the efficiency of economic system; thus, the significance of location study becomes clearer (Vinay and Chakra, 2005). Location could be defined in two ways: it might be used to refer to physical location of a special consumer or to refer to the difference between the desired specifications of a trade mark within a special consumer and the specifications of a trade mark that is purchased (Shy, 1995).

The selection of an optimum location for establishment of a firm or producing institution could play an essential role in its strategic decisions, and guarantee long-term profiting of the firm such that the life of firm or institution would be endangered in case of lack of these studies (Macoei et al., 2014). The decisions related to selection and acquisition of location specifications of a center could have great effect on the ability of profiting and preserving competitive advantage (Choo and Mazzrol, 2003). The investigation of early-return enterprises has specified that more than 50% of them would bankrupt in the first year and 30% after two years and choose another occupation. Since at the time of establishment of these occupations, all aspects of service providing are studied, inattention to the main issue of location would make the manufacturing unit do not reach the intended profit and lag behind goal achievement (Melaniphy, 1999). Competitive location is the most important branch of location problem that is used in game theory where the competitive enterprises seek to maximize their market share (Amiri et al., 2010). The main issue in this condition is optimum location of one or more new facility in the market where other competitors existed beforehand. By modeling this problem in form of a game and considering the location of existing facilities and decisions about the enterprises which are likely to enter the market in future, it is possible to obtain the optimum location through Nash equilibrium (Macoei et al., 2013). In classification of facilities location models, eight factors are effective including: geographical specifications, facilities' characteristics, objectives, solving method, demand patterns, kinds of supply chain, time horizon and input parameters (Huang et al., 2005). In this condition, the competitive enterprises try to attract the customers using the strategic decisions they choose. Thus, they are required to become familiar with costumers' behaviors (Abduli, 2011).

Various studies have investigated the selection of enterprises' location. Zamani (2009) studied the location of multi-story car parking in Qom using ANP. After data collection through questionnaire and calculation of final weight of criteria, the priority and consequently their significance became specified as follow: the criteria of "closeness to tourism attraction centers" and "accessibility" were at the first and second rank with a negligence difference, and "the specification of location" was at the third rank. Aminzadeh Goharrizi et al. (2012) studied methods of new cities location in three last decades and showed that there was relatively simplistic view in location of new cities such that the methods used in location of cities have not conformity with common scientific methods. Artle and Carruthers (1998) studied a bipolar competitive model of location in a linear market. They considered the entrance of customers simultaneously and sequentially with two players that were inclined toward demand by pricing policy and obtained Nash equilibrium. O'Kelly and Bryan (1998) showed that the location models of current center with assumption of independent costs not only wrongly forecast the total cost of network; they might mistakably select the location of optimum and allocation center. Hosun et al. (2003) showed that the best location for construction of factory was the place which was in better situation concerning the given facilities. Chen and Riordan (2007) considered the model of equal streets that is the generalization of circular city model. They considered N streets with equal length of 1/2 such that in all streets, the consumers are uniformly distributed with $\frac{2}{N}$ customer on each street. This model is normally the extension of Hotelling classic method (1929) to a set of streets with one center. They showed that the entrance of new enterprise changes the consumer surplus and social welfare through price, market development and their resultant and the free entrance of firms, the total production of market might be less or more compared to social optimum, and the equilibrium price is more than marginal cost even when the number of firms is higher. Ju (2008) showed the efficiency and consistency of location for several public facilities. He studied the simultaneous decisions of several players in the competitive line and environmental space for selection of the nearest facility with the aim of getting maximum profit and market share, and used Nash equilibrium concepts to solve them. Lin and Lee (2010) considered hub network in competitive and exclusive environment with imperfect information and operational cost. They used Cournot game and Nash equilibrium to solve their model. Granot et al. (2010) analyzed Hotelling model where the space of a city was along a line in competitive and exclusive environment. They showed the optimum location of firms seeking maximum profit using Nash equilibrium concepts.

Sáiz et al. (2011) considered a model where the quality decision variable was as a main variable of two-stage game in linear space and solved it through Nash equilibrium concepts. Shiode et al. (2012) considered optimum location policy in a linear market where the demand had uniform distribution and solved it through Nash pure strategy and Stackelberg equilibrium. Following Chen and Riordan, Reggiani and Lijesen (2013) proposed two questions and solved it: where is the location of enterprises and how effective are those sectors of market that have not been covered so far by the firm on the decision making of active firms in market? They analyzed the selection of location in spoke model endogenously and by changing transportation costs from linear form in Chen and Riordan model to quadratic form. They assumed that all streets were connected to city center, and for purchasing from a firm, if the firm was not in the consumer’s street or the consumer was located in a street without firm, the consumer should pass the city center. Țartavulea (2015) presented a model for determination of optimum residence location for improvement of performance in supply chain strategy (optimum location for a central warehouse) in Europe after 2007 crisis that led to stopping or optimum re-location for enterprises (this optimum location was located at the center of Europe, i.e. close to Germany and Austria border). The location issues have been considered from 1960s by the researchers, and classified to competitive and non-competitive categories, such that the share of non-competitive models was more than competitive models. Competitive location models were introduced in 1929 by Hotelling through game theory approach concerning the competition between two ice-cream vendors. Next studies are for improvement of one or several assumptions of Hotelling model and consideration of them in a more general level. The most important point in these generalizations is that the competitive location models are inherently unstable. In other words, a small change in an assumption or a parameter would yield completely different results (Macoei et al., 2014). Macoei et al. (2014) collected the works done on location from 1929 to 2013 that is a good source for removing the deficiencies or constrains in location area using game theory. One of the main deficiencies proposed by them was that the studies were inspired from methods with uniform distribution in modeling that could not be true in real world (these assumptions were solely for simplification). Thus, this study replaces the uniformity of consumers' distribution with better reality such as triangular distribution. One of these models presented by Chen and Riordan (2007) and extended by Lijesen and Reggiani (2013) was Spoke model that was the expansion of circular city one where N streets were considered with equal length of ½. All these streets are connected to city center and for purchasing from a firm, if the firm is not in the consumer's street or the consumer is located in a street without firm, the consumer should pass the city center. These hypotheses are just for simplification of the model and its result. Here, except the assumption of uniformity of consumers, all assumptions are maintained. One of the most important assumptions is simplification; while, in real world, the consumers are not equally and uniformly distributed in a street or city and the city centers are more crowded than suburb areas. Thus, it is tried to remove the weaknesses of Lijesen and Reggini model and present a more appropriate strategy concerning the location of firms in this structure. In following, the issue of products' consistency has been more precisely considered with assumption of triangular distribution. Results show that this method offers lower price, and a greater range of consumers could purchase the products in addition to having consistency with Rohlfs model.

This paper is organized in five sections. In section 2, the model is presented and then in section 3, the location is illustrated in a triangular model. The consistency is investigated in a triangular distribution through external consequences of the network in section 4, and then the conclusion and recommendations is presented in section 5.

2. Model

In this paper, spoke model presented by Chen and Riordan (2007) and studied by Lijesen and Reggiani (2013) are used with this difference that the consumers' distribution has not been uniform, rather it has triangular distribution function. The market includes N streets with equal length, each street has length of ½ on which the consumers are triangularly distributed. Following Chen and Riordan (2007), it is assumed that any consumer's evaluation of the desirability of products' purchase is equal to v. In addition, it is assumed that the consumer is inevitable to pass city center in case of purchasing from other enterprises located in other streets.

The distance between consumer in street s (X_s) and firm in street i (y_i) is shown by d (y_i, x_s). Its definition depends on whether the consumer and firm are located on the same or different streets. If they are both located on street *i*, then:

$$d\,(y_i\,, x_s) = |y_i - x_s| \qquad\qquad s = i$$

On the other hand, if the firms are located on different streets, we will have:

$$d\,(y_i\,, x_s) = \left(\tfrac{1}{2} - y_i\right) + \left(\tfrac{1}{2} - x_s\right) = 1 - y_i - x_s \qquad \forall\, s \neq i$$

In addition, it is assumed that in case the consumer purchases from other firms located on other street, he is obliged to pass the city center. Following Lijesen and Reggini (2013), it is assumed that the transportation costs are a ratio of the squared distance between consumer and firm. In other words, the transportation costs can be defined as follow:

$$T_{is}\,(y_i\,, x_s) = \begin{cases} td^2(y_i\,, x_s) = t\,(y_i - x_s)^2 & s = i \\ td^2(y_i\,, x_s) = t\,(1 - y_i - x_s)^2 & \forall\, s \neq i \end{cases}$$

Without losing the generality of the subject and for simplicity, it is assumed that t=1 and the final cost of production in the whole analysis process is zero (c=0).

3. Location in a Triangular Model

X random variable has triangular distribution and selects the values in S= [*a*, b]. Here, *a* is the starting point of the streets and *b* is their end point, and it is assumed that a point like *c* is the connection center of these streets. The probability of consumers' distribution in sub-interval [*a*,*c*] increases linearly. It means the more we become closer to center, the more the number of consumers located on that street increases. Moreover, in sub-interval [*c*, *b*], the probability of distribution of consumers decreases linearly and by becoming far from the center, the number of consumers on that street decreases. Thus, the density function of this variable has triangular form. The triangular distribution is shown by Tria (*a*, *c*, *b*) and its density function is obtained as:

$$PDF = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)} & a \le x < c \\ \frac{2}{b-a} & x = c \\ \frac{2(b-x)}{(b-a)(b-c)} & c < x \le b \end{cases}$$

In spoke model of Lijesen and Reggini (2013), all streets have length of ½ and connected to city center. Here, any street has length of ½; however, instead of uniform distribution of consumers on any street, in this state, the consumers are triangularly distributed on any street. By replacing *a*=0, *c*=1/2, *b*=1, the triangular density function will be as:

$$f(x) = \begin{cases} 4x & 0 \le x < \frac{1}{2} \\ 4-4x & \frac{1}{2} \le x \le 1 \end{cases}$$

In the following figure, the density function of triangular distribution is seen:

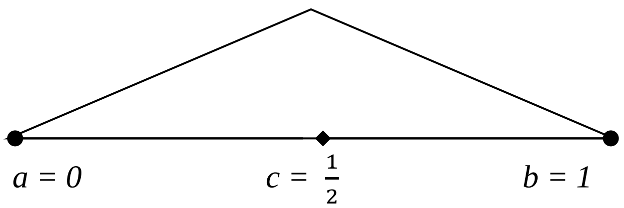

Figure1: Density Function of Triangular Distribution

In this condition, it is assumed that the consumers are distributed between 0 and 1, and the middle point that includes more consumers is *c* (the center of street). Following Chen and Riordan (2007), and Lijesen and Reggini (2013), it is assumed that any consumer has preferences for just two brands, and no problem would occur whether the firm is in the market or not. This assumption assumes that the consumer on any street prefers the products of the firm on his own street and as the second desired brand, he might prefer any of N-1 brands of other street (the brand of firm 2 or N-2 other firms that do not supply). Thus, generally, there are two types of consumers: 1. The consumers that realize both brands, 2. The consumers that prefer just one type of provided brands. Moreover, there are consumers that prefer both brands that are not provided by any firm. Thus, the whole market is not completely covered and the competition of firms is on first type consumers. A first type consumer is indifference in purchasing from firm i and j, when:

$$x_{ij} \text{ s.t. } T_{is}(y_i, x_{ij}) + p_i = T_{js}(y_j, x_{ij}) + p_j$$

Thus,

$$x_{ij} = \frac{1}{2} - \frac{1}{2}(y_i - y_j) - \frac{p_j - p_i}{2(1 - y_i - y_j)} \quad x_{ij} \in j \qquad \frac{1}{2} + \frac{1}{2}(y_i - y_j) - \frac{p_j - p_i}{2(1 - y_i - y_j)} \quad x_{ij} \in i$$

Assuming that the location of firm *i* is in y_i, we will begin from price subgame. The profit function of firm 1 will be as follow in respect to all consumers that could purchase from there:

$$\pi_1 = 4x\left(\frac{1}{N-1}p_1(1 - x_{12}) + \frac{1}{N-1}\sum_{j=3}^{N} p_1 x_{1j}\right) + \{4 - 4x\}\left(\frac{1}{N-1}p_1(1 - x_{12}) + \frac{1}{N-1}\sum_{j=3}^{N} p_1 x_{1j}\right)$$

In above formula, $\frac{1}{N-1}$ is a ratio of consumers that have the second desired brand and located on any of *s* streets. X_{12} is indifferent consumer in purchasing from any of 1 and 2 firms, and X_{1j} is the indifferent consumer in purchasing from 1 and j firm. The number of *j* firms is from 3 to N (all firms except firms 1 and 2).

After simplification, assuming that the indifferent consumer in purchasing from firms 1 and 2 (X_{12}) is located on street 2, the profit function of firm 1 will be obtained. Having profit function of firm 1, it is possible to obtain equilibrium prices. We continue our discussion by finding the equilibrium prices and the relation of these prices and the number of streets, and it is shown that increased number of streets leads to increased equilibrium prices. Based on the description provided in the triangular distribution, the following theorems will be described and proved. It should be noted that all the theorems and conclusions expressed in this research has been provided by the authors and proof of these theorems is the outcome of the study results.

Theorem 1

In a triangular distribution where the consumers are located in interval of zero and 1, there is a unique Nash equilibrium in prices. This equilibrium price is increasing in N.

Proof: since the profit function is defined as:

$$\pi_1 = 4x\left\{\left(\frac{p_1}{N-1}\right)\left(\frac{1}{2}+\frac{y_1-y_2}{2}+\frac{p_2-p_1}{2(1-y_1-y_2)}\right)+\frac{p_1(N-2)}{N-1}\right\}+$$
$$\{4-4x\}\left\{\left(\frac{p_1}{N-1}\right)\left(\frac{1}{2}+\frac{y_1-y_2}{2}+\frac{p_2-p_1}{2(1-y_1-y_2)}\right)+\frac{p_1(N-2)}{N-1}\right\}$$

There is a Nash equilibrium in prices and this equilibrium price is increasing in N.

By derivation of this function in respect to p_1 and simplification, the following results will be obtained:

$$\begin{cases} p_1 = -\frac{y_1^2+2y_1(N-2)-y_2^2+2y_2(N-1)-2N-p_2+3}{2} \\ p_2 = \frac{y_1^2+2y_1(1-N)-y_2^2+2y_2(2-N)+2N+p_1-3}{2} \end{cases}$$

By replacing p_1 and p_2, we have:

$$\Rightarrow \begin{cases} p_1 = -\frac{{y_1}^2+2y_1(3N-5)-{y_2}^2+2y_2(3N-4)-6N+9}{3} \\ p_2 = \frac{{y_1}^2+2y_1(4-3N)-{y_2}^2+2y_2(5-3N)+6N-9}{3} \end{cases}$$

$$\Rightarrow \frac{\partial p_1}{\partial N} = -2(y_1+y_2-1) = \frac{\partial p_2}{\partial N}$$

Where $y_1+y_2-1\leq 0$, this expression is always positive and the theorem is proved. Lijesen and Reggini (2013) showed that increased number of streets leads to increased equilibrium price. These results could be interpreted such that in respect to the location given to firms, when the number of empty streets is more than the firms, the equilibrium price increases. In other words, increased number of empty streets leads to increased equilibrium price concerning the increased demand. Now, the demand function of two firms will be derived when the consumers are triangularly distributed.

Theorem 2

In a triangular distribution where the consumers are distributed in interval zero and one, the demand of firm *1 and* firm *2 are as follow:*

$$\hat{X} = \frac{p_2 - p_1}{4\tau\,(2 - y_1 - y_2)} + \frac{1 - y_2}{2 - y_1 - y_2} \quad , \quad 1 - \hat{X} = \frac{p_1 - p_2}{4\tau\,(2 - y_1 - y_2)} + \frac{1 - y_1}{2 - y_1 - y_2}$$

Proof: if τ is linear transportation costs, in so far as the indifferent consumer to purchase from two firms with the given prices will become indifferent in a point, we seek to find this point.

Since in this point:

$$-p_1 - 4\tau x(x - y_1) = -p_2 - (4\tau - 4\tau x)(1 - y_2 - x)$$

We will obtain the following equation by simplification:

$$\hat{X} = \frac{p_2 - p_1}{4\tau\,(2 - y_1 - y_2)} + \frac{1 - y_2}{2 - y_1 - y_2} \quad , \quad 1 - \hat{X} = \frac{p_1 - p_2}{4\tau\,(2 - y_1 - y_2)} + \frac{1 - y_1}{2 - y_1 - y_2}$$

It is observed that the demand of firm 1 has reverse relation with the received price and transportation costs. It means that the more the price of firm increases or the more transportation costs increases, the less the demand for this firm will be. This condition is true for firm 2. In the Hotelling linear city model, where the consumers had been uniformly distributed, the results were obtained as follow:

$$\hat{X} = \frac{p_2 - p_1}{2\tau} + \frac{L - y_2 + y_1}{2} \quad , \quad L - \hat{X} = \frac{p_1 - p_2}{2\tau} + \frac{L + y_2 - y_1}{2}$$

Comparing the results shows that the parameter of transportation costs has greater effect on the demand of firms in triangular distribution approach compared to uniform distribution. This conclusion could be interpreted in this way that in triangular distribution, concerning the more population in city center compared to suburb areas, transportation costs will be significantly more. Moreover, it is observed that the effect of received prices of firms in their demands is the same in both methods. Now, we will deal with the investigation of equilibrium prices in triangular distribution method.

Theorem 3

In a triangular distribution where the consumers are distributed in interval zero and one, Nash-Bertrand equilibrium prices of firms *1 and 2 will be equal to:*

$$p_1 = \frac{4\tau\,(3 - y_1 - 2\,y_2)}{3} \quad , \quad p_2 = \frac{4\tau\,(3 - 2y_1 - y_2)}{3}$$

It means that the price significantly increases by increased transportation costs and increased distance of firms (price of firm 1 is more sensitive to location of firm 2 than its own location).

Proof:

$$\pi_1 = p_1\,\hat{X}$$

Replacing the related values in profit function and the first rank conditions, it becomes clear that:

$$\Rightarrow\; p_1 = \frac{p_2 + 4\tau - 4\tau y_2}{2} \quad , \quad p_2 = \frac{p_1 + 4\tau - 4\tau y_1}{2}$$

By replacing two equations, we simply obtain the following equation:

$$\Rightarrow p_1 = \frac{4\tau\,(3-y_1-2\,y_2)}{3} \quad , \quad p_2 = \frac{4\tau\,(3-2y_1-y_2)}{3}$$

Results clearly indicate that the price of two firms increases by increased transpiration costs and also increased firms' distance (price of firm 1 is more sensitive to location of firm 2 than its own location). Comparing these results with the results of Hotteling linear city which were as follows:

$$\mathrm{p}_1^h = \frac{\tau\,(3L - y_2 + y_1)}{3} \quad , \quad p_2^h = \frac{\tau\,(3L + y_2 - y_1)}{3}$$

It is observed that here the parameter of transportation costs have direct effect on the equilibrium costs of firms in triangular distribution approach the same as in uniform distribution. It means that the equilibrium price of firms increases with increase of transportation costs. If we put equilibrium prices for firm 1 in two methods equal, in the point where $L = \frac{12-5y_1+7y_2}{3}$, the equilibrium price will be equal in both. If $\mathrm{L} > \frac{12-5\mathrm{y}_1+7\mathrm{y}_2}{3}$, the equilibrium price obtained in Hotelling uniform distribution will be more than equilibrium price in triangular distribution method, and if $L < \frac{12-5y_1+7y_2}{3}$, the equilibrium price obtained in triangular distribution method will be more than equilibrium price in Hotelling uniform distribution. The other point in comparing the equilibrium prices obtained in these two methods is that in triangular distribution method, the price of firm1 has reverse relation with its own location, while in Hotelling uniform distribution method, this relation is direct. Since y_1 in both methods has been considered as the distance of firm1 from point zero, the result obtained in triangular distribution method is interpreted in the way that the more y_1 increases, the more this firm becomes far from the point zero, or becomes closer to the center it should lower its price. Moreover, by becoming far from the center and reaching point zero, the received price of firm increases which seems to be logical. However, in Hotelling method these results are reverse, and in Hotelling uniform distribution, the more y_1 increases, the more this firm becomes closer to middle points; so, it should increase its price more. Moreover, by becoming far from middle points and reaching point zero, the received price of firm decreases. If both firms are located in one point (homogenous productions), $p_1= p_2=0$ will be a unique equilibrium (Shy, 1996). On the other hand, in a Bertrand game with differentiated products, the profit of firms increase with differentiation of products. It means that the differentiation of the products increases the monopoly power of firms producing trademarks with decreasing the price competition between the firms producing trademarks (Shy, 1996).

Thus, the firms will be distanced from each other and the result of triangular distribution is confirmed. Miqer et al. (2014) considered the social optimum location in bilateral monopoly at the condition where the consumers are non-uniformly distributed. They calculated the increased welfare due to regulation of firms location and showed how this welfare will become different by distribution of consumers, while the regulations on the firms' location is enough for maximizing the welfare of consumers in symmetric distributions, and in asymmetric distributions, **the** price regulation is enough for supplying social optimum welfare. In following paragraphs, it is discussed that if in a triangular distribution, the situations of two firms are symmetric, we will have:

Theorem 4

In a triangular distribution where the consumers are distributed in interval [0, 1], if two firms have symmetric situation ($y_1=y_2$), then they will obtain the same share of market and call for same prices.

Proof:

By replacing $p_1 = \frac{4\tau\,(3-y_1-2y_2)}{3}$ *and* $p_2 = \frac{4\tau\,(3-2y_1-y_2)}{3}$ in $\hat{X}$ and replacing $y_1=y_2$, it becomes simply specified that:

$$\hat{X} = \frac{1-y_1}{2-y_1-y_2} \Rightarrow if\ y_1 = y_2 \Rightarrow \hat{X} = \frac{1}{2}\ , \qquad 1-\hat{X} = \frac{1}{2}$$

Thus, the market share of two firms will be the same and it becomes easily clear that they will call for the same prices. These results are the same as Hotelling model (1929). Nicolas (1989) studied the existence of equilibrium and optimization in analyzing a market for differentiation of products in terms of diversity. Results showed that in a three-stage game where the firms are entered in the first stage, select diversity in second stage and prices in the third stage, there is a subgame perfect equilibrium. In equilibrium, the products are distributed based on their symmetric specifications and the same prices are presented. Moreover, Lijesen and Reggini (2013) achieved similar results and showed that if the situations of two firms are symmetric, then they will both gain the same market share and receive similar prices. In following paragraphs, the profit function of firm and the influence it takes from the center is discussed.

Theorem 5

In a triangular distribution where the consumers are distributed in interval [0, 1], when firm 1 becomes closer to center, the profit of this firm will increase if the following conditions are true:

$$\begin{cases} \sqrt{3{y_2}^2-2y_2-1}\ -y_2+2\ \le y_1\ \le\ -\sqrt{3{y_2}^2-2y_2-1}\ -y_2+2 \\ -\sqrt{3{y_2}^2-2y_2-1}\ -y_2+2\ \le y_1\ \le\ \sqrt{3{y_2}^2-2y_2-1}\ -y_2+2 \end{cases}$$

Proof:

Since the profit function of firm 1 is:

$$\pi_1 = \frac{4\tau\,(3-4y_1-2y_2+\ {y_1}^2+2{y_2}^2)}{3\,(2-y_1-y_2)}$$

Its derivation in respect to *a* reveals that the following conditions should be presented to make positive this profit:

$$\frac{\partial \pi_1}{\partial y_1} \geq 0 \Leftrightarrow \begin{cases} \sqrt{3{y_2}^2 - 2y_2 - 1} - y_2 + 2 \leq y_1 \leq -\sqrt{3{y_2}^2 - 2y_2 - 1} - y_2 + 2 \\ -\sqrt{3{y_2}^2 - 2y_2 - 1} - y_2 + 2 \leq y_1 \leq \sqrt{3{y_2}^2 - 2y_2 - 1} - y_2 + 2 \end{cases}$$

It means that in each of y_1 and y_2 location, firm 1 could increase its profit to gain a greater market share by moving toward firm 2 (Principle of minimum differentiation) since the firms produce less differentiated products by moving toward center. The above conditions indicate that if firm 1 becomes closer to firm 2, no equilibrium would be achieved. Thus, for establishing equilibrium, the firms cannot be very close (Shy, 1996). Rajiani (2009) presented the condition for price-location equilibrium from spoke model in product delivery condition where all spokes are not occupied by firm. Results show that in equilibrium, one of the firms is supplier of all products while other firms focus in corners. Anderson and Nion (1986) showed that if the transportation costs are in convex form and demand is sufficiently high, a unit bilateral equilibrium will be at the center of linear market. Moreover, there is always a concentrated equilibrium at the center that doesn't depend on the number of firms. Meanwhile, if the transportation costs are sufficiently convex, the equilibrium includes some distinction between enterprises. Now, the question is studied that in what condition, the increased price of firms depends on their locations.

Theorem 6

If following conditions are established in a street where consumers have triangular distribution:

$$y_1 \leq 2 \quad , \quad y_1 \geq -1 \quad , \quad y_2 \leq 2 \ , \quad y_2 \geq -1$$

Then, with increase in y_1 and y_2, price of firms 1 and 2 are increased and vice versa.

Proof:

$$\frac{\partial p_1}{\partial y_1} = -\frac{2\,(y_1 + 3N - 5)}{3} \geq 0 \ \Leftrightarrow \ y_1 \leq 5 - 3N$$

$$\frac{\partial p_1}{\partial y_2} = \frac{2\,(y_2 - 3N + 4)}{3} \geq 0 \quad \Leftrightarrow \quad y_2 \geq 3N - 4$$

$$\frac{\partial p_2}{\partial y_1} = \frac{2\,(y_1 - 3N + 4)}{3} \geq 0 \quad \Leftrightarrow \ y_1 \geq 3N - 4$$

$\frac{\partial p_2}{\partial y_2} = -\frac{2\,(y_2+3N-5)}{3} \geq 0 \quad \Leftrightarrow \; y_2 \leq 5-3N$

Since these inequalities have no answer in respect to $N \geq 2$ (because length of each street is considered as $\frac{1}{2}$), if there is only one street, the results will be obtained as following with substituting N = 1:

$$y_1 \leq 2 \quad , \quad y_1 \geq -1 \quad , \quad y_2 \leq 2 \; , \quad y_2 \geq -1$$

Considering above results, it is clear that with above conditions, y_1 and y_2, price of firms 1 and 2 will increase and vice versa. Of course, results can be put in range $\left[0\,,\,\frac{1}{2}\right]$ with setting more limitations. These results show that the more firm 1 is far from the center, the more the price increases its, and price of firm 1 is also increased with taking more distance to firm 2 and vice versa. The same is true also for firm 2. Finally, as observed, with application of triangular distribution on the way of scattering consumers, as totally logical in real world, such results were obtained which were more compatible to realities. One is an inverse relationship between location of the firm and its price, which has inverse results, compare to Hotelling method. The other one is the price and its relationship with the number of streets, where results obtained from triangular distribution method were identical to results obtained from other methods. The other point was in demand function of firms. Effect of transportation costs is larger in triangular distribution method than Hotelling method. Minimal differentiation (tendency to center) under certain conditions and obtaining the identical market share at a symmetrical position relative to the center were other triangular distribution results which were consistent with other methods. In the following paragraphs, triangular distribution is applied in product compatibility area. It is one of the discussions which have not been ever addressed using triangular distribution method.

3. Compatibility in a Triangular Distribution in Network Externalities Method

In this section, basic model of network externalities is introduced in which consumer valuation of a brand is increased with increasing number of consumers using the same brand. One of the first attempts for modeling total demand for communication services was made by

Rolfes (1971). Starting point for this discussion is that subscribers gain utility from communication services which is increased with increasing number of subscribers. Gandal (1994) also provided some experimental evidence for proving presence of network externalities in computer software industry. Farol and Saloner (1987), David and Greenstein (1990), and Gabol (1991) also studied on compatibility issue. Rolfes considered a continuum of potential phone users in unit interval [0, 1] indexed by x in which, there is only one type of services and consumers with low x tend to subscribe in the phone system (tendency to pay high), and consumers with high x have low tendency to subscribe in the phone system (tendency to pay low). Total number of consumers which are currently subscribers of the phone system is shown by n, $0 \leq n \leq 1$, and price of phone system subscription is denoted by p. Consumer utility, x, $0 \leq x \leq 1$, is defined as follows:

$$U^x = \begin{cases} n\,(1-x) - p & \text{if consumer is subscribed in phone system} \\ 0 & \text{if consumer is not subscribed in phone system} \end{cases}$$

Thus, utility of each subscriber denotes network externalities, because consumer's utility x is increased with increase in n (number of people subscribed in the phone system). Rolfes found two intersection points with solving this equation calling them as $\hat{x}_0^H$ and $\hat{x}_0^L$, and stated $n = \hat{x}_0^L$ implies low number of subscribers and $n = \hat{x}_0^H$ implies high number of subscribers. According to Rolfes, only the point $\hat{x}_0^H$ is stable equilibrium of demand, because in intersection point $\hat{x}_0^L$, a small increase in the number of subscribers leads to more optimal phone subscription, and thus all consumers are subscribed in interval $[\hat{x}_0^L, \hat{x}_0^H]$. If final cost of adding one subscriber due to wiring all houses by Telecommunication Company is assumed as zero, with construction of profit function and first and second order conditions, it is simply specified that $\hat{x} = 0$ and $\hat{x} = \frac{2}{3}$ are extreme points. Since first order condition is positive for all $0 \leq \hat{x} \leq \frac{2}{3}$, thus $\hat{x} = \frac{2}{3}$ is absolute maximum point (Shy, 1996). In the following section, change in equilibrium price and consumer range in triangular distribution method is discussed.

Theorem 7

In a triangular distribution where consumers distributed within zero *and one interval, profit maximizing subscription price of monopoly firm is specified in such a way that the number of subscribers is more than half of consumer population but fewer than all the population. Triangular distribution includes lower price and larger range of consumers.*

Proof: Since density function of triangular distribution is as follows:

$$PDF = \begin{cases} \dfrac{2(x-a)}{(b-a)(c-a)} & a \le x < c \\ \dfrac{2}{b-a} & x = c \\ \dfrac{2(b-x)}{(b-a)(b-c)} & c < x \le b \end{cases}$$

With substitution of values a, b, and c, density function is as follows:

$$\text{f (x)}= \begin{cases} 4x & 0 \le x < \frac{1}{2} \\ 4-4x & \frac{1}{2} \le x \le 1 \end{cases}$$

Since,

$$U^x = \begin{cases} n\,(1-x) - p & \text{if consumer is subscribed in phone system} \\ 0 & \text{if consumer is not subscribed in phone system} \end{cases}$$

Thus indifferent consumer is specified as follows:

$$n\,(1-\hat{x}\,) - p = 0$$

Here since density function is as two-area, relations are obtained from two-valued function as follows:

$$U^x = \begin{cases} \left(\frac{n}{2}\left(x-\frac{1}{2}\right) - p\right) 4x & \text{if consumer is before center} \\ 0 & in\ case\ of\ no\ purchase \end{cases}$$

$$U^x = \begin{cases} \left(\frac{n}{2}\,(1-x) - p\right)(4-\ 4x) & \text{if consumer is after center} \\ \ 0 & in\ case\ of\ no\ purchase \end{cases}$$

In these equations, p is obtained and equality $n = \hat{x}$ is used, and solutions are obtained with calculation of profit function and first order conditions. By solving above equations, it is clear that first system's solution is 0 and $\frac{1}{3}$ and second system's solution is 0 and $\frac{2}{3}$ which is consistent to results found by Rohlfs (1974). Given the fact that here equilibrium price is $\frac{5}{36}$, and it cuts the curve in a lower point compared to Rohlfs method ($\frac{8}{36}$), by considering Fig. 2, triangular distribution covers greater range of consumers.

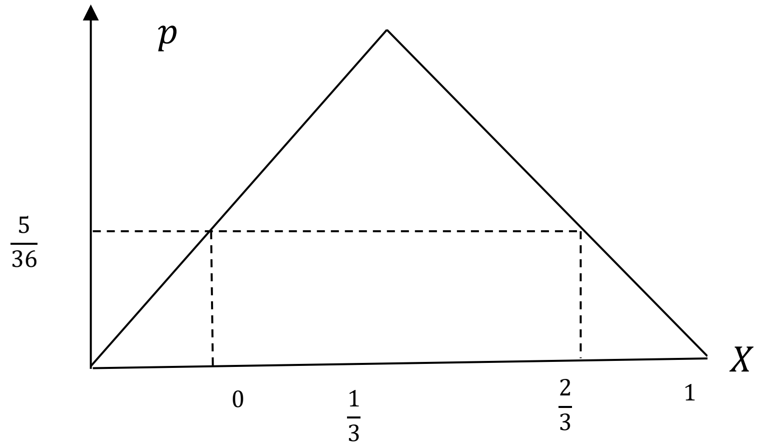

Figure 2: Deriving Demand for Communications Services in Triangular Distribution

It is observed that triangular distribution method results are consistent with results of uniform distribution by Rohlfs. Due to higher density of consumers at the center in triangular distribution method, lower costs are required for the distribution of products by the monopoly firm. Hence, the firm reduces price due to reduction in distribution costs and more number of consumers would buy the product. These results are perfectly consistent with real world conditions where firms seek for minimizing their distribution costs. With minimizing distribution costs, the firms reduce the price to gain more profit. It should be noted that price cannot be reduced by making uniform consumer distribution, because distribution costs increase directly with distancing consumers from firm location, and the firm cannot reduce price of its manufactured product.

4. Conclusion and Recommendations

Location is regarded as one of the key measures in the process of industrial or service unit's construction and paying attention to this factor is crucially important in success of the firms. Locating denotes selecting the location for one or more firms considering other firms and existing constraints in such a way that a special purpose is optimized. Such purpose can be transportation costs, obtaining greater profits, providing a fair service to customers, seizing greatest market (higher market share), etc. So, it is evident that locating should be taken into account in a totally logical and realistic view and the real world should be realistically modeled. One of the defects in the previous works regarding locating is simplistic assumption of consumer distribution on a single line or city, and using uniform distribution. Since distribution of consumers in real world is not within a single street or city, and usually the city centers are busier than margins, thus using triangular distribution of consumers more compete results were obtained. Triangular distribution results showed that price is increased with increasing number of streets and transportation costs. On the other hand, market price is increasingly grown with increasing distance between the firms, suggesting that the farther are firms, competition among them in the product market is reduced leading to higher price. If distance between two firms from the city center is the same, then both firms would obtain identical market share and the firms would tend to choose the location nearer to the center or with minimum differentiation. Also, with inclusion of triangular distribution in compatibility discussion, results shows that equilibrium price is lower, compared to Rohlfs model (1971) and larger range of consumers would buy the product. Finally, it is also recommended to use other statistical methods. Considering that in the real world the parameters are non-deterministic and external factors constantly influence the problem, paying attention to non-deterministic models seems necessary. The number of players in most papers is regarded as two; however, newer and more complicated problems can be defined with increasing number of players.

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