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Active control vibration of circular and rectangular plate with Quantitative Feedback Theory (QFT) Method

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Abstract

Natural vibration analysis of plates represents an important issue in engineering applications. In this paper, a new and simplify method for vibration analysis of circular and rectangular plates is presented. The design of an effective robust controller, which consistently attenuates transverse vibration of the plate caused by an external disturbance force, is given. The dynamics of the plate is modeled as a distributed parameter system. We have studied the control vibration of the plate using quantitative feedback theory method by determining the transfer functions between various factors of control system. In this method we have developed the general distributed parameter system method for uncertainty problem for simply supported rectangular plate and clamped circular plate. The quantitative feedback method is one of the robust control methods which is capable to solve problems despite structural and non-structural uncertainty. Quantitative Feedback Theory introduces the new technique to design one-point feedback controllers for distributed parameter systems. The results demonstrate that the control law provided a significant reduction in the plate vibration. The numerical simulation of the designed controller demonstrates that the QFT controller can consistently attenuate the vibration compared to a passive system.

Keywords: Vibration, control, rectangular plate, circular plate, quantitative feedback theory

1. Introduction

Plates are widely used as a major structural element in many fields such as mechanical, aerospace and civil engineering. As a result of many loadings are dynamical on plates, in many cases active control by the use of piezoelectric layers and piezoelectric patches have been used to reduce the vibration of plates. Active vibration control of stiffened structures is an important problem in various practical situations. In aircraft structures, the wings and fuselage consist of a skin with an array of stiffeningribs. Such structures are subjected to dynamic loads and their control is of paramount importance in safe and smooth functioning of the system. Bailey and Hubbard [1] proposed the use of distributed piezoelectric transducers (PZT) in active vibration control of beams. Anderson and Hagood [2] discussed issues related to simultaneous sensing and actuation in structural control. Tzou [3] gave exhaustive information on vibration control of beams, plates and shells using piezoelectric materials. Gaudenzi et al. [4] demonstrated the aspects of vibration control of beams with collocated PZT piezo patches using velocity and position feedback systems. Lam and Ng [5] presented theoretical formulation for laminated plates using classical plate theory and Navier solutions. A negative force-cum-moment

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feedback algorithm is used in controlling the dynamic response of the plates. Chandrashekhara and Agarwal [6] presented a finite element approach to active vibration control of laminated composite plates using piezoelectric devices. Investigations on vibration control of plates have been carried out and their reference is available in the literature [5, 7]. One of the important and complicated issues in the field of control is the analysis of uncertainty systems and designing a resistant controller for them. As many real systems have real structures and some uncertainties, control engineers show a lot of desire in designing and analyzing these systems. The method has expanded by Doyle [8] and the quantitative feedback theory method founded by Horowits [9-12] is among the methods used for solving such problems and designing controllers for them in the field of frequency. In this method, sensitivity transfer function matrix norms are utilized, and phase information diminishes while designing as closed loop system is used in this analysis [9] .But this method is one of the methods that provides designer with enough information about system phase despite structural and non-structural uncertainties. Concerning the design of controller with Quantitative Feedback Theory method (QFT) for linear multivariable systems, we can refer to works of Chain et al [14] and Cheng et al [15-16] in addition to Horowits [10-13] which have proposed a new formula for linear and uncertain multivariable procedures. Franchek has also used Gaussian elimination method to break a multivariable system to a single-variable system [17]. Rafeeyan has also proposed a new method for eliminating input and output disturbance. In their method, the multivariable procedure is first analyzed into a single-variable procedure. Preferable controllers are then designed using quantitative feedback theory method. Many works have also been pursued in control vibrations of plate. Using time delay method, Lang Xiang Chen controlled the vibrations of the flexible plate whose one end was harnessed [18]. In this controller, the influence of the time delay in the Mathematical dynamic system during designing has been taken into the account, and there is no estimation or hypothesis in derivative controller and the sustainability of the system is easily guaranteed. Chain and Chang controlled a square shaped plate using optical sensors and acoustic radiation. This method include the dynamic analysis of Uniform plates, considering the light path errors, nervous networks of control systems and experimental observations on acoustic radiations and the reduction of emission level from the plates using optical method [19]. In 2009, Kacar [20] used intelligent piezoelectric patches to control vibrations of multilayered plate and calculated the influence of the size of the patches installed on the plate for various border conditions on different layers of the layered plate. In 2003, Giovanni Caruso used some piezoelectric patches as sensors and stimulators to control the vibrations of the plate [21]. In 2004, TianXiong Liu by using the H_∞ method and intelligent patches with viscoelastic damping properties have been studied the control vibrations of plate [22]. In 2010, Kuzopa [23] conducted the active control vibrations of aluminum rectangular plate using piezoelectric patches and analyzed the influence of various parameters on the vibrations of the plate. Utilizing the quantitative feedback theory method and forming a model of transformation function with structural and nonstructural uncertainties on the plate, this research seeks to study the issue of recoiling the vibrations caused by external forces on the plate model, and it has the amplitude of variations of the plate has decreased by designing the appropriate controller.

2. Analysis of governing equations of motion

In this section, the Dynamic equations of plate with considering internal damping will be obtained. For this purpose, Potential energy of a rectangular plate with dimensions $a \times b$ and thickness h , due to the elastic deformation is expressed as follows [24-25]:

$$U = \frac{1}{2}\iint_R \left[\int_{-h/2}^{h/2} \begin{pmatrix} \sigma_{xx}\varepsilon_{xx} + 2\sigma_{xy}\varepsilon_{xy} \\ +\sigma_{yy}\varepsilon_{yy} \end{pmatrix} dz \right] dxdy \qquad (1)$$

Where the stress-strain relations of plate can be obtained the following equation. With consider the classical theory of plate the stress such as $\sigma_{zz} = \sigma_{zx} = \sigma_{zy} = 0$ and the strains in z direction will be zero.

$$\begin{aligned} \sigma_{xx} &= \frac{E}{1-\nu^2}\left[\left(\varepsilon_{xx} + \eta\dot{\varepsilon}_{xx}\right) + \nu\left(\varepsilon_{yy} + \eta\dot{\varepsilon}_{yy}\right)\right], \\ \sigma_{yy} &= \frac{E}{1-\nu^2}\left[\nu\left(\varepsilon_{xx} + \eta\dot{\varepsilon}_{xx}\right) + \left(\varepsilon_{yy} + \eta\dot{\varepsilon}_{yy}\right)\right], \\ \sigma_{xy} &= \frac{E}{1+\nu}\left[\varepsilon_{xy} + \eta\dot{\varepsilon}_{xy}\right], \end{aligned} \qquad (2)$$

And

$$\begin{aligned} \varepsilon_{xx} &= \frac{E}{1-\nu^2}\left(\sigma_{xx} - \nu\sigma_{yy}\right), \\ \varepsilon_{yy} &= \frac{E}{1-\nu^2}\left(\sigma_{yy} - \nu\sigma_{xx}\right), \quad \varepsilon_{xy} = \frac{\sigma_{xy}}{2G}, \end{aligned} \qquad (3)$$

Where $G = \dfrac{E}{2(1+\nu)}$ is the Shear modulus and η is the internal damping coefficient. The potential and

Kinetic energy due to applied the force (q)are expressed respectively as follows

$$V=-\iint_R q(x,y)w(x,y)\,dxdy, \qquad (4)$$

And

$$T=\frac{1}{2}\iint_R \rho h\dot{w}(x,y)\,dxdy, \qquad (5)$$

Where $w(x,y)$ is the displacement in the z direction. By substituting Eq. (2) into the Eq. (1), and simplifying, potential energy equation can be written in the new following form:

$$U=\frac{1}{2}\iint_R D\Bigg\{\left(\nabla^2 w\right)^2+2(1-\nu)\psi + \eta\frac{d}{dt}\left(\left(\nabla^2 w\right)^2+2(1-\nu)\psi\right)+ \eta^2\left(\begin{array}{l}\left[\frac{d}{dt}(\nabla^2 w)\right]^2+ \\ 2(1-\nu)\left(\begin{array}{l}\left[\frac{d}{dt}\left(\frac{\partial^2 w}{\partial x\partial y}\right)\right]^2 \\ -\left[\frac{d}{dt}\left(\frac{\partial^2 w}{\partial x^2}\right)\right]\left[\frac{d}{dt}\left(\frac{\partial^2 w}{\partial y^2}\right)\right]\end{array}\right)\end{array}\right)\Bigg\}dxdy, \qquad (6)$$

Where

$$\psi=\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)-\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right).$$

According to the Hamilton's principal for achieve the governing equation of plate by using the calculus of variations and its extreme application, the following equation will be obtained [26-27].

$$\delta(T-U-V)=\frac{1}{2}\int_{t1}^{t2}\iint_R\Bigg[\rho h\frac{\partial w}{\partial t}\frac{\partial\delta w}{\partial t}+q(x,y)\delta w-D\Big\{2\left(\nabla^2 w\right)\Lambda+2(1-\nu)\Pi+\eta\frac{d}{dt}\left(2\left(\nabla^2 w\right)\Lambda+2(1-\nu)\Pi\right)+\eta^2\Bigg(2\left[\frac{d}{dt}\right](\nabla^2 w)\frac{d\Lambda}{dt}+ 2(1-\nu)\left[2\frac{d}{dt}\left(\frac{\partial^2 w}{\partial x\partial y}\right)\frac{d}{dt}\left(\frac{\partial^2\delta w}{\partial x\partial y}\right)-\left(\frac{d}{dt}\left(\frac{\partial^2 w}{\partial x^2}\right)\right)\left(\frac{d}{dt}\left(\frac{\partial^2\delta w}{\partial y^2}\right)\right)-\left(\frac{d}{dt}\left(\frac{\partial^2\delta w}{\partial x^2}\right)\right)\left(\frac{d}{dt}\left(\frac{\partial^2 w}{\partial y^2}\right)\right)\right]\Bigg)\Big\}\Bigg]dxdydt, \qquad (7)$$

Where

$$\Pi=\left[\begin{array}{l}2\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2\delta w}{\partial x\partial y}+2\frac{\partial^2 w}{\partial y\partial x}\frac{\partial^2\delta w}{\partial y\partial x} \\ -\frac{\partial^2 w}{\partial x^2}\frac{\partial^2\delta w}{\partial y^2}-\frac{\partial^2\delta w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\end{array}\right], \qquad (8)$$

$$\Lambda=\left(\frac{\partial^2\delta w}{\partial x^2}+\frac{\partial^2\delta w}{\partial y^2}\right).$$

Using the integration of relation terms (7), the differential equation of a rectangular plate can be expressed in the following form

$$\rho h\frac{\partial^2 w}{\partial t^2}+D\nabla^4 w+\eta D\frac{\partial(\nabla^4 w)}{\partial t}+ \eta^2 D\frac{\partial^2(\nabla^4 w)}{\partial t^2}=q(x,y,t). \qquad (9)$$

3. Vibration Control of Rectangular Plate

In this section vibration control of rectangular plate due to apply disturbance external force on the plate will be studied by using the quantitative feedback theory method (QFT). The external disturbance force will be modeled by means of an impulse point load which is applied at the location (x_d, y_d)

$$\begin{aligned}&q(x,y,t)=F\delta(t)\delta(x-x_i)\delta(y-y_i),\\&F\delta(t)\delta(x-x_i)\delta(y-y_i)=\\&\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}F_{nm}(t)\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b},\end{aligned} \qquad (10)$$

In which δ is Dirac's delta function. Using the Fourier series expansion and integration of Eq. (10), the force disturbance will be written as follow

$$F_{nm}(t) = \frac{4F}{ab}\int_0^{\pi}\int_0^{\pi}\delta(t)\delta(x-x_i)\delta(y-y_i)$$
$$\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}dxdy \tag{11}$$
$$= \frac{4F}{ab}\delta(t)\sin\frac{n\pi x_i}{a}\sin\frac{m\pi y_i}{b}$$

The governing boundary conditions for simply supported plate are considered by following relationships

$$\begin{aligned} w(x,y,t)\big|_{x=0,a} &= 0, \quad M_x\big|_{x=0,a} = 0, \\ w(x,y,t)\big|_{y=0,b} &= 0, \quad M_y\big|_{y=0,b} = 0. \end{aligned} \tag{12}$$

By using the levy's solution [24-27] and assuming the deflection of plate with the series of Eq. (13) and substituting the Eq. (11) and Eq. (13) in to Eq. (9) the governing equation of rectangular plate can be obtained as

$$w(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{nm}(t)\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}, \tag{13}$$

And

$$\rho h\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A''_{nm}(t)\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}$$
$$+D\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\varphi_{nm}\begin{pmatrix} A_{nm}(t)+\eta A'_{nm}(t) \\ +\eta^2 A''_{nm}(t)\end{pmatrix}\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b} \tag{14}$$
$$=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} F_{nm}(t)\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b},$$

In which

$$\varphi_{nm} = \left[\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right]^2.$$

Equation (14) can be rewritten as the following form

$$\rho h A''_{nm}(t) + D\varphi_{nm}\begin{pmatrix} A_{nm}(t)+\eta A'_{nm}(t) \\ +\eta^2 A''_{nm}(t)\end{pmatrix} = F_{nm}(t). \tag{15}$$

Applying Laplace operator to Eq. (15) leads to

$$\rho h(s^2 L\{A_{nm}(t)\}) +$$
$$D\varphi_{nm}\left((L\{A_{nm}(t)\} + \eta(sL\{A_{nm}(t)\}) +\right.$$
$$\left.\eta^2(s^2 L\{A_{nm}(t)\})\right) = \tag{16}$$
$$\frac{4F}{ab}\delta(t)\sin\frac{n\pi x_i}{a}\sin\frac{m\pi y_i}{b}.$$

Consequently, the transverse displacement of the plate can be achieved as follows

$$W(x,y,s) =$$
$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\frac{4F}{ab}\sin\frac{n\pi x_i}{a}\sin\frac{m\pi y_i}{b}}{\left[\rho h+\eta^2 D\varphi_{nm}\right]s^2+\left[\eta D\varphi_{nm}\right]s+D\varphi_{nm}}\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b} = P_{x_i y_i} \tag{17}$$

In Eq. (17), n is a finite value for model accuracy representing the unstructured uncertainty of the system. In this section, robust control QFT [9-12] will be used for active vibration control of the plate. The main objectives in this section are to design a controller in a way that it could stabilize the system and reduce the vibration's domain.

2.1 A quantitative robust control approach for distributed parameter systems

QFT based robust control is one of well-known effective frequencial techniques for controlling different types of practical processes [9-12]. As the most important properties of QFT, the procedure of controller design is transparent and systematic, and it is relatively easy to include uncertain factors in the performance's specifications. Thus, the quantitative formulation of plant uncertainty and different performance specifications are essential for the feedback control. The basic developments with QFT theory are focused on the control design problem for uncertain Linear Time Invariant (LTI) systems. Consider the system of the Fig (1) as the basic structure. Suppose a linear multi inputs-single output *DPS* with partial differential equation (*PDE*) with constant coefficient and time variable $t > 0$ have been defined. In which P_{X2X1} is transfer function between input x_1 and output x_2 that obtained with Eq. (17). *DPS* and feedback loop which have been depicted in Fig (1) explain general distribution which includes sensor, actuator, disturbances and control factor in different points x_o , x_d , x_a , x_s respectively [30-32].

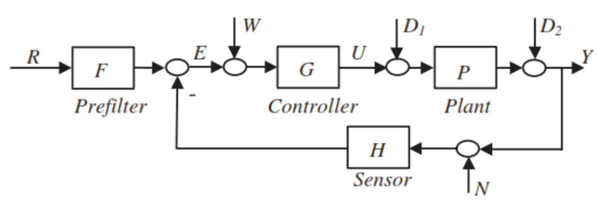

Fig 1. Two degrees of freedom *QFT* structure.

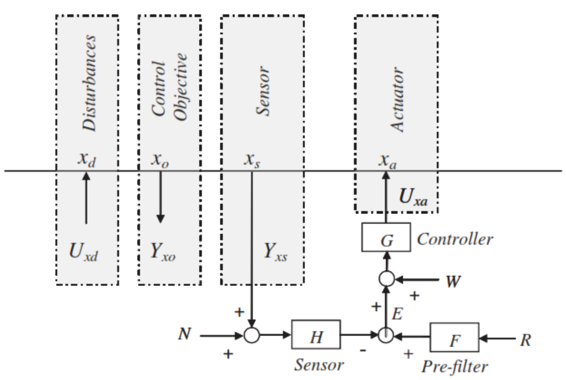

Fig 2. Control system and *DPS*

It should be noted that, for situation which $x_d = x_a$, $x_o = x_s$ that shows in Fig (2) the transfer functions between inputs and outputs will be written such as $P_{xsxa} = P_{xoxs} = P_{xoxd} = P_{xsxd}$. By considering Fig (2) the dynamic equations of *DPS* are explained with Eq (18)

$$\begin{aligned} Y_{xo} &= P_{xoxd}\, U_{xd} + P_{xoxa}\, U_{xa} \\ Y_{xs} &= P_{xsxd}\, U_{xd} + P_{xsxa}\, U_{xa} \\ U_{xa} &= G[FR + W - H(N + Y_{xs})] \\ E &= FR - HN - HY_{xs} \end{aligned} \tag{18}$$

In which capital letters describe Laplace transfers and U_{xa}, U_{xd}, N, W, R which show the inputs are reference signal, reference disturbances, noise sensor, external disturbances and actuator signals respectively [30-32]. In which the transfer functions depend on compensator (*G*), pre-filter (*F*), dynamic sensor (*H*), distance distributions, distance between sensor and actuator, disturbances and control factor among the transfer functions. In this section using the quantitative feedback control method and choosing proper transfer functions between input and output of system. With choosing the position of sensor and actuator on the peak of vibration mode such as Fig (3) the disturbance rejection problem in a simply supported rectangular has been considered. In this problem length of plate $a = b = 5m$ and the location of each factor of control includes sensor, disturbance, actuator and object which has been considered as (x_i, y_j).

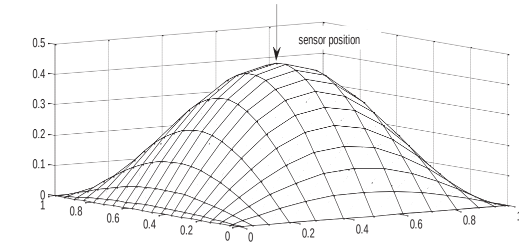

Fig 3. Mode shape of simply supported rectangular plate

Sensor and actuator position

It should be noted that (x_i, y_j) in each location of the plate can be located and they will not create limitation. Considered places to obtain different transfer functions among control factors have been considered as follow

$$(x_s, y_s) = (2,2), (x_o, y_o) = (2.5, 2.5),$$
$$(x_d, y_d) = (2.5, 2.5), (x_a, y_a) = (2,2),$$

The system performance specifications are defined with the following expressions:

(a) Stability. It is defined by the most restrictive condition of the following expressions:

$$\left| \frac{GP_{xoxa}}{1 + GP_{xsxa}} \right| \le 1.1 \quad \forall \omega \tag{19}$$

$$\left| \frac{GP_{xsxa}}{1 + GP_{xsxa}} \right| \le 1.1 \quad \forall \omega \tag{20}$$

(b) Disturbance rejection.

$$\left| \frac{Y_{xo}}{U_{xd}} \right| = \left| P_{xoxd} - \frac{GP_{xoxa} P_{xsxd}}{1 + GP_{xsxa}} \right| \le 0.3 |P_{xoxd}|, \quad \omega < 1.1\ rad/s \tag{21}$$

Templates are calculated from $P_{xoxd}, P_{xsxa}, P_{xsxd}, P_{xoxa}$ (transfer functions). Stability and disturbance rejection bounds are obtained by changing above equations as $\left| \frac{A + BG}{C + DG} \right| \le w_s$ and using quantitative feedback method [28-34]. Fig (4) shows bode diagram of Eq. (17) for transfer function between sensor and actuator which has acceptable approximation for different frequencies.

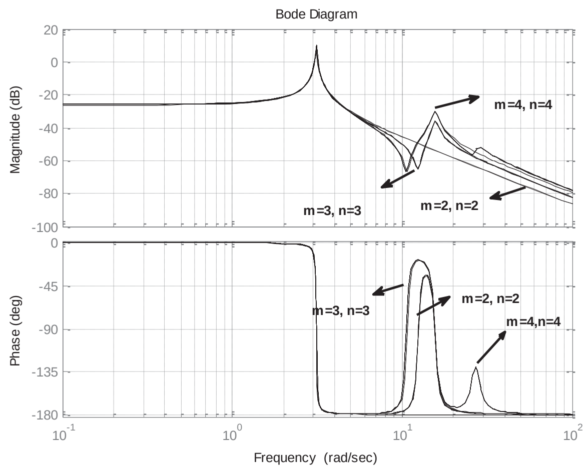

Fig 4. Bode diagram of plate models

For other transfer functions, similar diagrams will be obtained. The templates are calculated from the equivalent transfer functions $P_{xoxd}, P_{xsxa}, P_{xsxd}, P_{xoxa}$ and the robust stability and disturbance rejection bounds are obtained from the quadratic inequalities corresponding to Eq. (19), (20) and (21). The nominal open-loop expression is $L = GP_{xsxa}$. The G compensator (see Eq. (22)) is obtained by using a standard loop shaping QFT technique.

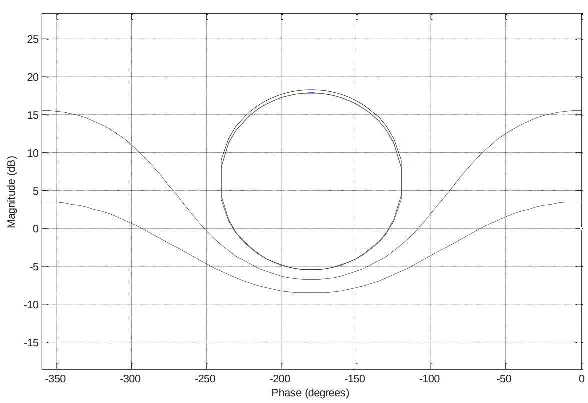

Fig 5. Stability bounds of plate models

According to Fig (6) Transfer function related to controller can be explained as follows:

$$G = \frac{2.5(s^2 + 8s + 4)(\frac{s}{0.0437} + 1)}{s(\frac{s}{22.9} + 1)(\frac{s}{25} + 1)(\frac{s}{30} + 1)}. \qquad (22)$$

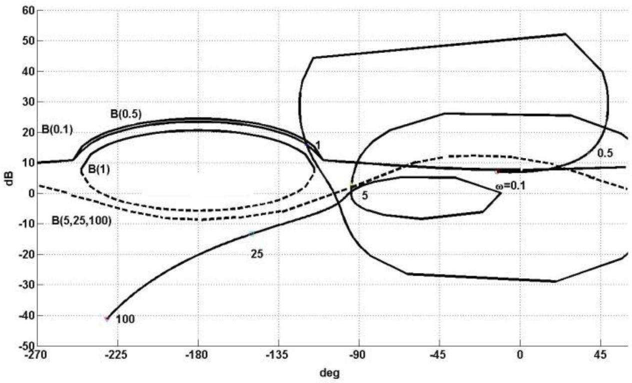

Fig 6. Loop shaping for open-loop controller system

Fig (7) shows the bode diagram for designed controller in Eq. (22)

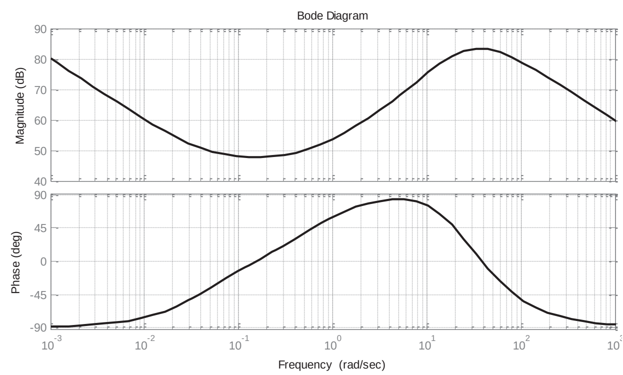

Fig 7. Bode diagram for controller of plate models

System's simulation shows that after disturbance rejection the vibrations domain of rectangular plate are reduced. The designed controller is well controlled the amplitude of transverse vibrations. In Fig (8a) vibration of plate with out using any controller has been shown. Fig (8b) shows the disturbance rejection of rectangular plate.

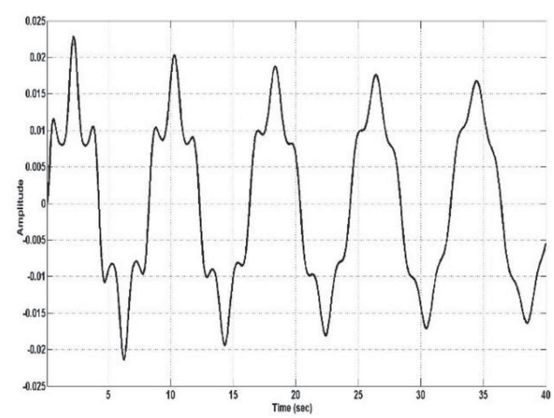

Fig 8a. Vibration of plate with out using any controller

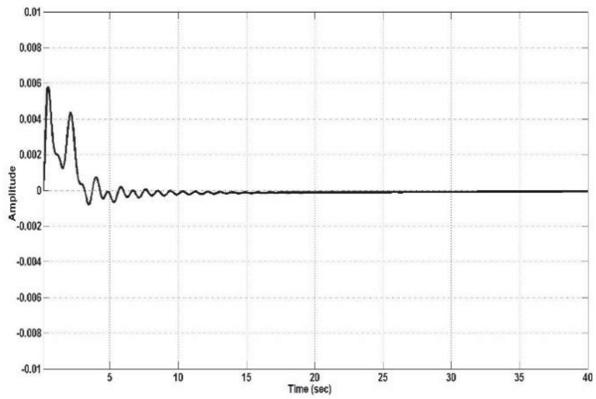

Fig 8b. Disturbance rejection of rectangular plate.

In addition, the proposed methodology in this paper is simpler, deals with the model uncertainty (structure, parameters and distribution) and is able to work with distributed specifications.

3 Active Control Vibration of Circular Plate using QFT

In this section control vibration of clamped circular plate has been studied such as the proposed method for controlling a rectangular plate [36]. The governing equation of circular plate can be written as follow:

$$D\left(1+\eta\frac{\partial}{\partial t}\right)\nabla^4 w + \rho h\frac{\partial w}{\partial t^2} = q(r,\theta,t) \tag{23}$$

Where η is Kelvin vogt damping coefficient [35-37]. The external force disturbance $q(r,\theta,t)$ will be modeled by means of an impulse point load which is applied at the location (r_i,θ_i)

$$q(r,\theta,t) = F_0\delta(t)\delta(r-r_0)\,\delta(\theta-\theta_0), \tag{24}$$

Using the Fourier series expansion and integration of Eq. (24) the disturbance force can be described as follow

$$F_0\delta(t)\delta(r-r_0)\delta(\theta-\theta_0) = \sum_{j=1}^{\infty}F_j(t)\sum_{n=1}^{\infty}\Phi(r,\lambda_{jn})\,f_n(\theta),$$
$$F_j(t) = F_0(t)\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}\frac{\Phi(r_0,\lambda_{jn})}{f_n(\theta_0)}\Phi(r,\lambda_{jn})\,f_n(\theta) \tag{25}$$

Boundary conditions for clamped circular plate are considered by following relationships [24, 26, and 27]

$$W = \frac{\partial W}{\partial r} = 0\Big|_{r=a}, \tag{26}$$

By using the levy's solution [24] and assuming the deflection of plate with the series of Eq. (27) and substituting the Eq. (25) and Eq. (27) in to Eq. (23) the governing equation of circular plate can be obtained as Eq. (30).

$$w(r,\theta,t) = W(r,\theta)g(t), \tag{27}$$

In which $w(r,\theta,t)$ for clamped circular plate can be obtained

$$W(r,\theta) = \sum_{j=1}^{\infty}\sum_{n=0}^{\infty}C_{jn}\Phi(r,\lambda_{jn})\,f_n(\theta), \tag{28}$$

Where $\lambda^4 = \dfrac{\omega^2\rho h}{D}$ is the natural frequency of circular plate that will be obtained by solving the following characteristic equation [28-29, 41].

$$J_n(\lambda_n r)I_{n+1}(\lambda_n r) + I_n(\lambda_n r)J_{n+1}(\lambda_n r) = 0. \tag{29}$$

And the governing equation of motion for circular plate rewrittin as follows

$$D\left(1+\eta\frac{\partial}{\partial t}\right)\left(\nabla^4\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}g_j(t)C_{jn}\Phi(r,\lambda_{jn})\,f_n(\theta)\right) + \rho h\frac{\partial}{\partial t^2}\left(\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}g_j(t)C_{jn}\Phi(r,\lambda_{jn})\,f_n(\theta)\right) = F_0(t)\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}\Phi(r_0,\lambda_{jn})\,f_n(\theta_0)\Phi(r,\lambda_{jn})\,f_n(\theta), \tag{30}$$

After simplifying the Eq. (30):

$$D\left(g(t)+\eta\dot{g}(t)\right)\left[A_{jn}\right]+ \rho h\ddot{g}(t)\left[B_{jn}\right]=F_0(t)H, \qquad (31)$$

Where

$$A_{jn}=\nabla^4\left(\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}C_{jn}\Phi(r,\lambda_{jn})\,f_n(\theta)\right),$$

$$B_{jn}=\left(\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}C_{jn}\Phi(r,\lambda_{jn})\,f_n(\theta)\right), \qquad (33)$$

$$H=\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}\Phi(r_0,\lambda_{jn})\,f_n(\theta_0)\Phi(r,\lambda_{jn})\,f_n(\theta),$$

Assuming zero initial conditions and Applying Laplace operator to Eq. (31) leads to

$$L\left\{g_j(t)\right\}=\frac{F_0(t)H}{\rho h\left[B_{jn}\right]s^2+D\eta\left[A_{jn}\right]s+D\left[A_{jn}\right]}. \qquad (33)$$

Consequently, the transverse displacement of the circular plate can be achieved as follows

$$W(r,\theta,s)=\sum_{n=1}^{\infty}\sum_{j=1}^{\infty}\frac{F_0(t)H\left(C_{jn}\Phi(r,\lambda_{jn})\,f_n(\theta)\right)}{\rho h\left[B_{jn}\right]s^2+D\eta\left[A_{jn}\right]s+D\left[A_{jn}\right]} \qquad (34)$$

In Eq. (34), n is a finite value for model accuracy representing the unstructured uncertainty of the system. In this section, robust control QFT will be used for active vibration control of the plate. The main objectives in this section are to design a controller in a way that it could stabilize the system and reduce the vibration's domain. It should be noted that (r_i,θ_i) in each location of the plate can be located and they will not create limitation. Considered places to obtain different transfer functions among control factors have been considered as follow

$$(r_s,\theta_s)=(0.65,\frac{2\pi}{3}),$$
$$(x_o,y_o)=(0.55,\frac{\pi}{6}),$$
$$(x_d,y_d)=(0.75,\frac{2\pi}{3}), \qquad (35)$$
$$(r_a,\theta_a)=(0.5,\frac{\pi}{4}),$$

The system performance specifications are defined with equations (19-21). Templates are calculated from $P_{xoxd}, P_{xsxa}, P_{xsxd}, P_{xoxa}$ (transfer functions). Stability and disturbance rejection bounds are obtained by changing above equations as $\left|\frac{A+BG}{C+DG}\right|\le w_s$ and using quantitative feed back method. Fig (9) shows bode diagram for transfer function between sensor and actuator which has acceptable approximation for different frequencies and other transfer functions among factors which control unique conditions figures.

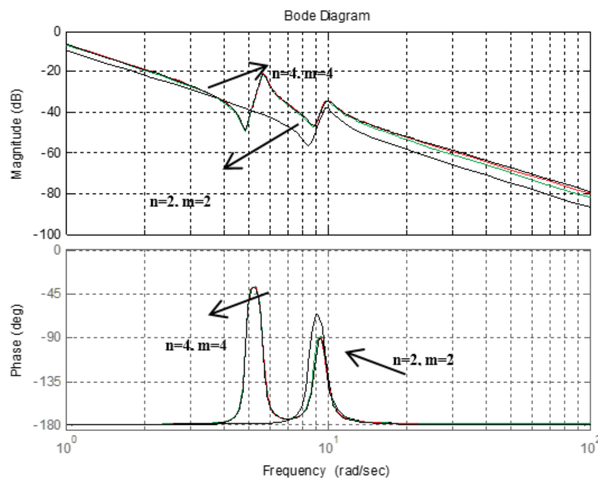

Fig 9. Bode diagram of circular plate models

For other transfer functions, similar diagrams will be obtained. According to Fig (11) and using the loop-shaping technique the transfer function related to controller can be explained as follows:

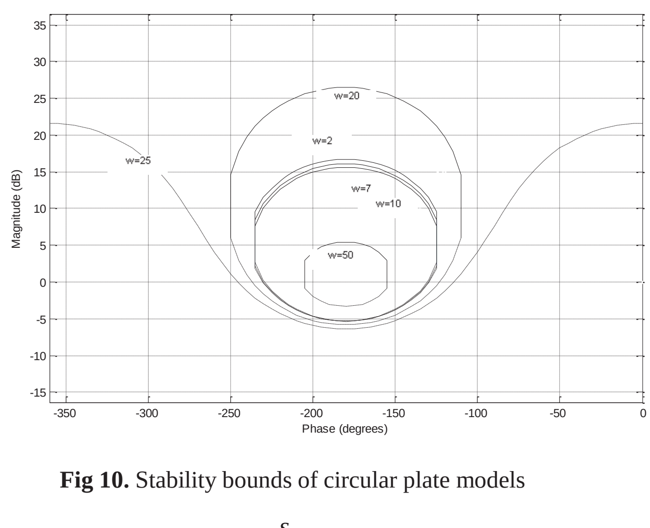

Fig 10. Stability bounds of circular plate models

$$G_{cp}=\frac{2.5(s^2+6s+2)(\frac{s}{0.2235}+1)}{s(\frac{s}{24.6}+1)(\frac{s}{22}+1)(\frac{s}{38}+1)}. \qquad (36)$$

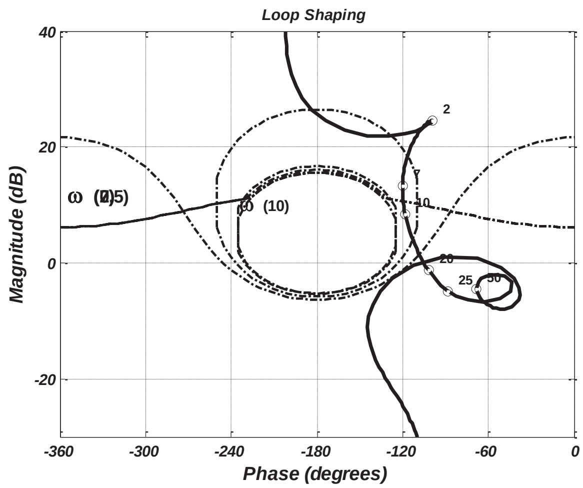

Fig11. Loop shaping for open-loop controller system

Fig (12) shows the bode diagram for designed controller in Eq. (36)

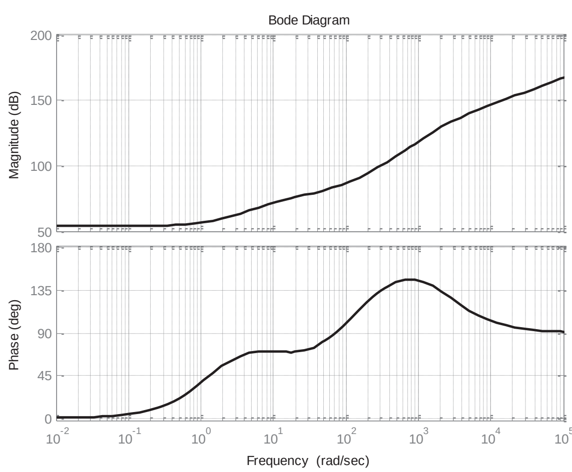

Fig12. Bode diagram for controller of circular plate models

In this section the effectiveness of the designed controller will be demonstrated by means of numerical simulations. Three parameters will be considered in simulations, namely: the response of the controlled system to disturbances, the response of the system without controller to disturbances, and the control effort or control force signal. Since a proper controller must suppress arbitrary disturbances, two different disturbances are used to ensure that the QFT controller has consistent controlling performance. System's simulation shows that due to this reason that, after disturbance rejection the reduction of vibrations domain of plate has been considered the designed controller reduces created vibration's domain against entry impact considerably. In Fig (13a) the displacement of circular plate the disturbance force extends on it is shown. In Fig (13b) shows the Actuator effort of circular plate on (r_a, θ_a) point. The desired output X0 shows a more damped behavior and a smaller tracking error when using the proposed methodology and the compensator G. The method can also deal with uncertainty in the model and the spatial distribution of the inputs and the outputs.

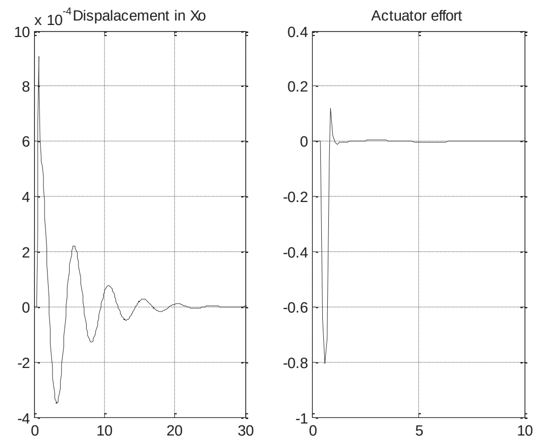

Fig13. a) Displacement of circular plate. **b)** Actuator effort.

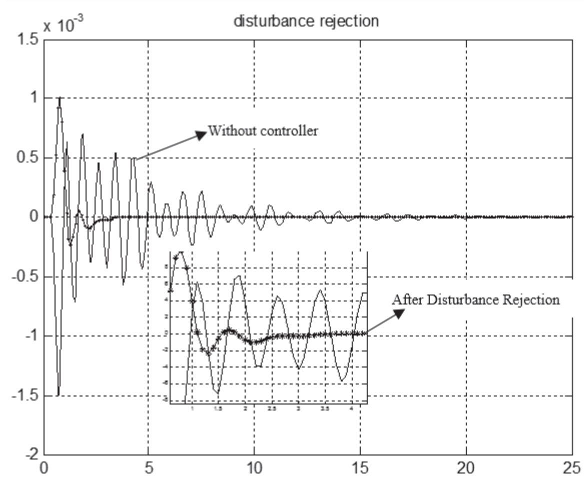

Fig.14 Simulation of result and disturbance rejection of circular plate.

A QFT robust controller is successfully implemented for vibration attenuation in the circular plate for a partial differential equation dynamic. The simulation results indicate that the controller performance is independent of the disturbance load, which is the necessary condition for any control system that can successfully suppress arbitrary external disturbances.

4 Conclusion

In this paper, the control vibrations of rectangular and circular plate have been studied using by the quantitative feedback theory method. The objective of this paper is to present quantitative design techniques for synthesizing a controller for two degrees of freedom plant in order to achieve desired closed loop specifications. Using the distributed parameter system method, the control issue has been analyzed and also reduced the amplitude vibration of the studied plate by designing the appropriate control. The method utilized based on the considered situations is stated for various factors of the control system, and it is capable of solving problems for different modes. There are no limitations for choosing spots on the plate in utilizing this method. As we can see in the outputs of the systems, the designed controller has been able the amplitude level of the plate's vibrations within the permitted range and increased the system's response. There is no need for very complex methodologies to design controllers for DPS. On the contrary, by using the approach presented in this paper, classical QFT methods based on the new bounds definition can be used to solve the problem. The paper considered a spatial distribution of the location where the actuator and the disturbances are applied and where the sensor and the control objectives are placed. From this topology new TFs, stability and performance specifications and quadratic inequalities for DPS were introduced.

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