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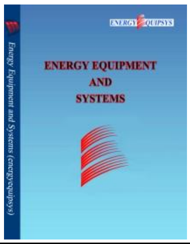


# A directional-based branches current method for transmission loss allocation in the pool-based electricity market

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**ABSTRACT**

*This paper proposes a new method for transmission loss allocation. The share of each bus in the transmission line losses is determined using transmission line loss equations with respect to bus-injected currents. Then, it is applied to the total network transmission lines. In the proposed method, comparing with other methods, a solution to remove the negative loss allocation has been introduced. This algorithm is based on the electric network relations and the injected power in various buses considering the network topology. The proposed method is studied on a typical three-bus network, and applied to the IEEE 14-bus networks. In comparison with other methods, a new solution for removing negative loss allocation is proposed.*

**Keywords:** Transmission Loss Allocation, Line Losses, Injected Power, Network Topology.

## 1. Introduction

In power networks, a small percentage of the transmission power is always lost. The main part of these losses is due to the flow of current in the ohmic resistance of transmission lines. In traditional power systems, all attempts are made to minimize the network losses in terms of costs. The overall cost of losses is added to other generation and transmission costs to form the total operation cost of the network. But in deregulated power systems, every player of the system possesses a separate legal character; therefore, it is independent in terms of income and costs. Thus, determining their share in the total network costs, including the losses, is unavoidable [1]. On the other hand, in deregulated power systems, regardless of loss optimization, another serious question is how the total cost of losses should be paid by the power market players. In the pool-based electricity market, the loss allocation helps to distinguish the share of each generation or consumption unit from the total network losses. So, the ISO could receive the cost of losses from each of the market participants and could return it to the generation companies [2].

In the markets that are based on bilateral contracts, the losses of each contract should be specified in the contract content and its support source should be determined. In spite of the high importance of loss allocation to the participants, technically and economically, due to complexity, the nonlinear nature, and high dependence of loss function on different variables, no comprehensive and precise method that can be practically employed has been presented hitherto. But due to the significance of this issue, various methods have been published in previous research

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papers, with most of them using simple assumptions. In the pro rata method [3], which is the most popular one, the loss is allocated to each generator or load with regard to their power injection to the network, rather than the total network power injection. In fact, this method does not consider the location of losses or network topology. So, a remote generator or load that certainly causes more power losses is treated the same as other near-network players.

The proportional sharing principle is based on a non-provable theorem that assumes the inflow powers are proportionally shared between the outflows power at each network bus [4]–[5]. This method uses an additional assumption: those losses of each branch allocate 50 per cent to its sending and ending nodes.

Ref [6] suggests a radial equivalent network for transmission systems so that each generator may have an individual connection to all loads and, in this way, makes it possible to allocate system loss. But total losses may not equal real system loss; moreover, it is too complicated for real power systems.

References [7–9] trace losses back from the network branch to the load. These strategies generally involve an algorithm to determine how the losses are attributed to generators or loads as one traverse through the network. Either the algorithm allows loss attribution to be specified according to a user-defined formula, or a loss-sharing formula is implicitly included.

Cooperative game theory was utilized to allocate transmission costs to wheeling transactions [10]. A method based on circuit theory has been proposed to trace power from either the seller's and/or the buyer's point of view [11]. In [12], line power flows are first unbundled into a sum of components, each corresponding to a bilateral transaction. In these schemes, the coupling terms among the components appearing in the line losses could be allocated to individual bilateral transactions. In [13], a process is used whereby individual bilateral transactions are gradually incremented along a given path of variation. Each bilateral transaction might elect to have its losses supplied by a separate slack generator.

In [9], starting from an AC load flow solution, the contributions of all generators to the flow in each circuit are evaluated and the same proportion is used to share circuit losses among them. The Z-bus loss allocation uses the total system loss formula and tries to write it in the summation form of each bus complex current injection [14].

In [15], a loss-allocation method has been introduced in bilateral markets. In order to apply the loss-allocation to contracts, this method uses the branch current circuit equations. In this paper, each contract contains a sending bus (seller) and several receiving buses (buyers). Ref [16] presents a loss-allocation method based on the circuit theories and the concept of orthogonal projection for pool-based electricity markets.

Min et al. in [17] present a new algorithm of accurate bus-wise transmission loss allocation based on path integrals. With rigorous theoretical analysis, a new path integral method is developed by integrating the partial differential of the system loss along a path reflecting the transaction strategy.

The transmission loss is allocated according to a mathematical relationship between the node power and the line power flow for a DC power network in [18]. By this proposed method, the authors can not only allocate total loss to either generator or load node, respectively, but also distribute it to both generator and load nodes conveniently.

Satyaramesh in [19] presented a usage-based methodology of transmission loss allocation in deregulated power systems under open access. This new approach calculates the portion of real power transmission loss contributions from the generators and, simultaneously, the portion of real power transmission loss allocated to the loads using their contract obligations with the generators in the open-access environment.

The loss allocation problem in multi-area transmission networks is studied in [20].

It should be noted that a suitable method should have the following properties in order to ensure a proper and fair loss allocation:

1. The allocated share to each of the buses of the network should be a real reflection of the losses of that bus.
2. The method can be performed with the load flow results.
3. The method does not allocate the negative loss to network buses.

In this paper, using the loss equations of a transmission line, with respect to bus-injected currents, the share of each bus from the mentioned transmission line losses has been determined. Then, the share of any buses from the total network losses has been acquired by applying the proposed method to the total network transmission lines. In addition, a solution to remove the negative loss allocation has been introduced. This algorithm is based on the main network relations besides the injected power in various buses, and it considers the network topology. In comparison with other methods, in this paper a new solution for removing loss allocation is proposed.

A three-bus test system is employed to show the main steps of the proposed technique. Numerical results obtained from the IEEE 14-bus test system illustrate the quality of the loss allocation determined via the proposed methodology.

## 2.Proposed Subsidized Transmission Loss Allocation

In power networks, the total loss is due to the power flows in transmission lines. In fact, the total loss is the sum of the losses of all transmission lines. Assume the power flow results of the network are available. The connected transmission line between the ith and jth buses is considered as shown in Fig. 1.

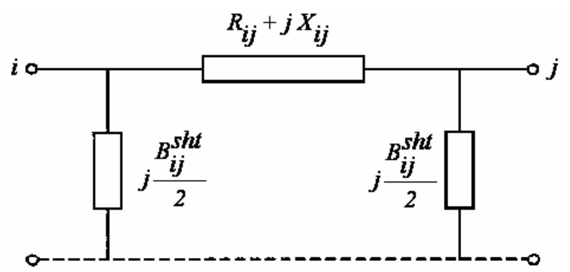


**Fig.1.** Diagram of transmission line

The equation of the line current flow with respect to the network impedance and admittance matrix, the voltage across the line, and injected currents to buses could be written as follows:

$$I_{ij} = y_{ij} \times (V_i - V_j)$$
$$I_{ij} = \Re\{I_{ij}\} + j\Im\{I_{ij}\} \quad (1)$$

In addition, the bus voltage equations with respect to the injected bus currents could be calculated as:

$$\begin{cases} V_i = \sum_{k=1}^{n} z_{ik} I_k \\ V_j = \sum_{k=1}^{n} z_{jk} I_k \end{cases} \Rightarrow V_i - V_j$$
$$= \sum_{k=1}^{n} z_{ik} I_k - \sum_{k=1}^{n} z_{jk} I_k = \sum_{k=1}^{n} (z_{ik} - z_{jk}) I_k \quad (2)$$

Inserting the above equation in (1) yields:

$$I_{ij} = \sum_{k=1}^{n} y_{ij} \times (z_{ik} - z_{jk}) I_k$$
$$= \sum_{k=1}^{n} (a_{ijk} + jb_{ijk}) \times (I_{kp} + jI_{kq}) \Rightarrow$$
$$a_{ijk} = \Re(y_{ij} \times (z_{ik} - z_{jk}))$$
$$b_{ijk} = \Im(y_{ij} \times (z_{ik} - z_{jk}))$$
$$I_{ij} = \sum_{k=1}^{n} (a_{ijk} I_{kp} - b_{ijk} I_{kq}) + j\sum_{k=1}^{n} (a_{ijk} I_{kq} + b_{ijk} I_{kp}) \quad (3)$$

The real and imaginary parts of the line current could be written as (4):

$$\Re\{I_{ij}\} = \sum_{k=1}^{n} (a_{ijk} I_{kp} - b_{ijk} I_{kq})$$
$$\Im\{I_{ij}\} = \sum_{k=1}^{n} (a_{ijk} I_{kq} + b_{ijk} I_{kp}) \quad (4)$$

The share of bus k from the line admittance current is as follows:

$$\Re\{I_{ij}\}^k = a_{ijk} I_{kp} - b_{ijk} I_{kq}$$
$$\Im\{I_{ij}\}^k = a_{ijk} I_{kq} + b_{ijk} I_{kp} \quad (5)$$

In addition, the transmission line loss can be written as:

$$Ploss_{ij} = r_{ij} \times |I_{ij}|^2$$
$$I_{ij} = \Re\{I_{ij}\} + j\Im\{I_{ij}\} \quad (6)$$

According to the real and imaginary parts of the line current, we have:

$$|I_{ij}|^2 = |\Re\{I_{ij}\}|^2 + |\Im\{I_{ij}\}|^2$$
$$|\Re\{I_{ij}\}|^2 = \left|\sum_{k=1}^{n} (a_{ijk} I_{kp} - b_{ijk} I_{kq})\right|^2$$
$$= \sum_{k=1}^{n} (a_{ijk} I_{kp} - b_{ijk} I_{kq})$$
$$\times \sum_{k=1}^{n} (a_{ijk} I_{kp} - b_{ijk} I_{kq})$$
$$= \sum_{k=1}^{n} \left\{ \begin{array}{l} (a_{ijk} I_{kp} - b_{ijk} I_{kq})^2 \\ + (a_{ijk} I_{kp} - b_{ijk} I_{kq}) \\ \times \sum_{m=1 \neq k}^{n} (a_{ijm} I_{mp} - b_{ijm} I_{mq}) \end{array} \right\}$$
$$|\Im\{I_{ij}\}|^2 = \left|\sum_{k=1}^{n} (b_{ijk} I_{kp} + a_{ijk} I_{kq})\right|^2$$
$$= \sum_{k=1}^{n} (b_{ijk} I_{kp} + a_{ijk} I_{kq}) \times \sum_{k=1}^{n} (b_{ijk} I_{kp} + a_{ijk} I_{kq})$$
$$= \sum_{k=1}^{n} \left\{ \begin{array}{l} (b_{ijk} I_{kp} + a_{ijk} I_{kq})^2 \\ + (b_{ijk} I_{kp} + a_{ijk} I_{kq}) \\ \times \sum_{m=1 \neq k}^{n} (b_{ijm} I_{mp} + a_{ijm} I_{mq}) \end{array} \right\} \quad (7)$$

$$\left|\Re\{I_{ij}\}\right|^2 = \left|\sum_{k=1}^{n}\left(a_{ijk}I_{kp} - b_{ijk}I_{kq}\right)\right|^2$$

$$= \sum_{k=1}^{n}\left(a_{ijk}I_{kp} - b_{ijk}I_{kq}\right)\times\sum_{k=1}^{n}\left(a_{ijk}I_{kp} - b_{ijk}I_{kq}\right)$$

$$= \sum_{k=1}^{n}\left\{\begin{array}{l}\left(a_{ijk}I_{kp} - b_{ijk}I_{kq}\right)^2 \\ +\left(a_{ijk}I_{kp} - b_{ijk}I_{kq}\right) \\ \times\sum_{m=1\neq k}^{n}\left(a_{ijm}I_{mp} - b_{ijm}I_{mq}\right)\end{array}\right\}$$

$$\left|\Im\{I_{ij}\}\right|^2 = \left|\sum_{k=1}^{n}\left(b_{ijk}I_{kp} + a_{ijk}I_{kq}\right)\right|^2$$

$$= \sum_{k=1}^{n}\left(b_{ijk}I_{kp} + a_{ijk}I_{kq}\right)\times\sum_{k=1}^{n}\left(b_{ijk}I_{kp} + a_{ijk}I_{kq}\right)$$

$$= \sum_{k=1}^{n}\left\{\begin{array}{l}\left(b_{ijk}I_{kp} + a_{ijk}I_{kq}\right)^2 \\ +\left(b_{ijk}I_{kp} + a_{ijk}I_{kq}\right) \\ \times\sum_{m=1\neq k}^{n}\left(b_{ijm}I_{mp} + a_{ijm}I_{mq}\right)\end{array}\right\}$$

$$\left|\Im\{I_{ij}\}\right|^2 = \left|\sum_{k=1}^{n}\left(b_{ijk}I_{kp} + a_{ijk}I_{kq}\right)\right|^2$$

$$= \sum_{k=1}^{n}\left(b_{ijk}I_{kp} + a_{ijk}I_{kq}\right)\times\sum_{k=1}^{n}\left(b_{ijk}I_{kp} + a_{ijk}I_{kq}\right)$$

$$= \sum_{k=1}^{n}\left\{\begin{array}{l}\left(b_{ijk}I_{kp} + a_{ijk}I_{kq}\right)^2 \\ +\left(b_{ijk}I_{kp} + a_{ijk}I_{kq}\right) \\ \times\sum_{m=1\neq k}^{n}\left(b_{ijm}I_{mp} + a_{ijm}I_{mq}\right)\end{array}\right\}$$

So, Eq. (6) can be written as:

$$Ploss_{ij} = r_{ij}\times\left|I_{ij}\right|^2$$

$$= r_{ij}\left|\Re\{I_{ij}\}\right|^2 + r_{ij}\left|\Im\{I_{ij}\}\right|^2$$

$$= r_{ij}\times\sum_{k=1}^{n}\left\{\begin{array}{l}\left(a_{ijk}I_{kp} - b_{ijk}I_{kq}\right)^2 \\ +\left(a_{ijk}I_{kp} - b_{ijk}I_{kq}\right) \\ \times\sum_{m=1\neq k}^{n}\left(a_{ijm}I_{mp} - b_{ijm}I_{mq}\right)+ \\ \left(b_{ijk}I_{kp} + a_{ijk}I_{kq}\right)^2 + \left(b_{ijk}I_{kp} + a_{ijk}I_{kq}\right)\times\sum_{m=1\neq k}^{n}\left(b_{ijm}I_{mp} + a_{ijm}I_{mq}\right)\end{array}\right\} \tag{8}$$

Furthermore, for bus k, we have:

$$S_k = V_k I_k^* \Rightarrow I_k = \frac{P_k - jQ_k}{V_k^*}$$

$$= \frac{\left(P_k - jQ_k\right)\times\left(Cos\delta_k + jSin\delta_k\right)}{\left|V_k\right|}$$

$$I_k = \frac{P_k Cos\delta_k + Q_k Sin\delta_k}{\left|V_k\right|} + j\frac{P_k Sin\delta_k - Q_k Cos\delta_k}{\left|V_k\right|}$$

$$I_{kp} = \frac{P_k Cos\delta_k + Q_k Sin\delta_k}{\left|V_k\right|}$$

$$I_{kq} = \frac{P_k Sin\delta_k - Q_k Cos\delta_k}{\left|V_k\right|} \tag{9}$$

Therefore, the share of bus k from the ijth transmission line loss can be formulated as follows:

$$Ploss_{ij}^{\ k} = r_{ij}\times\left\{\begin{array}{l}\left(a_{ijk}I_{kp} - b_{ijk}I_{kq}\right)^2 \\ +\left(a_{ijk}I_{kp} - b_{ijk}I_{kq}\right) \\ \times\sum_{m=1\neq k}^{n}\left(a_{ijm}I_{mp} - b_{ijm}I_{mq}\right)+ \\ \left(b_{ijk}I_{kp} + a_{ijk}I_{kq}\right)^2 \\ +\left(b_{ijk}I_{kp} + a_{ijk}I_{kq}\right) \\ \times\sum_{m=1\neq k}^{n}\left(b_{ijm}I_{mp} + a_{ijm}I_{mq}\right)\end{array}\right\} \tag{10}$$

According to (10), some buses may have negative loss allocation. In the next section, a solution to remove the negative loss allocation is proposed.

## 3.Proposed Unsubsidized Transmission Loss Allocation Based on the Branches Current Direction

In the previous section, according to the network topology, and the real and imaginary parts of the injected currents to buses, a new method for loss allocation has been proposed. In the proposed method, some buses might have negative loss allocation. The negative loss allocation to some buses is due to the flow of their currents in the opposite direction with respect to the dominant flow in some transmission lines. In fact, in an appointed transmission line, the injected current to a bus may flow in the opposite direction with respect to the total injected current resulting from the other buses. So, the loss-allocation equation of these buses has a negative result. Here, a solution to remove negative loss allocation is proposed. In this solution, the decreasing role of such buses that reduce the line losses is considered. According to Eqs.

(4) and (5), the loss-allocation solution to remove the negative values is as follows:

$$if \quad \Re\{I_{ij}\} \times \Re\{I_{ij}\}^k \leq 0 \quad Ploss_{i-j}^k = 0 \qquad (11)$$

The above equation shows that if the real part of the current in line i-j, contributed by bus k, is in the opposite direction with respect to the real part of the current contributed by other buses, the allocated loss to the bus k will be zero. However, if the real part of the current in line i-j, contributed by bus k, is in the same direction with respect to the real part of the current contributed by other buses, the modified real part of line i-j current should be expressed with respect to the real part of the currents contributed by buses that are in the same direction. In fact, the real parts of such currents that are in the opposite direction with respect to the real part of the line i-j current are negligible (see Eq. (12)).

$$if \quad \Re\{I_{ij}\} \times \Re\{I_{ij}\}^k \geq 0$$
$$\Rightarrow \Re\{I_{ij}\}_{\text{modified}} = \sum_{k=1}^{N_d} \Re\{I_{ij}\}^k \qquad (12)$$

So, the transmission loss due to the real part of the bus currents in line i-j can be written as follows:

$$Ploss_{i-j\Re\{I_{ij}\}_{\text{modified}}} = r_{ij} \times \left|\Re\{I_{ij}\}_{\text{modified}}\right|^2$$
$$= r_{ij} \times \left|\sum_{k=1}^{N_d} \Re\{I_{ij}\}^k\right|^2 =$$
$$r_{ij} \times \left(\sum_{k=1}^{N_d} \Re\{I_{ij}\}^k\right) \times \left(\sum_{k=1}^{N_d} \Re\{I_{ij}\}^k\right)$$
$$= r_{ij} \times \sum_{k=1}^{N_d} \left( \begin{array}{c} \left(\Re\{I_{ij}\}^k\right)^2 \\ + \Re\{I_{ij}\}^k \times \sum_{m=1\neq k}^{N_d} \Re\{I_{ij}\}^m \end{array} \right) \qquad (13)$$

Therefore, the contribution of bus k in the losses of line i-j due to the real part of currents is as follows:

$$Ploss^k{}_{i-j\Re\{I_{ij}\}_{\text{modified}}} = \left(\Re\{I_{ij}\}^k\right)^2$$
$$+ \Re\{I_{ij}\}^k \times \sum_{m=1\neq k}^{N_d} \Re\{I_{ij}\}^m \qquad (14)$$

In addition, the above equations can be obtained in a similar way for the imaginary part of the line i-j current. So, the contribution of bus k in the losses of line i-j due to the imaginary part of the current can be calculated as Eq. (15):

$$Ploss^k{}_{i-j\Im\{I_{ij}\}_{\text{modified}}} = \left(\Im\{I_{ij}\}^k\right)^2$$
$$+ \Im\{I_{ij}\}^k \times \sum_{m=1\neq k}^{N_d} \Im\{I_{ij}\}^m \qquad (15)$$

Therefore, the allocated loss value to the bus k is as follows:

$$Ploss^k{}_{i-j\{I_{ij}\}_{\text{modified}}} = Ploss^k{}_{i-j\Re\{I_{ij}\}_{\text{modified}}}$$
$$+ Ploss^k{}_{i-j\Im\{I_{ij}\}_{\text{modified}}} \qquad (16)$$

According to Eq. (12), we have:

$$Ploss_{i-j\{I_{ij}\}_{\text{modified}}} \neq Ploss_{i-j} \qquad (17)$$

Finally, by normalizing the allocated loss values, the normalized contribution of bus k can be calculated as follows:

$$Ploss^k{}_{(i-j)_{normalized}} = \frac{Ploss^k{}_{i-j\{I_{ij}\}_{\text{modified}}}}{\sum_{k=1}^{N_d} Ploss^k{}_{i-j\{I_{ij}\}_{\text{modified}}}} \times Ploss_{i-j} \qquad (18)$$

## 4.Numerical Results

A simple example without fixed losses is selected to show the application of the proposed allocation method. Figure 2 shows a three-bus system and Table 1 shows its transmission line data, which is used for this purpose. A generator (located at bus 1) supplies the power demand located at buses 2 and 3.

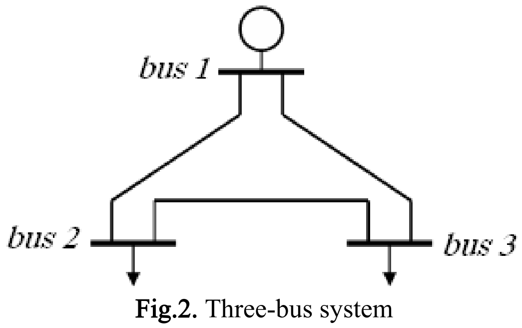


**Fig.2.** Three-bus system

Table 2 summarizes the power flow solution using the Newton–Raphson method. Columns 2, 3, 4, 5, 6, and 7 show bus magnitude voltages, bus angle voltages, active generated powers, reactive generated powers, active demand powers, and reactive demand powers, respectively.

Loss allocation to each bus of the typical three-bus network is illustrated in Table 3. As shown in Fig. 2, bus 3 injects the current in the opposite direction with respect to the resultant current of the network in line 2–3.

**Table 1.**Transmission-line data

| Line from Bus to Bus | R (%) | X (%) | B (%) |
|---|---|---|---|
| 1–2 | 0.0200 | 0.040 | 0.025 |
| 1–3 | 0.0100 | 0.030 | 0.025 |
| 2–3 | 0.0125 | 0.025 | 0.025 |

**Table 2.** Three-bus system power flow results

| Bus No. | Vol. | Ang. | PG (MW) | QG (MVAr) | PD (MW) | QD (MVAr) |
|---|---|---|---|---|---|---|
| 1 | 1.050 | 0.0000 | 409.224 | 172.963 | 0.000 | 0.000 |
| 2 | 0.984 | -3.539 | 0.0000 | 0 | 256.6 | 110.2 |
| 3 | 1.003 | -2.892 | 0.0000 | 0 | 138.6 | 45.20 |
| Total Sum | | | 409.224 | 172.963 | 395.2 | 155.40 |

**Table 3.** The allocated loss of transmission lines to each bus of the typical three-bus network

| Line | Line loss (MW) | Share bus 1 | Share bus 2 | Share bus 3 |
|---|---|---|---|---|
| 1–2 | 8.3400 | 5.0339 | 3.2328 | 0.0663 |
| 1–3 | 4.8943 | 3.5211 | 0.3369 | 1.0363 |
| 2–3 | 0.7946 | 0.1896 | 1.0811 | -0.4760 |

So, the allocated loss of the line 2–3 to bus 3 has a negative value. The negative allocated loss to bus 3 is due to its decreasing role in reduction of the network losses. However, if this bus increases the network losses, it receives the positive loss allocation cost.

For more descriptions, the active load of all buses has increased. For each case, the loss allocation by using the proposed method has been done. The variations of allocated loss to each bus and the network line losses due to the load increase in bus 2 from 0 MW to 1,000 MW have been illustrated in Figs. 3 and 4, respectively.

As shown in Figs. 3 and 4, by increasing the load in bus 2, the power flows in lines and, proportionally, the network line losses have increased. Thus, the allocated losses to buses 1 and 2 from the line losses have increased. By increasing the load of bus 2, the power flow in line 3–2 from bus 3 toward bus 2 has increased. Therefore, the load of bus 3 has a decreasing role in the flowing power of line 3-2. So, the share of bus 3 in the allocated loss should be constant, as has been yielded by the proposed method. Table 4 shows the allocated losses to the buses of the typical three-bus network using the proposed method.

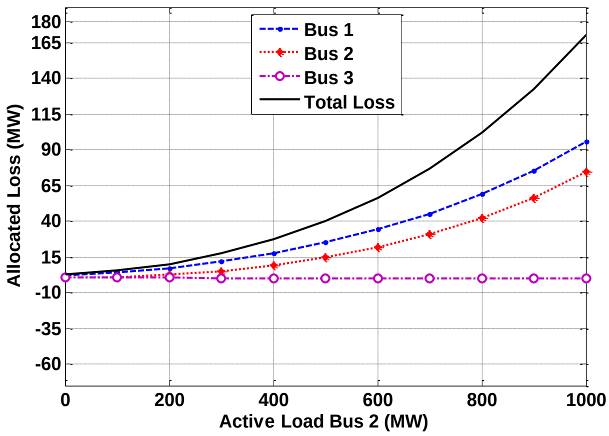


**Fig.3.** Variation of allocated loss to each bus due to the load increase of bus 2

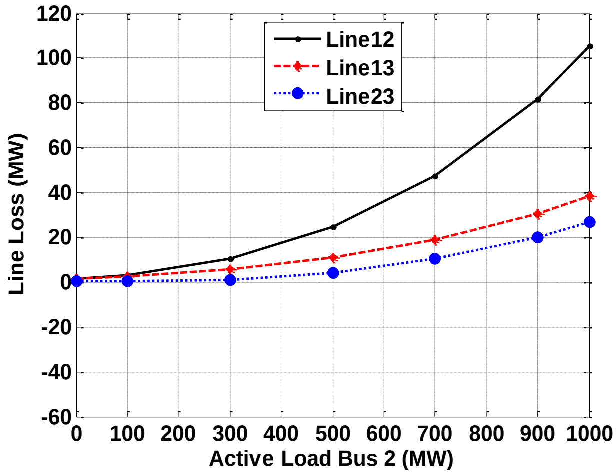


**Fig.4.** Variation of the network line losses due to the load increase of bus 2

**Table 4.** The allocated losses to the buses of a typical three-bus network

| Line | Line loss (MW) | Share bus 1 | Share bus 2 | Share bus 3 |
|---|---|---|---|---|
| 1–2 | 8.3400 | 5.0339 | 3.2328 | 0.0663 |
| 1–3 | 4.8943 | 3.5211 | 0.3369 | 1.0363 |
| 2–3 | 0.7946 | 0.1151 | 0.6795 | 0 |

As illustrated in row 3 of Table 4, the allocated loss to bus 3 is zero. This zero allocated loss to bus 3 is due to the opposite direction of the bus 3 injected current to line 3-2 with respect to the current contributed by other buses. Although the share of bus 3 is zero in the loss-allocation process, due to the usage of the network to transmit the power in line 3–2, it should pay the transmission service costs.

The proposed method has been tested on a set of networks of different sizes, and it has been compared to some of most well-known alternative algorithms described in the literature. In this paper, the IEEE 14-bus is used to show the result of the proposed method in comparison with the pro-rata method (PR) based on complex power injection and the incremental transmission loss method (ITL) as the most referenced loss-allocation methods and Z-bus method.

As can be seen in Fig. 5, the IEEE 14-bus system has five controlled buses including two generator buses, in which bus 1 is considered the slack bus. According to the power flow results, bus 1 provides 13.54 MW, which should be divided between market players.

Table 5 shows the results of the proposed method adjacent to the other methods.

The proposed method, similar to the impedance matrix method, emphasizes the location of buses and network topology. Bus 1, which provides about 85 per cent of the total generation, always has the highest contribution to the loss allocation in all the methods. In addition, bus 3, which comprises 36 per cent of system load, after bus 1, receives the highest loss allocation from all methods. Bus 2, because of its appropriate location in the network, has the least loss allocation value.

In order to analyse the effect of distributed generation and consumption, a 100 MW generator is added to bus 8.

Table 6 shows the main variation of transmission system losses, which has led to a 50 per cent decrease in total network losses. Therefore, the allocated loss to the buses has changed and the share of bus 1 decreased from 62 per cent in the previous state to 42 per cent in this condition. But, due to the high distance of bus 3 from the generation centre, its contribution to the allocated loss has no main variations.

By varying the generation of bus 8 from zero to 300 MW, as illustrated in Fig. 6, the network losses first decrease and then increase. In addition, the allocated loss, which first decreases and then increases, shows the

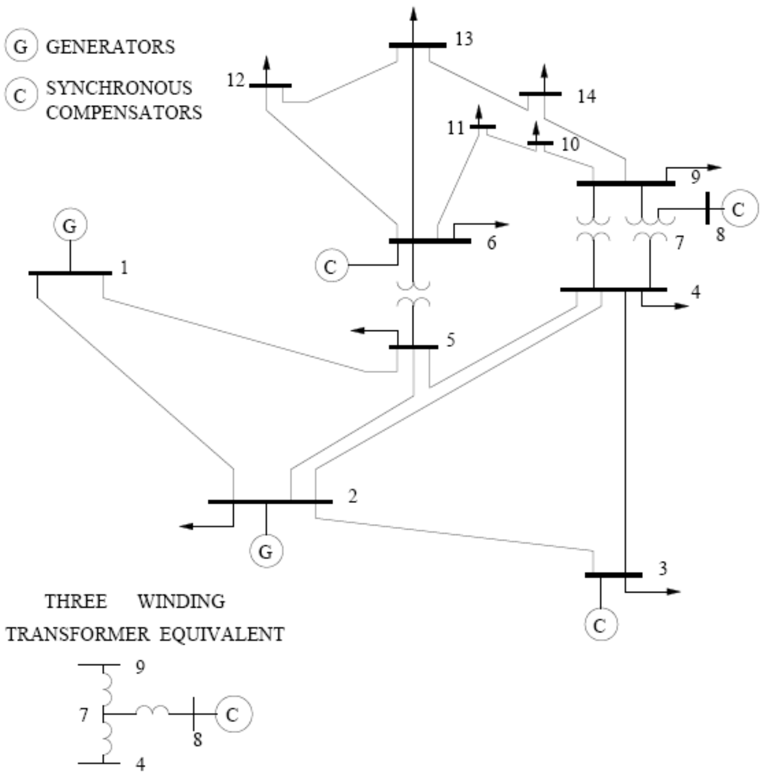


**Fig.5.** 14-bus IEEE test system

**Table 5.** Proposed method vis-à-vis the other methods

| Bus. No. | Z-bus Method | ITL Method | Pro-Rata Method | Proposed Method |
|---|---|---|---|---|
| 1 | 7.8000 | 6.14 | 6.46 | 8.5202 |
| 2 | 0.1550 | 0.96 | 0.50 | 0.373 |
| 3 | 2.6980 | 2.92 | 2.62 | 2.2054 |
| 4 | 0.9056 | 1.26 | 1.36 | 0.8369 |
| 5 | 0.0903 | 0.18 | 0.22 | 0.0359 |
| 6 | 0.6783 | 0.32 | 0.32 | 0.0899 |
| 7 | 0.0000 | 0.00 | 0.00 | 0.0000 |
| 8 | 0.0258 | 0.00 | 0.00 | -0.0644 |
| 9 | 0.4484 | 0.68 | 0.82 | 0.4740 |
| 10 | 0.1690 | 0.20 | 0.24 | 0.1141 |
| 11 | 0.0620 | 0.08 | 0.10 | 0.0221 |
| 12 | 0.1385 | 0.18 | 0.16 | 0.0542 |
| 13 | 0.3412 | 0.32 | 0.38 | 0.3310 |
| 14 | 0.4689 | 0.32 | 0.42 | 0.5549 |
| Total Sum | 13.54 | 13.54 | 13.54 | 13.54 |

**Table 6.** Results of proposed method vis-à-vis the other methods, adding a 100 MW generator to bus 8 of the IEEE 14-bus network

| Bus. No. | Z-bus Method | ITL Method | Pro-Rata Method | Proposed Method |
|---|---|---|---|---|
| 1 | 2.32 | 1.80 | 1.60 | 2.5838 |
| 2 | 0.08 | 0.56 | 0.24 | 0.1588 |
| 3 | 2.48 | 1.58 | 1.20 | 2.3168 |
| 4 | 0.26 | 0.50 | 0.62 | 0.5281 |
| 5 | 0.02 | 0.08 | 0.10 | 0.0331 |
| 6 | 0.46 | 0.20 | 0.14 | 0.1418 |
| 7 | 0.00 | 0.00 | 0.00 | 0.000 |
| 8 | -0.18 | 0.88 | 1.28 | -0.8595 |
| 9 | 0.06 | 0.14 | 0.38 | 0.2956 |
| 10 | 0.06 | 0.06 | 0.12 | 0.1055 |
| 11 | 0.02 | 0.04 | 0.04 | 0.0221 |
| 12 | 0.10 | 0.10 | 0.08 | 0.0664 |
| 13 | 0.22 | 0.16 | 0.18 | 0.2605 |
| 14 | 0.30 | 0.10 | 0.20 | 0.3850 |
| Total Sum | 6.03 | 6.03 | 6.03 | 6.03 |

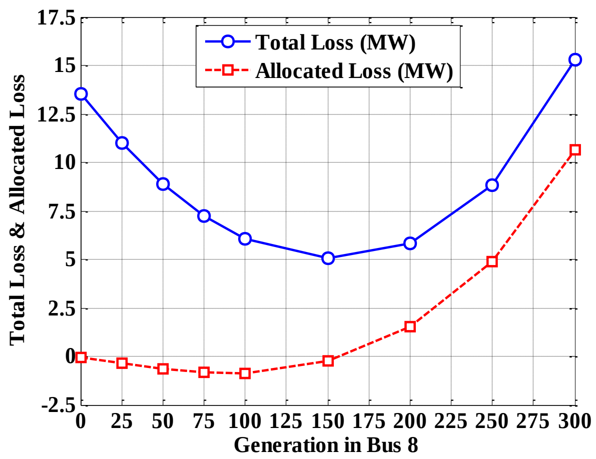


**Fig.6.** The total network losses and the allocated loss to bus 8 by changing its generation from zero to 300 MW

proposed method has considered the network topology and the injected currents to the network buses.

The loss allocation results for the IEEE 14-bus system using the modified method adjacent to the other methods have been shown in Table 7.

As can be seen, bus 8 has negative loss allocation in Table 7 with 0.0296 MW of total losses using the modified method. The modified method, in addition to considering the decreasing role of the generators in the network, considers the location of generators in the network and removes the negative loss allocation values.

In real power systems, some loads with a low power factor cause an increase in the network losses and decrease transmission line capacity. The proposed method can consider these conditions. Assume the reactive load of bus 14 increases from 5 MVAr to 50 MVAr. The effect of this increment on the connected line, line 13–14, and the allocated loss to bus 14 has been studied. As can be seen in Fig. 7, the losses of line 13–14 and the allocated loss to bus 14 is increased with an increase in the reactive load of bus 14. This study shows the modified method can consider the effect of loads with a low power factor in the increment of losses in network.

## 5.Conclusion

In this paper, a new method for loss allocation is proposed. First, using the relations of a transmission line loss with respect to bus-injected currents, the contribution of each bus to the mentioned transmission line losses is determined. Then, this method is applied to

**Table 7.** Loss-allocation results using the modified method vis-à-vis the other methods

| Bus No. | Z-bus Method | ITL Method | Pro-Rata Method | Proposed Modified Method |
|---|---|---|---|---|
| 1 | 7.800 | 6.14 | 6.46 | 7.6822 |
| 2 | 0.155 | 0.96 | 0.50 | 0.2857 |
| 3 | 2.698 | 2.92 | 2.62 | 2.1428 |
| 4 | 09056 | 1.26 | 1.36 | 0.8433 |
| 5 | 0.0903 | 0.18 | 0.22 | 0.0435 |
| 6 | 06783 | 0.32 | 0.32 | 0.0725 |
| 7 | 0.000 | 0.000 | 0.0000 | 0.0000 |
| 8 | 0.0258 | 0.000 | 0.0000 | 0.0296 |
| 9 | 0.4484 | 0.68 | 0.82 | 0.7053 |
| 10 | 0.1690 | 0.20 | 0.24 | 0.0921 |
| 11 | 0.0620 | 0.08 | 0.10 | 0.2599 |
| 12 | 0.1385 | 0.18 | 0.16 | 0.2374 |
| 13 | 0.3412 | 0.32 | 0.38 | 0.8594 |
| 14 | 0.4689 | 0.32 | 0.42 | 0.2864 |
| Total Sum | 13.54 | 13.54 | 13.54 | 13.54 |

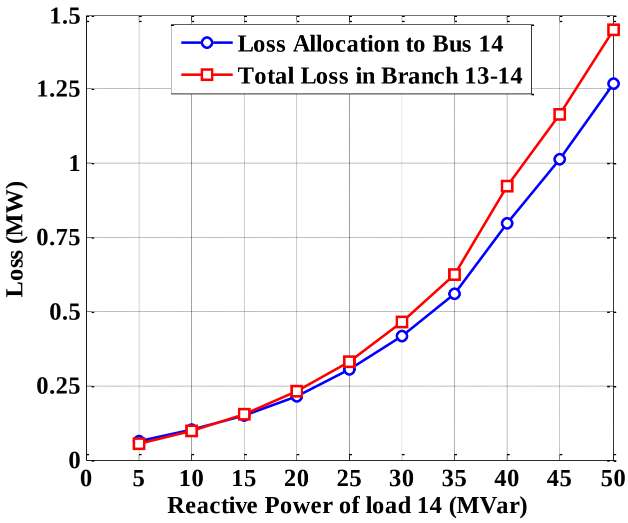


**Fig.7.** Losses of line 13–14 and the allocated loss to bus 14 by changing the reactive load of bus 14

the total network transmission lines. In the proposed method, a solution to remove the negative loss allocation is introduced. This algorithm is based on the main network relations and the injected power in various buses.

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