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Strategic Technology Adoption under Technological Uncertainty[1]

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Abstract

This paper studies technology adoption in a duopoly where the unbiased technological change improves production efficiency. Technological progress is exogenous and modeled as a jump process with a drift. There is always a Markov perfect equilibrium in which the firm with more efficient technology never preempts its rival. Also, a class of equilibria may exist that lead to a smaller industry surplus. In these equilibria either of the firms may preempt its rival in a set of technology efficiency values. The first investment does not necessarily happen at the boundary of this set due to the discrete nature of the technology progress. The set shrinks and eventually disappears when the difference between firms' efficiencies increases.

Keywords: uncertainty, strategic investment, technology adoption, investment timing, preemption.

1- Introduction

This paper studies the effect of competition between two firms on timing of a new technology adoption. I investigate whether firms with different production efficiencies adopt a new technology when it is optimal, or whether threat of being preempted makes them adopt a new technology inefficiently early. It is assumed that the efficiency of cutting edge

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technology improves over time, and there is uncertainty about when a breakthrough in the technology advancement will happen and how large it will be. This assumption for evolution of technology is used by Boyarchenko and Levendorskii [3] to study technology adoption by a single firm. I model technology process in a similar way to study the effect of competition between two firms.

A related work is a paper by Huisman and Kort [11]. This paper analyzes a new technology adoption game between two firms. [11] finds that three types of equilibrium outcomes are possible: preemption, war of attrition and simultaneous adoption. Only one of the mentioned types are possible according to [11], depending on the expected time of arrival of the superior technology.

Only two possible technology levels are considered in [11]'s model, described as one technology being superior to the other. The superior technology will not be available until a random point in time. In my model it is assumed that cutting edge technology improves over time and unpredictable breakthroughs may arrive at any time. I believe this is a more realistic assumption than considering only two technology levels, as also assumed by [3]. One possible outcome of competition is preemptive behavior. Such a behavior results in inefficiently early investment due to the threat of being preempted. I investigate whether or not preemptive behavior is possible in adoption of a new technology. I assume that the efficiencies of the firms are different. Intuitively it is expected that the firm with less efficient technology adopts a new technology first. Because the firm with less efficient technology would gain more than the firm with more efficient technology from adoption of a new technology.

In contrast to [11], I find that two types of Markov perfect equilibrium are possible based on the assumptions of my model. In the first type, the firm with less efficient technology adopts first. In this type of equilibrium, the firm with more efficient technology has no incentive to preempt. Hence, the firm that adopts first does not choose the adoption time under the threat of being preempted. In the second type, the preemptive behavior exists. The threat of being preempted makes firms adopt inefficiently earlier when compared to the first type of equilibrium. I use the terms sequential investment and preemptive investment to refer to these two types of equilibria.

The first type of equilibrium exists for all possible values of parameters in my model.[1] However, the second type does not always exist. It exists only when the efficiencies of the initial technologies of the firms are close enough and the loss caused by adoption of a new technology by the rival is large enough. This suggests the possibility of both of these equilibria for certain parameter values. This result is different from [14], [11] and [4]. They find that only one type of equilibrium can exist for any specific parameter values. However, multiplicity of equilibria is not unique to my study. [18], [13] and [7] also find that it is possible to have multiple types of equilibria, one with simultaneous investment and the other one with preemption. Because I allow asymmetry in efficiency of initial technologies, instead of simultaneous investments in the aforementioned works, the first type of equilibrium involves sequential investment.

Hoppe and Lehmann-Grube[9] study a continuous time innovation timing game between two firms. They offer an algorithm to analyze a set of intractable problems. They point out the possibility of multiple equilibria when the payoff function for leader is not continuous. I also find possibility of multiple equilibria, but not because of discontinuous leader's payoff.

I use the real options approach to solve the problem of optimal timing of the new technology adoption. Here, the adoption of new technology is viewed as a call option that can be exercised at any time. This approach is similar to [6], [1] and [3]. The aforementioned works study adoption of a new technology by a single firm. This study considers competition between two firms in adopting new technology which is an improvement to the mentioned works. Progress of technology is modeled similar to one of my previous works [5]. The technology process consists of jumps and a drift. The size of jumps is distributed exponentially. The jump process is characterized by two parameters: the average size of jumps and the intensity of jumps.

1-1- Related Literature

Investment timing games have been extensively studied in the literature. Two famous early works on investment games are by Reinganum [5], and

1- This result is different from [14] where non-preemptive equilibrium does not always exist.

Fudenberg and Tirole [7]. In the past decade, researchers have also studied the effects of uncertainty on strategic investment by competing firms. Related examples of such works are Huisman and Kort [11], Mason and Weeds [13], Pawlina and Kort [14], Boyarchenko and Levendorskii [4], Thijssen [16], Weeds [18], Thijssen, Huisman, and Kort [17]. Huisman [10] collects some game theoretic problems for more than one firm or more than one technology in his book.

In a discrete time environment, where action by one player is optimal but simultaneous action is not, a symmetric mixed strategy equilibrium exists. An example is a game called "grab-the-dollar game," as discussed in [7]. However, in a continuous time setting, such equilibrium fails to exist. To address this problem, in [7], the strategy space is extended to allow a sequence of atoms as part of the player's strategy. Each player chooses the intensity of this sequence. Thijssen, Huisman, and Kort [17] use a similar method. In contrast, [18] uses Markov strategies, and [16] uses private signals for players to coordinate their actions and defines Markov correlated equilibrium. In this study, I define a Markov strategy that consists of the intensity of a sequence of atoms. The idea of using a sequence of atoms is previously used to define closed loop strategies in [7], [17] and [4].

In [14], [16], [13], [18] and [4], the time the first mover invests does not affect the value of the second mover. This is because the impact of the investment of the first mover on the profit flow of the second mover is fixed and does not depend on when the first mover invests. This makes the value of the second mover independent of when the first mover invests. In [11], the value of the follower depends on whether the other firm has adopted the low efficiency technology or the superior technology. However, because the level of technology does not improve over time, the value of the follower is independent of the threshold at which the leader adopts. In their model, the uncertainty in profit flow is due to factors other than efficiency of cutting edge technology.

In contrast to the works just mentioned, in my study the follower's payoff stream is affected by the efficiency of the technology that the leader adopts. As a result, the leader's strategy for the timing of the investment affects the value of the follower. Preemption zone is defined to be the set of all the efficiency levels where both firms strictly prefer to move first. I find that the sequential investment is always an equilibrium, regardless of the existence of the preemption zone. This result is different from some other works that study the timing of investment, such as [14]. In their work, the sequential investment cannot happen if the preemption zone exists. In [14], investment outside the preemption zone is not optimal for the leader, given that the rival invests at the follower's investment threshold. Thus, the leader would invest inside the preemption zone, if the rival would invest as a follower. However, it is not possible to have the sequential investment as an equilibrium if the leader invests inside the preemption zone. Because this would create incentive for the other firm to preempt and the equilibrium would collapse.

In contrast, in my study, the optimal investment threshold for the less efficient firm is outside the preemption zone, given that the firm with the more efficient technology invests as a follower. In other words, the intersection of the stopping set for the less efficient firm and the preemption zone is empty. This implies that the more efficient firm has no incentive to be the first mover and sequential investment remains an equilibrium.

This study is also different from [14], [11], [13] and [18] in another way. In the mentioned studies, the time or state variable changes continuously. Hence, the value of the state variable reaches the stopping sets at their boundaries.

In contrast, in this study, the uncertain change in efficiency of the cutting edge technology is modeled as a jump process. It can be the case that the value of the state variable jumps over the preemption zone. It is also possible that the efficiency of the new technology jumps into the preemption zone without continuously crossing its boundaries. Therefore, even before reaching the preemption zone, the threat of being preempted in the next moment exists. Boyarchenko and Levendorskii [4] study a preemption game under Levy uncertainty and arrive at this finding as well. In this study, because of the jumps, I find that in the preemptive investment the less efficient firm may invest prior to the time the preemption zone is reached in order to guarantee the adoption as a leader.

The remaining of this paper is organized as follows. In section 2, I provide a brief description of the model. In addition, the conditions for existence of the preemption zone is investigated. In section

3 the strategy space is defined and equilibria of the game are characterized. In section

5, the value functions and the optimal adoption thresholds are derived.

2- The Model

The players of the game analyzed in this paper are two risk-neutral firms, indexed by $i \in \{1,2\}$. Each firm has a single irreversible opportunity to upgrade to a more efficient technology in its unlimited lifetime.[1] The efficiency of the technologies of the firms are different before adoption. This is the only source of asymmetry between the firms. The efficiency of a technology is assumed to be measured by cost of production. Let c_i^0 denote the initial production cost of firm i. Adoption of a new technology is assumed to improve the efficiency which would reduces the production cost.[2] The instantaneous payoff function for firm i is assumed to be

$$\Pi(c_i, c_j) = \frac{A - c_i + \gamma c_j}{B} \tag{1}$$

where c_i denotes the production cost of firm i and c_j denotes the production cost of its rival, firm j. The parameters **A** and **B** are positive constants. The parameter $\gamma \in [0,1]$, controls how the profit is affected by the rival's production cost.[3] When a firm adopts a more efficient technology, its instantaneous payoff increases while its rival undergoes a loss.

The sunk cost of adoption of a new technology is $I > 0$ which is assumed to be identical for both firms. The sunk cost includes both the price the firm pays for a new technology as well as any other adjustment costs it faces while switching to new technology is underway. I assume $I < \frac{c_i^0}{rB}$, otherwise investment is never profitable. The parameter r is the discount rate. When firm i invests, it adopts the best available technology.

The efficiency of the cutting edge technology improves over time. Let C_t denote the production cost of the cutting edge technology at time t. The value of the random variable C_t is exogenous to the firm and uncertain.

1- Outcomes of the strategic adoption with multiple future adoption opportunities can be obtained in a single investment setting, as Pawlina and Kort [14] discuss in their paper, unless market structure allows for complex interactions, such as two consecutive preemptive investments by a single firm.

2- Here I have assumed that adoption of a better technology reduces production cost. However, the results cover cases where the new technology improves other factors that linearly improve the reduced form profit function.

3- In general, if the demand and competition in the product market are such that the reduced form profit (instantaneous payoff) has a functional form other than (1), an approximation would yield (1).

Uncertainty is modeled by a family of strong Markov process on a probability space (Ω, f, P_c). The filtration $(\mathscr{F}_t)_{t\geq 0}$ is right continuous and $\mathscr{F}_e$ includes all the P_c-null sets of $\mathscr{F}$. The state space is $\mathbb{R}_+$. The random variable C_t is such that $\ln(C_t)$ follows a stochastic process with negative jumps. It is the solution to the following stochastic differential equation under an equivalent Martingale measure (EMM) given by the market:

$$d \ln (C_t) = \mu dt + dJ_{\eta,\lambda,t} \qquad (2)$$

Here, μ is the negative drift. The jump process, $J_{\eta,\lambda,t}$, is a compound Poisson process with Levy density (density of jumps) $F(dy) = \eta\lambda e^{\lambda y} 1(-\infty.0)(y)dy$. The jump process is characterized by two parameters: η and λ. Here, η corresponds to the intensity of the jumps, while $\frac{1}{\lambda}$ is the average jump size in (2). The drift represents gradual improvements of existing technology, while the compound Poisson process relates to innovations that cause a breakthrough in technology or introduce new products that affect the profit to a great extent. This model for technological innovations is not unique to this study and it is based on [3].

As mentioned before, the stochastic process is exogenous to the firm and is dependent on research performed by other firms that specialize in development of new technologies. The adopting firm benefits from the innovations and enhances its performance.

One example is adoption of industrial control systems. These systems use electronic devices. The control systems are used for automation of electromechanical processes in various industries such as oil refining plants, metallurgical process plants, chemical plants, pharmaceutical manufacturing, water management systems, as well as amusement rides. Innovations in micro controllers, microprocessors, memory chips, or sensors will result in a better control system and improves the technology. However, the research that delivers these innovations is carried out outside the plants that adopt control systems.

Other examples are robotics technology and technologies that improve energy efficiency in power plants. In these examples innovations are exogenous to the adopting firm. The parameters of the innovation process are controlled by research centers and firms that develop the technology. The government can also affect the innovations through policies and direct investment.

The model can be used to compare different strategies for R&D investment by research firms or the government as well. One strategy is to raise the intensity of jumps denoted by η. This can be achieved through diversification and investment in separate fields of research. For example, projects that aim to develop better memory chips are separate from projects focused on improving sensors. If both of these fields of research are financed, more jumps are expected. Another strategy is to invest more in each field of research. This can raise the average size of jumps roughly, without an increase in the intensity. Note that the average size of jumps in the logarithm of production cost is $\frac{1}{\lambda}$. These two strategies have different impacts on the expected time that the firm waits before the adoption.

The game of adoption of a new technology ends as soon as one of the firms (firm i) adopts a new technology. Consider that firm i adopts as a leader when the value of the state variable (realization of Ct) is equal to c. The remaining firm (firm j) would face a standard optimal stopping time problem. The optimal adoption time for the follower (firm j) is the solution to this problem. Let Fj (c) denote the value of firm j (the follower) evaluated when firm i adopts. The value of firm i evaluated when it adopts a new technology as a leader is denoted by Li(c).

As I will later show, in both types of equilibrium that I introduce, at some values of the state variable the leader (firm i) adopts a new technology when it is optimal to do so without being preempted by its rival (firm j). At some other values of the state variable, both firms adopt a new technology at the same time. Let S(c) denote the value of each firm when simultaneous adoption happens. Only in one of the types of the equilibrium that I will introduce, at some values of the state space both firms wish to preempt each other. In the same type equilibrium that preemption happens, there may exist a new subset of the state space that the leader adopts a new technology without being preempted by its rival. This set can exist only in the equilibrium where preemption happens. This set is different from the set where leader adopts in the equilibrium that preemption does not happen.

2-1- Payoff Functions

If both firms adopt a new technology simultaneously, their payoff would be $S(c) = r^{-1}\Pi(c, c) - I$, where r is the discount rate and c is the value of the state variable when the adoption happens.

If simultaneous adoption of a new technology does not happen, then one of the firms (firm i) adopts as a leader. Let c be the value of the state variable when firm i adopts. The value of the follower (firm j) when the leader adopts is the solution to the stopping time problem

$$F_j(c) = \sup_{Tj \in M} E^c \left[\int_0^{Tj} e^{-rs} \Pi(c_j^0, c) ds + \int_{Tj}^{+\infty} e^{-rs} \Pi(C_{Tj}, c) ds - e^{-rrj} I \right] \tag{3}$$

where M is the set of stopping times adapted to $(\mathscr{F}_t)_{t \geq 0}$. The solution to the above problem characterizes the optimal stopping set for the follower denoted by S_j^F. This set consists of the values of the state variable when it is optimal for the follower to adopt a new technology. In other words, the optimal value of T_j is the same as $\inf\{t \geq 0 | C_t \in S_j^F\}$.

Lemma 2-1 Given that firm i has already adopted a new technology and has the production cost c, the optimal time for firm j to adopt a new technology is when C_t is less than or equal to the adoption threshold

$$h(c_j^0) = \frac{c_j^0}{r^{-1}(r - \Psi(1))} \tag{4}$$

See subsection

5-2 for derivation of $h(c_j^0)$. It is equivalent to the above lemma that $S_j^F = (0, h(c_j^0))$. In (4), $\Psi(z)$ is the Levy exponent of a process defined by $E_0\left[e^{z \ln C_t}\right] = e^{t\Psi(z)}$ for $z > -\lambda$, the value of $\Psi(z)$ is

$$\Psi(z) = \mu z - \frac{\eta z}{\lambda + z} \tag{5}$$

See subsection 5-1 for more detail. It follows that the value of the follower is

$$F_j(c)=\begin{cases} r^{-1}\Pi(c_j^0,c)-\left(\frac{c_j^0}{rB}-I\right)\sum_{k=1,2}\frac{a_k}{\beta_k(\beta_k-1)}\left(\frac{c}{h(c_j^0}\right)^{\beta_k} & c>h(c_j^0) \quad c>h(c_j^0) \\ r^{-1}\Pi(c,c)-I & c\le h(c_j^0 \end{cases} \quad (6)$$

where

$$\beta_1=\frac{1}{2\mu}\left(r-\mu\lambda+\eta+\sqrt{(r-\mu\lambda+\eta)^2+4r\mu\lambda}\right)$$

$$\beta_1=\frac{1}{2\mu}\left(r-\mu\lambda+\eta-\sqrt{(r-\mu\lambda+\eta)^2+4r\mu\lambda}\right) \quad (7)$$

are the zeros of $r-\Psi(z)$ and $\beta_1<-\lambda<\beta_2<0$. Also $a_1=\frac{r(\lambda+\beta_1)}{\mu(\beta_1-\beta_2)}$ and $a_2=\frac{r(\lambda+\beta_2)}{\mu(\beta_2-\beta_1)}$ and both are negative. See subsection

5-2 for derivation of $F_j(c)$. The value function $F_j(c)$ consists of two parts: 1) the present value of perpetual payoff that firm receives using the initial technology $(r^{-1}\Pi(c_j^0,c))$ and 2) the expected net present value of improvement caused by adoption. This net expected value is known as the value of the option to adopt a new technology.

When the leader (firm i) adopts a new technology at c, her value is

$$L_i(c)=E^c\left[\int_0^{Tj}e^{-rs}\Pi(c,c_j^0)ds+\int_{Tj}^{+\infty}e^{-rs}\Pi(c,C_{Tj})ds\right]-I \quad (8)$$

Firm i assumes that firm j would behave optimally as a follower according to what Lemma predicts. At time T_j, when the value of the state variable falls in the stopping set S_j^F, the follower will adopt a new technology. The first integral shows the value the leader receives before the follower adopts. The follower uses the old technology and has the production cost c_j^0 before adoption. After adoption the follower would have the production cost C_{Tj} forever. The second integral shows the value the leader receives after the follower adopts. There are two random variables, C_{Tj} and T_j in the above expression. As derived in subsection

5-3, the value of the leader at the adoption time is

$$L_i(c)=\begin{cases} r^{-1}\Pi(c,c_j^0)-I+\frac{}{rB}\sum_{k=1,2}a_k\left(\frac{c_j^0-rIB}{\beta_k-1}-\frac{c_j^0}{\beta_k}\right)\left(\frac{c}{h(c_j^0)}\right)^{\beta_k} & c>h(c_j^0) \\ r^{-1}\Pi(c,c)-I & c\le h(c_j^0) \end{cases} \quad (9)$$

If firm j was not able to preempt firm i, then firm i would adopt as a leader assuming that firm j would behave optimally as a follower. In this situation the leader (firm i) would solve her own optimal stopping time problem and her value before adoption would be

$$\hat{L}_i(c)=\sup_{T_i\in M}E^c\left[\int_0^{T_i}e^{-rs}\Pi(c_i^0,c_j^0)ds+e^{-rT_i}L_i(C_{T_i}\right] \quad (10)$$

One may suggest that the optimal time for firm i would be when the function $L_i(c)$ reaches its maximum. However, this is not true. The function $L_i(c)$ gives the value of the leader after adoption. A required step to determine the optimal time to adopt a new technology is to consider the value of waiting. This requires an analysis similar to derivation of optimal adoption time for the follower.

Lemma 2-2 If $h(c_j^0)\le h(c_i^0)$, the optimal time for firm i to adopt as a leader is when C_t less than or equal to the adoption threshold $h(c_i^0)$.

As a result, the optimal stopping set for the leader is $S_i^L=(0,h(c_i^0)$. Note that $S_i^L=S_i^F$. This is due to the assumption that the instantaneous payoff is an additively separable function of c_i and c_j. Hence, the amount of improvement in profit flow of firm i resulted from adoption of a new technology is independent of c_j. Thus, the benefit of adoption and benefit of waiting do not depend on whether the rival has adopted or not. If the decision of firm i is not changing firm j's decision, the problem of firm i is reduced to the problem of a single firm adopting a new technology. Hence, the optimal adoption time for a firm is the same regardless of its role as a leader or a follower. Lemma suggests this is the case.

2-2- Preemption Zone

It is possible for firm j to preempt firm i. Consider the situation that firm i adopts a new technology as a leader at c. If adopting as a leader pays off more than being a follower at c, firm j would prefer to be a first mover and preempt its rival. Let S_j^P denote the set of values of the state variable when firm j wishes to preempt its rival. Define $d_j(c)=L_j(c)-F_j(c)$. Then $S_j^P=\left\{c\in R_+\middle|d_j(c)>0\right\}$. It is possible that for certain values of c, this situation occurs for both firms. That is both firms prefer to be the first mover given that their rival adopts at c. In such circumstances, firm i will be better off preempting firm j and vice versa. Preemption zone is defined to be the set of these values of c.

Definition 2-1 Preemption zone denoted by P is the set of values of c that satisfy $d_i(c) > 0$ for both i= 1.2.

In other words, $P = S_1^P \cap S_2^P$.

Lemma 2-3 For $c \geq h(c_i^0), d_i^{"}(c) < 0$, and $\lim_{c \to +\infty} d_i(c) = -\infty$.

From lemma , it follows that the sets S_i^P for i=1,2 are bounded convex sets. As a result, preemption zone is also a bounded convex set. Hence, when P is nonempty, I can define it as $P = (p_1, p_2)$.

I start with characterizing the preemption zone when initial production costs are the same, that is $c_1^0 = c_2^0 = c^0$.

Lemma 2-4 For the case when $c_1^0 = c_2^0 = c^0$, $d_i(h(c^0)) = 0$. If $\gamma > 0$ then $d_i^{'}(h(c^0)) > 0$ and if $\gamma = 0$ then $d_i^{'}(h(c^0)) = 0$.

Here is the intuition for the fact that $d_i\ (h(c^0)) = 0$. Consider that one of the firms adopts at $c = h(c^0)$ as a leader. The optimal follower's adoption threshold is $h(c^0)$ and as a result the other firm would adopt at the same time. Hence, there is no difference between adoption at $h(c^0)$ as leader or as a follower if the rival is adopting at $h(c^0)$. Hence, we have that $d_i(h(c^0)) = 0$.

For symmetric firms, it follows from (6) and (9) that $d_i(c) = 0$ at any $c \in (0, h(c^0)]$. From Lemma , it follows that $d_i(c) > 0$ for some values of c if and only if $\gamma > 0$. Then $d_i(c) > 0$ if and only if $c \in (h(c^0), p_2)$ where $h(c^0)$ and p_2 are the only zeros of $d_i(c)$ among values of $c \geq h(c^0)$. This concludes the following proposition.

Proposition 2.1 *For $c_1^0 = c_2^0 = c^0$, preemption zone exists (P is nonempty) if and only if $\gamma > 0$. Then $P = (p_1, p_2)$ where $p_1 = h(c^0)$ and p_2 is the only solution for $d_i(c) = 0$ among the values of $c > h(c^0)$.*

Now I characterize the preemption zone when initial production costs are different. Without loss of generality, assume $c_1^0 > c_2^0$. Since $h(.)$ is an increasing function (see (4)), it follows that $h(c_1^0) > h(c_2^0)$. For any

$c \leq h(c_2^0)$, from (6) and (9) it follows that $L_i(c) = F_i(c)$, hence $d_i(c) = 0$ for $i = 1, 2$. In conclusion $(0, h(c_2^0)] \cap S_i^P = \emptyset$ for $i = 1, 2$.

Lemma 2.5 *For the case when* $c_1^0 > c_2^0$, $d_1(h(c_1^0)) > 0$ *and* $d_2(h(c_1^0)) < 0$.

The inequality $d_1(h(c_1^0)) > 0$ suggests that S_1^P is nonempty. The inequality $d_2(h(c_1^0)) < 0$ suggest that if firm 1 (high production cost) is adopting at $h(c_1^0)$, it is not in the interest of firm 2 (low production cost) to preempt. This reassures us that in the equilibrium which will be introduced in subsection

3-2, the more efficient firm does not wish to preempt the less efficient firm.

Here is the intuition behind the fact that $d_2(h(c_1^0)) < 0$. This claim compares two scenarios for firm 2. In the first scenario firm 1 adopts at $h(c_1^0)$ as a leader and firm 2 waits until $h(c_2^0)$ is reached and adopts as a follower. In the second scenario, firm 2 adopts as soon as $h(c_1^0)$ is reached. In the second scenario firm 1 would adopt at the same time that firm 2 adopts, because $h(c_1^0)$ is the optimal threshold for firm 1 to adopt as a follower.

The claim that $d_2(h(c_1^0)) < 0$ suggests that firm 2 strictly prefers the first scenario. Here is the reason. The profit flow of firm 2 is an additively separable function of c_1 and c_2. Hence, the amount of improvement in profit flow of firm 2 resulted from adoption of a new technology is independent of c_1. Thus, the benefit of adoption and benefit of waiting do not depend on whether the rival has adopted or not. If the decision of firm 2 is not changing firm 1's decision, the problem of firm 2 is reduced to the problem of a single firm adopting a new technology. Hence, the optimal adoption time for a firm is the same regardless of its role as a leader or a follower. In conclusion the optimal decision for firm 2 would be to invest when $h(c_2^0)$ is reached, hence the first scenario is preferred. In fact $d_2(c) < 0$ for any $c \in (h(c_2^0), h(c_1^0)]$. If firm 2 adopts in this set as a leader, firm 1 would simultaneously adopt. Hence, as expressed in (9) adoption as a leader yields a payoff equal to the payoff from simultaneous adoption. But waiting and adopting optimally as a follower would yield a higher payoff than simultaneous adoption. In conclusion, $L_2(c) \leq F_2(c)$, hence $(h(c_2^0), h(c_1^0)] \cap S_2^P = \emptyset$.

Lemma 2-6 *For the case when $c_1^0 > c_2^0$, for any value of $c > h(c_1^0)$ it is true that $d_1(c) - d_2(c) > 0$.*

It was established before that $(0, h(c_1^0)] \cap S_2^P = \emptyset$. Adding this to lemma yields that $S_2^P \subset S_1^P$. As a result $P = S_2^P = \{c \in (0, \infty) | d_2(c) > 0\}$. The set P is non-empty if and only if $d_2(c) > 0$ for some $c > h(c_1^0)$. This concludes the following proposition.

Proposition 2-2 *For $c_1^0 > c_2^0$, if $d_2(c) > 0$ for some $c \geq h(c_1^0)$, then preemption zone exists (P is nonempty). Then $P = (p_1, p_2)$ where p_1 and p_2 are the only solutions for $d_2(c) = 0$ among values of $c \geq h(c_1^0)$.*

Note that $p_1 \geq \max\{h(c_i^0)\}_{i=1,2}$. Hence, $S_i^L \cap P = \emptyset$ for $i = 1, 2$. As a result, no firm wishes to adopt as a leader in the preemption zone if the rival has no plan to adopt in the preemption zone.

The following proposition states that preemption zone exists (P in nonempty) if efficiencies of the technologies of the firms are close enough.

Proposition 2-3 *If $\gamma > 0$, for $c_1^0 > c_2^0$, the preemption zone described in proposition exists if and only if $c_2^0 > \check{c}$, where $\check{c}$ is a function of parameters of the model excluding c_2^0. The size of the preemption zone increases when c_2^0 increases.*

The following proposition states that the preemption zone would not exist if the loss caused by adoption of a new technology by the rival is smaller than a certain level.

Proposition 2-4 Assume that a preemption zone exists (P is nonempty). There exists $\check{\gamma}$, that if γ is reduced to a value below $\check{\gamma}$, a preemption zone will not exist. The size of the preemption zone increases when γ increases.

3- Equilibrium

In the game of adoption of a new technology, at any point in time, each firm can either adopt or wait. As soon as one of the firms adopts, the game ends and the follower (firm i) waits until the set S_i^F is reached.

I assume that the strategies of the firms are stationary Markovian. This means that the strategy of a firm does not change over time and depends only on the value of C_t. Because the process for C_t is Markov, the payoffs only depends on C_t. Hence, it is weakly dominated to choose strategies that are not Markovian. Markovian strategies include all the factors that affect the payoffs. (Refer to Maskin and Tirole [12], Fudenberg and Tirole [8], and Mason and Weeds [13], for more discussion.)

3-1- Strategy Space and Equilibrium Concept

Definition 3.1 A Markov strategy for player i is a pair $\sigma_i = (\mathcal{S}_i, \alpha_i)$, where $\mathcal{S}_i \in \mathbb{R}_+$ is a stopping set and $\alpha_i(c) : \mathbb{R}_+ \mapsto [0,1]$ is an intensity function. The intensity function α_i satisfies the following condition: If $\alpha_i(c) = 0$ and $c = \sup\{u < c | \alpha_i(u) > 0\}$, then the left derivative of $\alpha_i(c)$ exists and is negative. The intensity function $\alpha_i(c)$ is adapted to $(\mathscr{F}_t)_{t\geq 0}$.

The strategy for a player is defined similar to [16]. Choice of stopping set would indicate when a firm acts. Choice of intensity function would indicate how a firm acts. The strategy σ_i describes the behavior of firm i when firm j has not adopted the new technology yet. As soon as one of the firms adopts, the game ends and the remaining firm adopts as a follower as described in Lemma .

The intensity function has been introduced as part of a strategy in similar timing games, such as [7] and [17]. I define intensity to be a function of efficiency of the cutting edge technology. At any time t, $\alpha_i(C_t)$ is the intensity of a sequence of atoms in a very tiny interval $[t, t+dt)$, and allows for coordination between the firms when adoption by one firm is optimal, but simultaneous adoption is not. Let c denote realization of C_t. At any c, if both of the firms wish to adopt with a positive probability, each firm adopts with intensity $\alpha_i(c)$, and they repeat this game until at least one of them adopts. It is assumed that this process takes no time.

For example, assume that each player randomly generate a sequence of 0s and 1s. The player that produces the first 1 in the sequence invests. If it happens that in both of the sequences, the first 1 is produced at the same time, both firms invest. In this example, the value of the intensity function indicates the probability that 1 is generated as a member of the sequence. The condition for the intensity function mentioned in the definition is not necessary, but simplifies the definition of payoffs.

The set Σ denotes set of Markov strategies. Strategy space is $\Sigma \times \Sigma$ and a pair of Markov strategies is $(\sigma_1, \sigma_2) \in \Sigma \times \Sigma$. Define $T_i = \inf\{t > 0 | \alpha_i(C_t) > 0\}$ and $T = \min\{T_1, T_2\}$. The expected payoff for player i is

$$V_i(c,\sigma_i,\sigma_j) = E^c \left[\int_0^T e^{-rs} \Pi(c_i^0, c_j^0) ds \right.$$
$$\left. + e^{-rT} \left(\mathbb{1}_{(T_i<T_j)} L_i(C_T) + \mathbb{1}_{(T_i>T_j)} F_i(C_T) + \mathbb{1}_{(T_i=T_j)} . Y_i(C_T) \right) \right] \tag{11}$$

If for some $i \in \{1,2\}$, $\alpha_i(c) > 0$, then

$$Y_i(c) = \frac{\alpha_i(c)(1-\alpha_j(c))L_i(c) + (1-\alpha_i(c))\alpha_j(c)F_i(c) + \alpha_i(c)\alpha_j(c)S(c)}{\alpha_i(c) + \alpha_j(c) - \alpha_i(c)\alpha_j(c)}. \tag{12}$$

Here, the payoff that each firm gets at the simultaneous adoption is denoted by $S(c) = r^{-1}\Pi(c,c) - I$, where r is the discount rate. If $\alpha_i(c) = \alpha_j(c) = 0$, then by l'Hôpital's rule, the limit is

$$Y_i(c) = \frac{\alpha_i'(c-)L_i(c) + \alpha_j'(c-)F_i(c)}{\alpha_i'(c-) + \alpha_j'(c-)}. \tag{13}$$

Definition 3-2 A Markov perfect equilibrium is a pair of strategies (σ_1^*, σ_2^*) such that for all $c \in \mathbb{R}_+$: $V_i(c, \sigma_i^*, \sigma_j^*) \geq V_i(c, \sigma_i, \sigma_j^*)$ for all $\sigma_i \in \Sigma$ and for all $i \in \{1,2\}$.

3-2- Sequential Adoption

Proposition 3-1 Pair of Markov strategies $(\sigma_1^{seq}, \sigma_2^{seq})$ where $\mathcal{S}_i^{seq} = (0, h(c_i^0)]$ and

$$\alpha_i^{seq}(c) = \begin{cases} 0 & \text{if } h(c_i^0) < c \\ 1 & \text{if } 0 < c \leq h(c_i^0) \end{cases}, \tag{14}$$

always constitutes a Markov perfect equilibrium.

Note that firms adopt simultaneously at any $c \in (0, \min_i\{h(c_i^0)\}]$.

3-3- Preemptive Adoption

Here I give strategies for symmetric firms in an equilibrium that involves preemptive adoption. Since there is symmetry, consider that $c_i^0 = c^0$, $d_i(.) = d(.)$ and $L_i(.) = L(.)$ for $i \in \{1, 2\}$. Remember from subsection

2-2 that p_2 is the least upper bound of the preemption zone. The variable m would indicate the value of the intensity function at p_2. The variable $p \in P$ would indicate the intersection of the stopping set and the preemption zone. As it will be shown later, in a preemptive adoption it is not required for the stopping set to include all the preemption zone. The variable p would indicate a subset of the preemption zone that is included in the stopping set. Let the stopping set be $\mathcal{S}_{m,p}^{sym} = (0, h(c^0)] \cup (p, p_2]$ and the intensity function be

$$\alpha_{m,p}^{sym}(c) = \begin{cases} 0 & \text{if } p_2 < c \\ m & \text{if } c = p_2 \\ \dfrac{d(c)}{L(c) - S(c)} & \text{if } p < c < p_2 \\ 0 & \text{if } h(c^0) < c \leq p \\ 1 & \text{if } c \leq h(c^0) \end{cases} . \quad (15)$$

The pair $\sigma_{m,p}^{sym} = (\mathcal{S}_{m,p}^{sym}, \alpha_{m,p}^{sym})$ is a Markov strategy. I define Γ^{sym} to be a set of pairs of strategies as defined above, such that, if $\gamma > 0$,

$$\begin{aligned} \Gamma^{sym} = &\{(\sigma_{m,p}^{sym}, \sigma_{0,p}^{sym}) | m \in [0,1], p \in [h(c^0), p_2)\} \cup \\ &\{(\sigma_{0,p}^{sym}, \sigma_{m,p}^{sym}) | m \in [0,1], p \in [h(c^0), p_2)\}, \end{aligned} \quad (16)$$

and if $\gamma = 0$, then Γ^{sym} is empty.

Proposition 3.2 *If $\gamma > 0$, any pair of strategies $(\sigma_1^*, \sigma_2^*) \in \Gamma^{sym}$ constitutes a Markov perfect equilibrium.*

In the case when both firms have the same production cost, none of the firms has incentive to adopt at any $c > p_2$ and it is optimal for both of them to wait. However, in the presence of asymmetry, the less efficient firm (firm 1) strictly prefers to adopt as a leader at the least upper bound of the preemption zone, p_2, because $d_1(p_2) > 0$. Even at $c > p_2$, because of jumps, there is always the possibility that the preemption zone is reached in the next moment. Hence, the less efficient firm always faces the threat of being preempted in the next moment. As a result, the less efficient firm wishes to secure adopting as a leader by adopting much earlier than when the preemption zone is reached.

I assume that firm 1 is less efficient than firm 2 (ie. $c_1^0 > c_2^0$). I also assume that for $i = 1, 2$, firm i's strategy at any $c < p_2$ is such that

$$\alpha_{i,p}^{asym}(c) = \begin{cases} \dfrac{d_j(c)}{L_j(c) - S(c)} & \text{if } p < c < p_2 \\ 0 & \text{if } h(c_i^0) < c \leq p \\ 1 & \text{if } c \leq h(c_i^0) \end{cases} . \tag{17}$$

Also I assume that $\alpha_{2,p}^{asym}(c) = 0$ for $c \geq p_2$. The leader (firm 1), at $c \geq p_2$, solves the optimal stopping time problem

$$V_w(c,p) = \sup_{\tau_w(p) \in \mathcal{M}} E^c \left[\int_0^{\tau_w(p)} e^{-rs} \Pi(c_1^0, c_2^0) ds + e^{-r\tau_w(p)} V_\Omega(C_{\tau_w(p)}, p) \right] . \tag{18}$$

In the above expression for the value function, the first term is the value the leader receives from production while both firms are using old technologies. The function $V_\Omega(.)$ is the continuation payoff at $\tau_w(p)$. Hence, the second term is the discounted value of the leader at $\tau_w(p)$. Because of jumps, there are three scenarios for $C_{\tau_w(p)}$. First, it may jump into the set $(0, p]$. If this happens, according to the strategies in (17), firm 1 assumes the leader role in that region. Second, it may fall inside (p, p_2) and firm 1's continuation payoff is equal to $F(C_{\tau_w(p)})$ for this case. Note that given firm 2's strategy described in (17), firm 1 is indifferent between any action inside (p, p_2), hence the expected payoff is equal to the follower's payoff. A third possible scenario is $C_{\tau_w(p)} \geq p_2$ and firm 1 adopts immediately. Hence, the continuation payoff is

$$V_\Omega(c,p) = \begin{cases} L_1(c) & \text{if } p_2 \leq c \\ F_1(c) & \text{if } p < c < p_2 \\ \hat{L}_1(c) & \text{if } c \leq p \end{cases} . \tag{19}$$

Lemma 3-1- Consider asymmetric firms where $c_1^0 > c_2^0$. Assume that conditions in Proposition are satisfied and the preemption zone exists. Also assume that at any $c \in (0, p_2)$, firms play strategies introduced in (17).

There exists a function $h_w(p)$, where $h_w(p) \geq p_2$ and the following holds true. Firm 1 (the less efficient firm), adopts a new technology when $C_t \in [p_2, h_w(p)]$ and waits when $C_t > h_w(p)$. If

$$rIB + r^{-1}(r - \Psi(1))p_2 - c_1^0 + r^{-1}\eta M(p)p_2^{-\lambda} \geq 0 \tag{20}$$

$h_w(p)$ is enqual to p_2. Otherwise, $h_w(p)$ is equal to the value of c that solves

$$rIB + r^{-1}(r - \Psi(1))c - c_1^0 + r^{-1}\eta M(p)c^{-\lambda} = 0, \tag{21}$$

where

$$\begin{aligned}
M(p) = {}& p_2^{\lambda}\left(rIB + \frac{\lambda(1+\gamma)}{1+\lambda}p_2 - \gamma c_2^0 - c_1^0\right) \\
& + p^{\lambda}\gamma\left[c_2^0 - \frac{\lambda p}{1+\lambda} + \sum_{k=1,2}\frac{a_k\lambda}{\beta_k+\lambda}\left(\frac{r^{-1}(r-\Psi(1))}{\beta_k-1}h_2 - \frac{c_2^0}{\beta_k}\right)\left(\frac{p}{h_2}\right)^{\beta_k}\right] \\
& - p_2^{\lambda}\sum_{k=1,2}\frac{a_k\lambda}{\beta_k+\lambda}\left[\left(\frac{r^{-1}(r-\Psi(1))}{\beta_k-1}h_1 + \frac{rIB - c_1^0}{\beta_k}\right)\left(\frac{p_2}{h_1}\right)^{\beta_k}\right. \\
& \left. +\gamma\left(\frac{r^{-1}(r-\Psi(1))}{\beta_k-1}h_2 - \frac{c_2^0}{\beta_k}\right)\left(\frac{p_2}{h_2}\right)^{\beta_k}\right].
\end{aligned} \tag{22}$$

See subsection 5.5 for derivation of $h_w(p)$. For asymmetric firms, where $c_1^0 \neq c_2^0$, if the conditions in Proposition are satisfied and a preemption zone exists, I define Markov strategies for the firms as follows. Without loss of generality assume that $c_1^0 > c_2^0$. Let $\mathcal{S}_{1,p}^{asym} = (0, h(c_1^0)] \cup (p, h_w(p)]$ and

$$\alpha_{1,p}^{asym}(c) = \begin{cases} 0 & \text{if } h_w(p) < c \\ 1 & \text{if } p_2 \leq c \leq h_w(p) \\ \dfrac{d_2(c)}{L_2(c) - S(c)} & \text{if } p < c < p_2 \\ 0 & \text{if } h(c_1^0) < c \leq p \\ 1 & \text{if } c \leq h(c_1^0) \end{cases}. \tag{23}$$

The pair $\sigma_{1,p}^{asym} = (\mathcal{S}_{1,p}^{asym}, \alpha_{1,p}^{asym})$ is a Markov strategy for firm 1. Similarly, let $\mathcal{S}_{2,p}^{asym} = (0, h(c_2^0)] \cup (p, p_2)$ and

$$\alpha_{2,p}^{asym}(c) = \begin{cases} 0 & \text{if } p_2 \le c \\ \dfrac{d_1(c)}{L_1(c) - S(c)} & \text{if } p < c < p_2 \\ 0 & \text{if } h(c_2^0) < c \le p \\ 1 & \text{if } c \le h(c_2^0) \end{cases} . \tag{24}$$

The pair $\sigma_{2,p}^{asym} = (\mathcal{S}_{2,p}^{asym}, \alpha_{2,p}^{asym})$ is a Markov strategy for firm 2. I define Γ^{asym} to be a set of pairs of strategies as defined above, such that, if the preemption zone exists,

$$\Gamma^{asym} = \{(\sigma_{1,p}^{asym}, \sigma_{2,p}^{asym}) | p \in [p_1, p_2)\}, \tag{25}$$

and if the preemption zone does not exist, then Γ^{asym} is an empty set.

Proposition 3-3 For asymmetric firms, where $c_1^0 > c_2^0$, if the conditions in Proposition are satisfied and if the preemption zone exists, any pair of strategies $(\sigma_1^*, \sigma_2^*) \in \Gamma^{asym}$ constitutes a Markov perfect equilibrium.

3-4- Welfare Comparison

Proposition 3-4 Welfare is weakly higher under sequential investment than preemptive investment.

If the initial value of C_t is in $(0, \inf P)$, then both equilibria give the same payoff. Otherwise, the expected aggregate payoff is strictly larger in sequential investment.

4- Numerical Example

Consider the following parameter values: $\mu = -.003, \lambda = 50, \eta = .3, I = 20, r = .01, A = 1, B = 3, \gamma = .5, c_1^0 = .9$ and $c_2^0 = .8$. It follows that $h(c_1^0) = 0.158$ and $h(c_2^0) = 0.106$. The boundaries of the preemption zone are $p_1 = 0.171$ and $p_2 = 0.281$. For $p = p_1$, the optimal adoption threshold for the less efficient firm before reaching the preemption zone is $h_w(p_1) = 0.295$.

5- Derivation of Value Functions and Adoption Thresholds

5-1- Infinitesimal Generator, Levy Exponent and EPV Operators

See [2] for more detail on definitions and derivations presented in this section. Let $x = \ln(c)$ and $X_t = \ln(C_t)$. I define operators for process X. To use the operators with any function $g(c)$, it needs to be transformed to $u(x) = g(e^x)$ first. Let L be the infinitesimal generator of the process X. The action of this operator for the specific process introduced in (2) is defined as follows:

$$Lu(x) = \mu \frac{\partial}{\partial x} u(x) + \int_{-\infty}^{+\infty} (u(x+y) - u(x)) F(dy) \tag{26}$$

Let $\Psi(z)$ be the Levy exponent of the process defined by $E_0\left[e^{zX_t}\right] = e^{t\Psi(z)}$. The relationship between the Levy exponent and the infinitesimal generator is such that $Le^{zx} = \Psi(z)e^{zx}$. For the process given by (2), I can derive $\Psi(z)$ from (26). For $z > -\lambda$, I have:

$$\Psi(z) = \mu z + \int_{-\infty}^{0} \eta\lambda \left(e^{zy} - 1\right) e^{\lambda y} dy = \mu z - \frac{\eta z}{\lambda + z} \tag{27}$$

To find value functions, I need to solve for integro-differential Bellman equations of the form: $(r - L)V(x) = g(x)$. In order to do this, I first solve for the roots of $r - \Psi(z) = 0$.

$$r - \Psi(z) = \frac{-\mu}{\lambda + z}\left(z^2 - \frac{r - \mu\lambda + \eta}{\mu} z - \frac{\lambda r}{\mu}\right) = 0 \tag{28}$$

The value of $r - \Psi(0)$ is positive and as z approaches $-\lambda$ from the right, $r - \Psi(z)$ becomes negative. Hence, there is a negative root for the characteristic equation which is larger than its pole $-\lambda$. Since $-\frac{\lambda r}{\mu}$ is positive, the other root is negative as well and should be smaller than $-\lambda$. Let β_1 and β_2 be the roots, such that: $\beta_1 < -\lambda < \beta_2 < 0$.

$$\beta_1 = \frac{1}{2\mu}\left(r - \mu\lambda + \eta + \sqrt{(r - \mu\lambda + \eta)^2 + 4r\mu\lambda}\right)$$
$$\beta_2 = \frac{1}{2\mu}\left(r - \mu\lambda + \eta - \sqrt{(r - \mu\lambda + \eta)^2 + 4r\mu\lambda}\right) \tag{29}$$

To have a well-defined problem, I need $r - \Psi(1)$ to be positive. It is always true, because $r - \Psi(z)$ has no positive zero and it is positive at

$z=0$. This is the necessary and sufficient condition for the value of the firm to be finite. Let $\mathcal{E}$ be the normalized expected present value operator or EPV-operator. This operator gives the present value of a stream of payoffs. The value depends on the present observed x (or c), and it is normalized such that $\mathcal{E}$ of a stream of payoff that gives one unit at any time is equal to one. Note that $\mathcal{E}=r(r-L)^{-1}$. See [2] for details. I also define $\mathcal{E}^{+}$ to be the normalized EPV-operator under the supremum process $\overline{X}_t=\sup_{0\leq s<t}X_s$ and $\mathcal{E}^{-}$ to be the normalized EPV-operator under the infimum process $\underline{X}_t=\inf_{0\leq s<t}X_s$. Let $a_1=\frac{r(\lambda+\beta_1)}{\mu(\beta_1-\beta_2)}$ and $a_2=\frac{r(\lambda+\beta_2)}{\mu(\beta_2-\beta_1)}$, I derive the action of operators for supremum and infimum processes to be

$$\begin{cases} \mathcal{E}^{+}u\left(x\right)=u(x) \\ \mathcal{E}^{-}u\left(x\right)=-a_1\int_{-\infty}^{0}e^{-\beta_1 y}u(x+y)dy-a_2\int_{-\infty}^{0}e^{-\beta_2 y}u(x+y)dy \end{cases} . \tag{30}$$

Refer to [2] for more details on how to derive the EPV-operators. For exponential functions they reduce to

$$\begin{cases} \mathcal{E}^{+}e^{zx}=e^{zx} \\ \mathcal{E}^{-}e^{zx}=-a_1\int_{-\infty}^{0}e^{-\beta_1 y}e^{z(x+y)}dy-a_2\int_{-\infty}^{0}e^{-\beta_2 y}e^{z(x+y)}dy=\left[\frac{a_1}{\beta_1-z}+\frac{a_2}{\beta_2-z}\right]e^{zx} \end{cases} \tag{31}$$

I define $\kappa^{+}\left(z\right)$ and $\kappa^{-}\left(z\right)$ such that $\mathcal{E}^{\pm}e^{zx}=\kappa^{\pm}(z)e^{zx}$. It also holds that

$$\begin{cases} \kappa^{+}\left(z\right)=1 \\ \kappa^{-}\left(z\right)=\frac{r}{r-\Psi(z)}=\frac{a_1}{\beta_1-z}+\frac{a_2}{\beta_2-z} \end{cases} . \tag{32}$$

According to Wiener-Hopf factorization formula, the normalized EPV operator can be expressed as $\mathcal{E}=\mathcal{E}^{+}\mathcal{E}^{-}$. Hence, $\frac{r}{r-\Psi(z)}=\kappa^{+}(z)\kappa^{-}(z)$. See [2] for more details on Wiener-Hopf factorization, EPV-operators and how they are derived. $\mathcal{E}=\mathcal{E}^{+}\mathcal{E}^{-}$. Hence, $\frac{r}{r-\Psi(z)}=\kappa^{+}(z)\kappa^{-}(z)$. See [2] for more details on Wiener-Hopf factorization, EPV-operators and how they are derived.

5-2- Follower's Problem

In this subsection, I assume that firm j has already adopted technology c_j. I solve the follower's, i.e. firm i's problem. When the firm adopts, it loses the opportunity to adopt better technologies that possibly become available in the future. Hence, if firm i adopts technology c, the continuation payoff would be $W_F(c, c_j) = r^{-1}\Pi(c, c_j)$, where r is the discount rate.

Here I derive the value of firm i (the follower) before adoption. Let τ_i denote the random time of adoption for firm i. Assuming that firm i adopts when threshold h_i is reached, it holds that $\tau_i = \inf\{t \geq 0 | C_t \leq h_i\}$. Let c denote the realization of C_t. At any $c > h_i$, the follower's value is

$$\begin{aligned} V_F(c, c_i^0, c_j, h_i) &= E^c\left[\int_0^{\tau_i} e^{-rs}\Pi(c_i^0, c_j)ds + \int_{\tau_i}^{+\infty} e^{-rs}\Pi(C_{\tau_i}, c_j)ds - e^{-r\tau_i}I\right] \\ &= E^c\left[\int_0^{+\infty} e^{-rs}\Pi(c_i^0, c_j)ds + \int_{\tau_i}^{+\infty} e^{-rs}\left(\Pi(C_{\tau_i}, c_j) - \Pi(c_i^0, c_j)\right)ds - e^{-r\tau_i}I\right] \\ &= r^{-1}\Pi(c_i^0, c_j) + E^c\left[e^{-r\tau_i}\left(r^{-1}\Pi(C_{\tau_i}, c_j) - r^{-1}\Pi(c_i^0, c_j) - I\right)\right]. \end{aligned} \quad (33)$$

Note that the above value function is more general than $F_i(c)$. The function $V_F(.)$ gives the value of the follower at anytime before the follower adopts, while $F_i(c)$ gives the value of the follower only for the time the leader adopts. It holds that $F_j(c) = V_F(c, c_j^0, c, h(c_j^0))$.

The value function expressed in (33) consists of two parts: 1) the present value of perpetual payoff that firm receives using the initial technology and 2) the expected net present value of improvement caused by adoption. This net expected value, also known as the option value to adopt, equals to

$$V_F^{opt}(c, c_i^0, c_j, h_i) = E^c\left[e^{-r\tau_i}\left(r^{-1}B^{-1}\left(c_i^0 - C_{\tau_i}\right) - I\right)\right], \quad (34)$$

and it is independent of c_j. It holds that

$$V_F(c, c_i^0, c_j, h_i) = r^{-1}\Pi(c_i^0, c_j) + V_F^{opt}(c, c_i^0, c_j, h_i). \quad (35)$$

Since $r^{-1}B^{-1}\left(c_i^0 - c\right) - I$ is decreasing in c, the option value to adopt resembles the value of a put option. Hence, assuming the firm adopts at h_i, as suggested by Boyarchenko and Levendorskii [2], the value of this option for $c > h_i$ is

$$V_F^{opt}(c, c_i^0, c_j, h_i) = \mathcal{E}^- \mathbb{1}_{(-\infty, h_i]}(\ln(c))(\mathcal{E}^-)^{-1}\left[\left(r^{-1}B^{-1}\left(c_i^0 - c\right) - I\right)\right]. \quad (36)$$

Following some steps, the option value to adopt for $c > h_i$ is

$$V_F^{opt}(c, c_i^0, c_j, h_i) = -r^{-1}B^{-1}\sum_{k=1,2} a_k \left(\frac{r^{-1}(r - \Psi(1))}{\beta_k - 1}h_i + \frac{rIB - c_i^0}{\beta_k}\right)\left(\frac{c}{h_i}\right)^{\beta_k}. \quad (37)$$

Let $h(c_i^0)$ be the value of c that solves $(\mathcal{E}^-)^{-1}\left[\left(r^{-1}B^{-1}\left(c_i^0 - c\right) - I\right)\right] = 0$. Based on what Boyarchenko and Levendorskii [3] have shown, the optimal adoption time for the follower is when C_t falls below $h(c_i^0)$. This value is

$$h(c_i^0) = \frac{c_i^0 - rIB}{r^{-1}(r - \Psi(1))}. \quad (38)$$

Adoption does not necessarily happen at the threshold. It can be the case that C_t receives a negative jump and the follower adopts at a value of c below the threshold. The value of the follower at the moment of adoption at $c \leq h_i$ is

$$V_F(c, c_i^0, c_j, h_i) = W_F(c, c_j) - I = r^{-1}\Pi(c, c_j) - I. \quad (39)$$

5-3- Leader's Payoff at the time of Adoption

In this subsection, it is assumed that none of the firms has adopted yet. Each firm evaluates the decision to adopt, knowing what the other firm would do as a follower. I solve for the continuation payoff for firm i if it adopts as a leader. Let $W_L(c, c_j^0, h_j)$ be the value of the leading firm immediately after adopting the cutting edge technology that yields the production cost c. This function depends on the initial production cost of the follower, c_j^0, and the threshold at which it plans to adopt, h_j. The value of $L_i(c)$ would be $W_L(c, c_j^0, h(c_j^0)) - I$.

If firm i adopts at any $c \leq h_j$, the follower would adopt immediately after the leader's adoption of a new technology. This is because the value of c is smaller than the follower's adoption threshold. In this model, this is how simultaneous adoption of a new technology happens. The value of the leader that adopts at any c smaller than h_j is as follows:

$$W_L(c, c_j^0, h_j) = r^{-1}\Pi(c, c) \quad (40)$$

However, if the leader adopts at $c > h_j$, its value looks different. The follower delays the adoption until the state variable crosses h_j. Hence, the value of the leader adopting at any c larger than the follower's threshold is

$$W_L(c, c_j^0, h_j) = E^c \left[\int_0^{\tau_j} e^{-rs} \Pi(c, c_j^0) ds + \int_{\tau_j}^{+\infty} e^{-rs} \Pi(c, C_{\tau_j}) ds \right]. \tag{41}$$

The first integral shows the value the leader receives before the follower adopts. The follower is still using the old technology c_j^0, but at time τ_j, which is not known at the time the leader adopts, the follower will adopt a new technology and produce with the cost C_{Tj} forever. The second integral shows the value the leader receives after the follower adopts. There are two random variables, C_{Tj} and Tj. The value of the leader is the expectation conditional on c, which is the realization of C_t. From (41) it follows that:

$$\begin{aligned} W_L(c, c_j^0, h_j) &= \Pi(c, c_j^0) E^c \left[\int_0^{+\infty} e^{-rs} \mathbb{1}_{(h_j, +\infty)}(C_s) ds \right] + r^{-1} E^c \left[e^{-r\tau_j} \Pi(c, C_{\tau_j}) \right] \\ &= \Pi(c, c_j^0) r^{-1} \mathcal{E} \mathbb{1}_{(h_j, +\infty)}(c) + r^{-1} \mathcal{E}^- \mathbb{1}_{(0, h_j]}(c) (\mathcal{E}^-)^{-1} \Pi(c^*, c) \end{aligned} \tag{42}$$

The integral in the first line is nothing but the EPV operator applied on the indicator function. Also $r^{-1} E^c \left[e^{-r\tau_j} \Pi(c, C_{\tau_j}) \right]$ is similar to value of a call option exercised at τ_j, and can be calculated using the EPV operators as shown above. There is no uncertainty with respect to the first argument in $\Pi(c, C_{\tau_j})$. I define a new variable c^* which is equal to c and replace $\Pi(c, C_{\tau_j})$ with $\Pi(c^*, C_{\tau_j})$. I applied the EPV operators as defined in (30), then substituted c for c^* in order to derive $W_L(.)$ as expressed in (43).

Lemma 5-1 Assume that firm j would adopt as a follower at h_j. The value of firm i evaluated immediately after adopting a new technology at $c > h_j$ is

$$W_L(c, c_j^0, h_j) = r^{-1} \Pi(c, c_j^0) + \gamma r^{-1} B^{-1} \sum_{k=1,2} a_k \left(\frac{r^{-1}(r - \Psi(1))}{\beta_k - 1} h_j - \frac{c_j^0}{\beta_k} \right) \left(\frac{c}{h_j} \right)^{\beta_k}. \tag{43}$$

5-4- Value of the Leader before Adoption

In this subsection I derive the value of firm i (the leader) before adoption. I assume that firm j would adopt a new technology when C_t falls below the threshold h_j. Fixing firm j's adoption threshold, h_j, let $V_L(c, c_i^0, c_j^0, h_i, h_j)$

denote the value of firm i at c before it adopts as a leader. This value function is more general than $\hat{L}_i(c)$. The value function $\hat{L}_i(c)$ would be equal to $V_L(c, c_i^0, c_j^0, h(c_i^0), h(c_j^0))$. At any value of $c \le h_i$, the value of the leader before adopting, is equal to $W_L(c, c_j^0, h_j) - I$. Because the state variable has crossed the threshold, firm i adopts immediately and will receive $W_L(.)$ net of the investment cost.

However, at any c larger than h_i, the leader waits until the proper time comes. Let τ_i denote the random time of adoption for firm i. Assuming that firm i adopts when threshold h_i is reached, it holds that $\tau_i = \inf\{t \ge 0 | C_t \le h_i\}$. Let c denote the realization of C_t. At any $c > h_i$, the leader's value is

$$V_L(c, c_i^0, c_j^0, h_i, h_j) = E^c\left[\int_0^{\tau_i} e^{-rs}\Pi(c_i^0, c_j^0)ds\right] + E^c\left[e^{-r\tau_i}\left(W_L(C_{\tau_i}, c_j^0, h_j) - I\right)\right]. \quad (44)$$

In the above expression for the value function, the first term is the value the leader receives from production while both firms are using old technologies. The second term is the discounted value of the leader at the time of adoption of a new technology. Before the leader's adoption, knowing h_i is not enough to predict c_i, the production cost of the leader after adoption. It is because of the possibility that a jump pushes C_t below h_i. For this reason, the random variable C_{τ_i} appears in the above expression.

The function $V_L(.)$ described in (44), is the solution to the following boundary value problem:

$$\begin{cases} (r - L)V_L(c, c_i^0, c_j^0, h_i, h_j) = \Pi(c_i^0, c_j^0) & c > h_i \\ V_L(c, c_i^0, c_j^0, h_i, h_j) = W_L(c, c_j^0, h_j) - I & c \le h_i \end{cases} \quad (45)$$

Lemma 5-2 Value of leader, firm i, before reaching the adoption threshold (ie. $c > h_i$) is

$$\begin{aligned} V_L(c, c_i^0, c_j^0, h_i, h_j) = & r^{-1}\Pi(c_i^0, c_j^0) \\ & - r^{-1}B^{-1}\sum_{k=1,2} a_k\left(\frac{r^{-1}(r - \Psi(1))}{\beta_k - 1}h_i + \frac{rIB - c_i^0}{\beta_k}\right)\left(\frac{c}{h_i}\right)^{\beta_k} \\ & + \gamma r^{-1}B^{-1}\sum_{k=1,2} a_k\left(\frac{r^{-1}(r - \Psi(1))}{\beta_k - 1}h_j - \frac{c_j^0}{\beta_k}\right)\left(\frac{c}{h_j}\right)^{\beta_k}. \end{aligned} \quad (46)$$

Note that for $c > \max\{h_i, h_j\}$ it holds that

$$\hat{V}_F(c, c_i^0, c_j^0, h_i, h_j) = V_L(c, c_i^0, c_j^0, h_i, h_j), \tag{47}$$

suggesting that the expected payoff before any adoption is continuous in adoption thresholds at $h_i = h_j$.

5.5 The leader's Problem before Reaching the Preemption Zone

In this subsection I consider asymmetric firms and assume that the threat of being preempted in the preemption zone exists. I assume that firm 1 is less efficient than firm 2 (ie. $c_1^0 > c_2^0$). I also assume that for $i = 1, 2$, firm i's strategy at any $c < p_2$ is such that

$$\alpha_{i,p}^{asym}(c) = \begin{cases} \dfrac{d_j(c)}{L_j(c) - S(c)} & \text{if } p < c < p_2 \\ 0 & \text{if } h(c_i^0) < c \leq p \\ 1 & \text{if } c \leq h(c_i^0) \end{cases}. \tag{48}$$

Also I assume that $\alpha_{2,p}^{asym}(c) = 0$ for $c \geq p_2$. I study the problem of the leader before the preemption zone is reached. (ie. for $c \geq p_2$)

Let $\tau_w(p)$ denote the random time that either $C_t < p_2$ for the first time or firm 1 chooses to adopt a new technology. Assuming that adoption of technology by firm 1 happens when threshold $h_w(p)$ is reached, it holds that $\tau_w(p) = \inf\{t \geq 0 | C_t \leq h_w(p)\}$. Let c denote the realization of C_t. The value of firm 1 before $\tau_w(p)$, is a function of $h_1 = h(c_1^0)$ and $h_2 = h(c_2^0)$. The variables h_1 and h_2 are the adoption thresholds for the case when the interval (p, p_2) is bypassed by a jump. At any $c > h_w(p)$, the value of firm 1 is

$$V_w(c, c_1^0, c_2^0, h_1, h_2, p) = E^c\left[\int_0^{\tau_w(p)} e^{-rs}\Pi(c_1^0, c_2^0)ds + e^{-r\tau_w(p)}V_\Omega(C_{\tau_w(p)}, c_1^0, c_2^0, h_1, h_2)\right]. \tag{49}$$

In the above expression for the value function, the first term is the value the leader receives from production while both firms are using old technologies. The function $V_\Omega(.)$ is the continuation payoff when $h_w(p)$ is crossed. Hence, the second term is the discounted value of the leader at

$\tau_w(p)$. Before the leader's adoption, knowing $h_w(p)$ is not enough to predict C_{τ_i}. It is because of the possibility that a jump pushes C_t below $h_w(p)$. For this reason, the random variable C_{τ_i} appears in the above expression.

The function $V_\Omega(.)$ is the continuation payoff when $h_w(p)$ is crossed. Because of jumps, there are three scenarios for $C_{\tau_w(p)}$. First, it may jump into the set $(0, p]$. If this happens, according to the equilibrium strategies, firm 1 assumes the leader role in that region. Second, it may fall inside (p, p_2) and firm 1's continuation payoff is equal to $V_F(C_{\tau_w(p)}, c_1^0, C_{\tau_w(p)}, h_1)$ in this case. Note that in equilibrium firm 1 is indifferent between any action inside (p, p_2). A third possible scenario is $C_{\tau_w(p)} \geq p_2$ and firm 1 adopts immediately. Hence, the continuation payoff is

$$V_\Omega(c, c_1^0, c_2^0, h_1, h_2, p) = \begin{cases} W_L(c, c_2^0, h_2) - I & \text{if } p_2 \leq c \\ V_F(c, c_1^0, c, h_1) & \text{if } p < c < p_2 \\ V_L(c, c_1^0, c_2^0, h_1, h_2) & \text{if } c \leq p \end{cases} . \tag{50}$$

It follows from (49) that the value function $V_w(.)$, is a solution to the following problem:

$$\begin{cases} (r - L)V_w(c, c_1^0, c_2^0, h_1, h_2, p) = \Pi(c_1^0, c_2^0) & c > h_w(p) \\ V_w(c, c_1^0, c_2^0, h_1, h_2, p) = V_\Omega(c, c_1^0, c_2^0, h_1, h_2, p) & c \leq h_w(p) \end{cases} \tag{51}$$

Lemma 5-3 If $c_1^0 > c_2^0$, for $c > h_w(p)$ value of the firm with less efficient technology (firm 1) is

$$V_w(c, c_1^0, c_2^0, h_1, h_2, p) =$$
$$r^{-1}\Pi(c_1^0, c_2^0) + \gamma r^{-1} B^{-1} \sum_{k=1,2} a_k \left(\frac{r^{-1}(r - \Psi(1))}{\beta_k - 1} h_2 - \frac{c_2^0}{\beta_k} \right) \left(\frac{c}{h_2} \right)^{\beta_k}$$
$$- r^{-1} B^{-1} \sum_{k=1,2} a_k \left(\frac{r^{-1}\eta M(p)}{\beta_k + \lambda} h_w(p)^{-\lambda} + \frac{r - \Psi(1)}{r(\beta_k - 1)} h_w(p) + \frac{rIB - c_1^0}{\beta_k} \right) \left(\frac{c}{h_w(p)} \right)^{\beta_k} \tag{52}$$

where

$$\begin{aligned}M(p) &= p_2^{\lambda}\left(rIB + \frac{\lambda(1+\gamma)}{1+\lambda}p_2 - \gamma c_2^0 - c_1^0\right)\\ &+ p^{\lambda}\gamma\left[c_2^0 - \frac{\lambda p}{1+\lambda} + \sum_{k=1,2}\frac{a_k\lambda}{\beta_k+\lambda}\left(\frac{r^{-1}(r-\Psi(1))}{\beta_k-1}h_2 - \frac{c_2^0}{\beta_k}\right)\left(\frac{p}{h_2}\right)^{\beta_k}\right]\\ &- p_2^{\lambda}\sum_{k=1,2}\frac{a_k\lambda}{\beta_k+\lambda}\left[\left(\frac{r^{-1}(r-\Psi(1))}{\beta_k-1}h_1 + \frac{rIB-c_1^0}{\beta_k}\right)\left(\frac{p_2}{h_1}\right)^{\beta_k}\right.\\ &\left.+\gamma\left(\frac{r^{-1}(r-\Psi(1))}{\beta_k-1}h_2 - \frac{c_2^0}{\beta_k}\right)\left(\frac{p_2}{h_2}\right)^{\beta_k}\right] \qquad (53)\end{aligned}$$

Lemma 5-4 If

$$rIB + r^{-1}(r-\Psi(1))p_2 - c_1^0 + r^{-1}\eta M(p)p_2^{-\lambda} \geq 0 \qquad (54)$$

the optimal value of $h_w(p)$ that maximizes V_w is equal to p_2. Otherwise, the optimal $h_w(p)$ is equal to the value of c that solves

$$rIB + r^{-1}(r-\Psi(1))c - c_1^0 + r^{-1}\eta M(p)c^{-\lambda} = 0. \qquad (55)$$

6- Conclusion

In the past, adoption of a new technology under uncertainty about future innovations has been studied. Also, there are works that analyze the effect of competition on adoption strategies. This paper links these efforts and studies strategic technology adoption when cutting edge technology improves over time and there is uncertainty about when a breakthrough will happen and how large it will be.

I find that two kinds of perfect equilibria may exist. First, an equilibrium where both firms adopt at optimal thresholds, and I find that it always exists. Second, an equilibrium in which the threat of being preempted exists. In the latter, the equilibrium strategy of the less efficient firm is to adopt much earlier than the optimal time to secure the position of being the first mover. The result that the less efficient firm wishes to be the first mover more than the more efficient firm is consistent with normal payoff functions. It is because the less efficient firm would gain much more than the more efficient firm from adoption of a new technology. By the same token, the less efficient firm would lose much more than the more efficient firm if preempted by the rival.

The model developed in this study can be extended to answer more questions regarding the adoption behavior of firms. Further research in this area could involve endogenizing the parameters of the technology evolution process to study the behavior of the firm or the whole industry regarding the R&D expenditures. Another extension of the model would be to allow the firms to adopt more than once.

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