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Effect of grain size on the natural frequencies of high-strength steel HT-80

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ABSTRACT

Ultrafine-grained steels offer the prospect of high strength compared with traditional steel. In this article, the vibration responses of a beam as a function of the grain size of the material in HT-80 steel are investigated by an analytical approach. First, the relation between Young's modulus and grain diameter in HT-80 steel is obtained based on the experimental results using curve fitting in the form of a mathematical equation. Then, governing equations of the cantilever beam and also associated boundary conditions are derived based on Hamilton's principle using obtaining the total kinetic and potential energies of the system. After that, the natural frequencies of the system are determined using an analytical approach. Finally, numerical results of the natural frequencies of the system are presented concerning different values of the system parameters such as thickness, width, length, and grain size of the material. The obtained results show that the grain diameter of the material and also the dimensions of the beam such as thickness in the micro-scale have significant effects on the vibration response of the system. The presented approach can be used to estimate the vibration characteristics of ultrafine-grained steels and also microsystems such as piezoelectric cantilever-based MEMS sensors.

Keywords: *Grain size; Natural Frequencies; Size-dependency; Free vibration.*

1. Introduction

Young's modulus, as one of the mechanical properties, is an important technological and structure-sensitive parameter that is dependent on the grain size in ultrafine-grained materials. For many years, research on ultrafine-grained metals carried out all over the world due to their excellent mechanical properties [1-4]. Tanaka and Iizuka [5] studied the effects of grain size and microstructure on Young's modulus and internal friction for a high-strength steel HT-80. They found that Young's modulus is large in both coarse-grained and ultrafine-grained specimens for sorbate and ferrite-pearlite structures. The microstructure of ultrafine-grained steel is characterized by X-ray diffraction, scanning electron microscopy, transmission electron microscopy, and Nanoindentation techniques [6]. Zhang et al. [7] examined the effect of nanoscale reinforcement on the mechanical behavior of ultrafine-grained composites. They concluded that the presence of nanoparticles enhances strength by interacting with dislocations, while simultaneously retarding grain growth.

The vibration characteristics study of Micro-Electro-Mechanical-System (MEMS) devices is very important to the design and optimization of micro components in small equipment. Nowadays, the manufacturing of MEMS sensors is possible through the development of fabrication technologies [8-11]. A few of the researchers investigated the effect of grain size on the natural frequencies and mechanical properties of materials in the macro/micro dimension. Saffar et al. [12] studied the correlation of the natural frequency of thin solar silicon wafers with material grain size and grain orientation using the nonlinear finite element method. They found that the natural frequency is a strong function of material orientation for an anisotropic single-crystal silicon wafer.

Su et al. [13] presented a core-shell model with a grain-boundary affected zone to study the grain-size effect of nanocrystalline polycrystal frequency. Hahn and Meyers [14] studied the theories of grain size-dependent mechanical behavior pertaining to the nanocrystalline regime. Gholami and Ansari [15] investigated the effect of grain size, grain surface energy, and small-scale effects on the nonlinear pull-in instability and free vibration of electrostatic nanoscale actuators made of nanocrystalline silicon (Nc-Si). They studied the influences of various parameters such as the length scale parameter, the volume fraction of the inclusion phase, density ratio, and average inclusion radius on the pull-in instability and free vibration of Nc-Si actuators.

Shatt [16] studied the effects of the inhomogeneity nature of NCMs on the bending behavior of NCM beams. They showed that the bending stiffness of the beam is agreed upon for the grain size effects and the microstructure rigid rotation effects.

In the present article, the vibration behavior of a cantilever beam with respect to the grain size of the material in HT-80 steel is investigated by an analytical approach. First, the mathematical relation between grain diameter and Young's modulus in HT-80 steel is determined based on the experimental results which are presented in reference [5]. Then, governing equations of the cantilever beam and also related boundary conditions are derived using Hamilton's principle. The natural frequencies of the system are calculated using the analytical approach. Finally, numerical results of the natural frequencies of the system are presented concerning different values of the system parameters such as thickness, width, length, and grain size of the material. The results show that the grain diameter of the material and also the dimensions of the beam such as thickness in the micro-scale have significant effects on the vibration response of the system.

2. The relation between Young's Modulus and Grain Diameter

Fig. 1 shows the experimental results of the relation between Young's modulus and grain diameter of specimens in HT-80 steel [5]. It can be seen that there are different values for Young's module with respect to the average of the grain diameter.

Using experimental results which have been presented by Tanaka et al. [5], the mathematical equation between Young's modulus with respect to the grain diameter has been depicted in Fig. 2. According to Fig. 2, the equation between the grain diameter d_g and Young's module E using curve fitting can be written as follows:

$$E = 6.4812\,x^2 - 22.939x + 224.68 \qquad (1)$$
$$x = \mathrm{Log}_{10}(d_g)$$

Using Eq. 1, it can be seen that the minimum value of Young's modulus E will occur at the following point:

$$d_g\big|_{min} = 58.838\ \ \mu m \quad \Rightarrow \quad E_{min} = 204.3829\ \mathrm{GPa} \qquad (2)$$

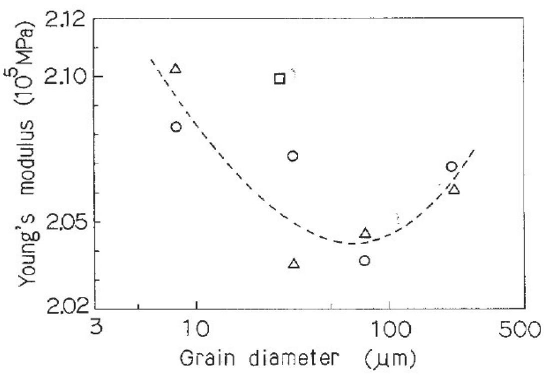

Fig. 1- Experimental results of the relation between grain size and Young's modulus in HT-80 steel; ○: Sorbite; Δ: ferrite-pearlite; □: as-received [5].

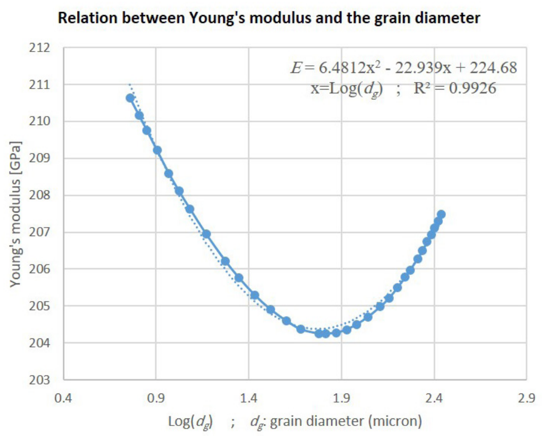

Fig. 2- The mathematical equation of the relation between the grain size and Young's modulus in HT-80 steel.

It should be noted that by increasing the grain size the single crystal material will be created.

It should be noted that in the general form of HT-80 steel, the mathematical equation of Young's modulus with respect to the grain diameter d_g can be expressed, approximately, as follows (please see Fig. 3):

$$\text{for } d_g < 5\mu m\,; \quad E=E_{S1} \qquad (3)$$
$$\text{for } 5\mu m < d_g < 300\mu m\,;$$
$$E=6.4812[\mathrm{Log}_{10}(d_g)]^2-22.939\mathrm{Log}_{10}(d_g)+224.68$$
$$\text{for } d_g > 300\mu m\,; \quad E=E_{S2}$$

where E_{S1} and E_{S2} are the maximum Young's modulus of the material with the ultrafine-grained and large grain size (for example single crystal) cases, respectively. Values of E_{S1} and E_{S2} are equal to 215.17 GPa and 211.88 GPa, respectively.

3. Definition and Modeling of the System

A cantilever beam is used to study the vibration behavior of a system with respect to the variation of the size grain diameter of the material. The modeling and geometry of the cantilever beam are shown in Fig. 4. The cantilever beam has length L, width b, thickness h, density ρ, Young's modulus E, and Poisson's ratio ϑ. Also, the coordinate system X-Y-Z has been shown in Fig. 4. It should be noted that in this research the cantilever beam has been made from HT-80 steel which the elasticity modulus of the material can be obtained from Eq. 3.

4. Governing Equations

The strain energy of a linear elastic isotropic material can be written as follows:

$$\pi_s = \frac{1}{2}\int_V (\sigma_{ij}\varepsilon_{ij})\, dV \quad ; \quad i = x, y, z \qquad (4)$$

where V denotes the volume of the system. In addition, the components of the stress tensor σ_{ij} and the strain tensor ε_{ij} can be expressed as follows:

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} \quad ; \quad \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \qquad (5)$$

In the above equations, u_i are components of the displacement vector, and the parameters λ and μ are Lamé constants. The Lamé constants can also be written regarding Young's modulus E and Poisson's ratio ϑ as follows:

$$\lambda = \frac{\vartheta E}{(1+\vartheta)(1-2\vartheta)} \quad ; \quad \mu = \frac{E}{2(1+\vartheta)} \qquad (6)$$

Consider that the w(x,t) denotes the transverse deflection of the neutral line of the beam at any point x along the length of the beam in the Z direction (please see Fig. 4). By using Euler-Bernoulli beam theory, the displacement field at any material point in the beam can be expressed as follows:

$$u_1 = z\frac{\partial w(x,t)}{\partial x} \quad ; \quad u_2 = 0 \quad ; \quad u_3 = w(x,t) \qquad (7)$$

Assuming small transverse deflection, the non-zero components of the strain and the stress tensors can be expressed as follows:

$$\varepsilon_{xx} = z\frac{\partial^2 w}{\partial x^2} \qquad (8)$$

$$\sigma_{xx} = E\varepsilon_{xx} = E\,z\frac{\partial^2 w}{\partial x^2} \qquad (9)$$

Therefore, from Eq. (4), the total strain energy of the system can be obtained as follows:

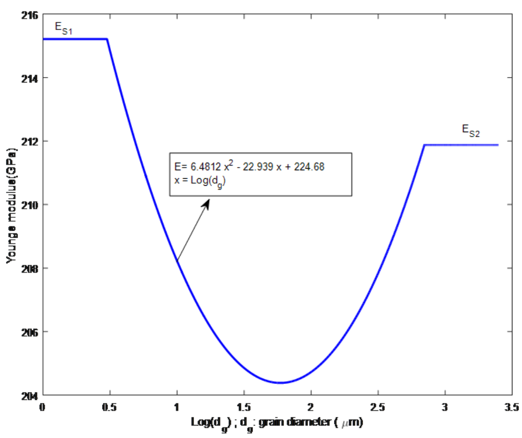

Fig. 3- General form of the mathematical equation of Young's modulus with respect to the grain diameter in HT-80 steel.

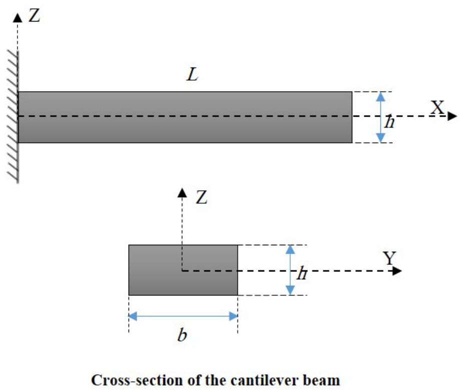

Fig. 4- The modeling and the geometry of the cantilever beam.

$$\pi_s = \frac{1}{2}\int_L \left[\int_A (\sigma_{ij}\varepsilon_{ij})\, dA\right] dx = \frac{1}{2}\int_L \left[\int_A (\sigma_{xx}\varepsilon_{xx})\, dA\right] dx$$
$$= \frac{1}{2}\int_0^L \left[(EI_{yy})\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right] dx \qquad (10)$$

where

$$I_{yy} = \frac{b\,h^3}{12} \quad ; \quad A = bh \qquad (11)$$

Therefore, for a uniform beam with an anisotropic elastic material and a rectangular section, the total strain energy of the system can be written as follows:

$$\pi_s = \frac{1}{2}(EI_{yy})\int_0^L \left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right] dx \qquad (12)$$

The kinetic energy of the system T can be written as follows:

$$T = \frac{1}{2}\int_0^L \left[(\rho A)\,\dot{w}^2 + (\rho\, I_{yy})\left(\frac{\partial \dot{w}}{\partial x}\right)^2\right] dx \qquad (13)$$

where the dot over variables is the derivative of the variable relative to time.

For free vibration analysis of the system without external non-conservative forces, Hamilton's principle is considered as follows:

$$\int_{t_1}^{t_2} \delta(T - \pi_s)\, dt = 0 \qquad (14)$$

By substituting the Eqs. (12) and (13) into (14), and then using variational calculus, governing equations of motion of the system can be derived as follows:

$$(EI_{yy})\frac{\partial^4 w}{\partial x^4} + (\rho A)\,\ddot{w} - (\rho\, I_{yy})\frac{\partial^2 \ddot{w}}{\partial x^2} = 0 \qquad (15)$$

By neglecting $\frac{\partial^2 \ddot{w}}{\partial x^2}$, the Eq. (15) can be simplified as follows:

$$(EI_{yy})\frac{\partial^4 w}{\partial x^4} + (\rho A)\,\ddot{w} = 0 \qquad (16)$$

The general solution of Eq. (16) can be expressed as follows:

$$w(x,t) = W(x)\sin(\omega t) \qquad (17)$$

By substituting Eq. (17) into Eq. (16) and to have some algebraic simplification, we have

$$\frac{d^4 W}{dx^4} - \beta^4 W(x) = 0 \qquad (18)$$
$$\beta^4 = \frac{(\rho A)\,\omega^2}{EI_{yy}}$$

where ω is the natural frequency of the system. Also, the general solution of Eq. (18) can be obtained as follows: (19)

$$W(x) = \tilde{B}_1\sin(\beta x) + \tilde{B}_2\cos(\beta x) + \tilde{B}_3\sinh(\beta x) + \tilde{B}_4\cosh(\beta x)$$

where $\tilde{B}_1$, $\tilde{B}_2$, $\tilde{B}_3$ and $\tilde{B}_4$ are constants.

Also, the boundary conditions of the cantilever beam system can be expressed as follows:

$$W(0) = 0 \;\; ; \;\; \frac{dW}{dx}(0) = 0 \qquad (20)$$
$$\frac{d^2W}{dx^2}(L) = 0 \;\; ; \;\; \frac{d^3W}{dx^3}(L) = 0$$

By substituting Eq. (20) into Eq. (19), a set of four algebraic equations resulting in matrix form can be obtained as follows: (21)

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ -\beta & 0 & -\beta & 0 \\ -\beta^2\sin(\beta L) & -\beta^2\cos(\beta L) & \beta^2\sinh(\beta L) & \beta^2\cosh(\beta L) \\ -\beta^3\cos(\beta L) & \beta^3\sin(\beta L) & \beta^3\cosh(\beta L) & \beta^3\sinh(\beta L) \end{bmatrix} \begin{Bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \\ \tilde{B}_3 \\ \tilde{B}_4 \end{Bmatrix} = 0$$

For the nontrivial solution of Eq. (21), the determinant of the matrix $[Q_{ij}]$ must be zero. Also, if the determinant of the matrix $[Q_{ij}]$ is zero, the result and the first, second, third, and fourth roots of it can be calculated as follows: (22)

$$\det[Q_{ij}] = 0 \;\Rightarrow\; \cos(\beta_n L)\cosh(\beta_n L) = -1; \;\; n = 1, \ldots, \infty$$
$$\beta_1 L = 1.87510; \qquad \beta_2 L = 4.69409; \qquad \beta_3 L = 7.85476;$$
$$\beta_4 L = 10.99554$$

Therefore, according to Eqs. (18), the natural frequencies of the cantilever beam system can be determined as follows:

$$\omega_n = (\beta_n L)^2\left[\frac{E\, I_{yy}}{\rho\, A\, L^4}\right]^{\frac{1}{2}} \qquad (23)$$

where E denotes Young's modulus of the material can be written with respect to the average of the grain diameter d_g in HT-80 steel as follows: (24)

$$E = 6.4812\left[\mathrm{Log}_{10}(d_g)\right]^2 - 22.939\left[\mathrm{Log}_{10}(d_g)\right] +$$
for $5\mu m < d_g < 300\mu m$

5. Results and Discussion

In this section, the effect of grain size in HT-80 on the natural frequencies of the system is studied. In numerical analyses of a macro system, the following nominal dimensions data and materials properties in Table 1 have been considered. Also, to study a microsystem, the following nominal dimensions data in Table 2 have been mentioned.

The first natural frequency of the macro cantilever beam (please see Table 1) versus the grain diameter (d_g) has been depicted in Fig. 5, for different values of the beam length L. The obtained results in Fig. 5 show that the first natural frequencies of the macro system will decrease and then increase with an increase in the grain diameter. Also, the first natural frequencies of the system will increase with an increase in the beam length L. It should be noted that there is interesting behavior on the natural frequencies with the increase of the grain diameter d_g which can be considered to identify the microstructure of material with the measurement of its natural frequencies. The above behavior for the second and third natural frequencies of the macro cantilever system can also be observed in Figs. 6 and 7, respectively.

In addition, the first and second natural frequencies of the microcantilever beam (microsystem) versus the thickness h have been presented in Figs. 8 and 9, respectively, for different values of the microbeam length L. The correction

Table 1- Nominal dimensions and material properties of the macro cantilever beam which has been made from HT-80 steel

Parameters description	Symbol	unit	Value
Length	L	mm	60
Width	b	mm	20
Thickness	h	mm	2
Young's modulus (E) (for $5\mu m < d_g < 300\mu m$)	E	GPa	$6.4812\,x^2 - 22.939x + 224.68$; $x = Log_{10}(d_g)$
Density [5]	ρ	Kg/m^3	7850
Poisson's ratio [5]	ϑ	---	0.28

Table 2- Nominal dimensions of the microcantilever beam which has been made from HT-80 steel

Parameters description	Symbol	unit	Value
Length	L	μm	800
Width	b	μm	300
Thickness	h	μm	17

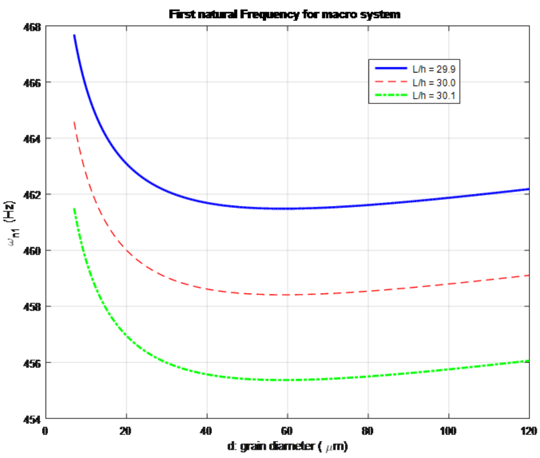

Fig. 5- Variation of the first natural frequency of the macro cantilever beam versus the grain diameter d_g for different values of the beam length.

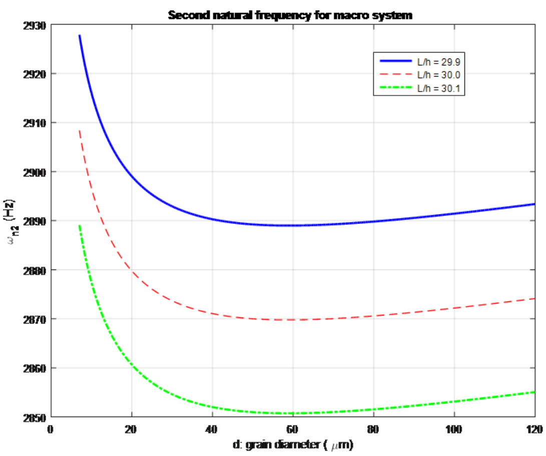

Fig. 6- Variation of the second natural frequency of the macro cantilever beam versus the grain diameter d_g for different values of the beam length.

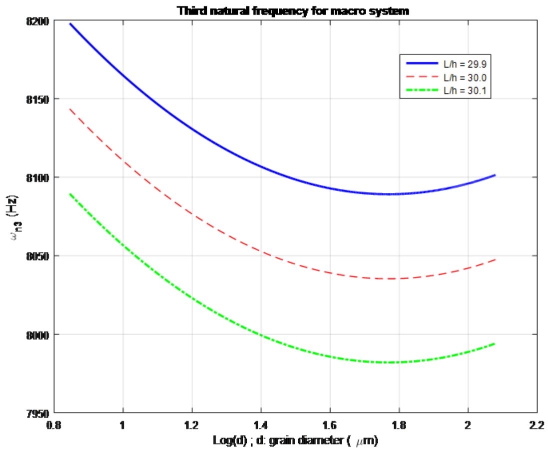

Fig. 7- Variation of the third natural frequency of the macro cantilever beam versus the grain diameter d_g for different values of the beam length.

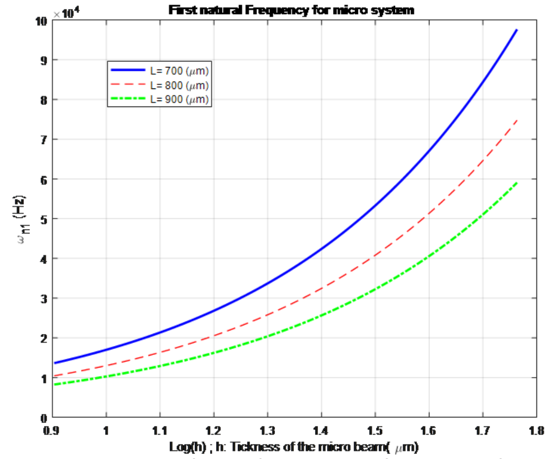

Fig. 8- Variation of the first natural frequency of the microcantilever beam versus the thickness h for different values of the beam length.

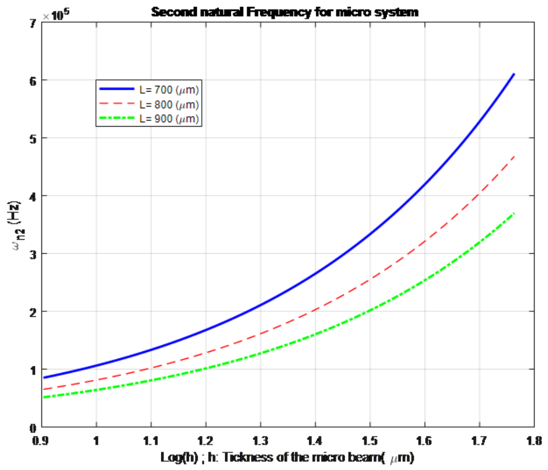

Fig. 9- Variation of the second natural frequency of the microcantilever beam versus the thickness h for different values of the beam length.

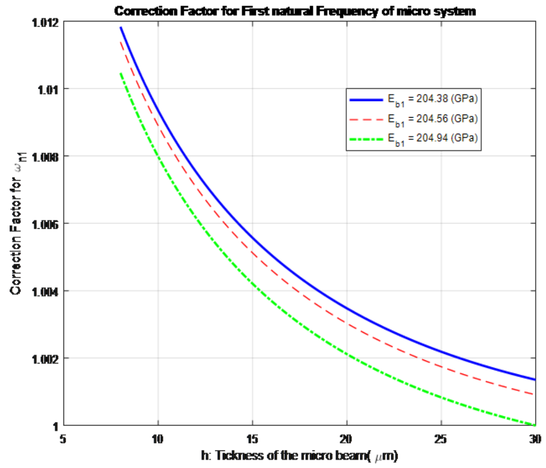

Fig. 10- Correction factor for first natural frequency of the microcantilever beam versus the thickness h for different values of the beam base Young's modulus E_b.

factor of the first and second natural frequencies of the microcantilever beam system versus the thickness h has been investigated in Figs. 10 and 11, respectively, for different values of Young's modulus E_b of the microbeam. The obtained results in Figs. 10 and 11 show that correction factor for first and second natural frequencies of the microsystem will decrease when the thickness h increases.

6. Summary and Conclusion

In this article, the vibration behavior of macro and microcantilever beams with respect to the grain size of the material in HT-80 steel was studied by an analytical approach. To capture size effects and also to predict the natural frequency behavior of the microsystem, governing equations

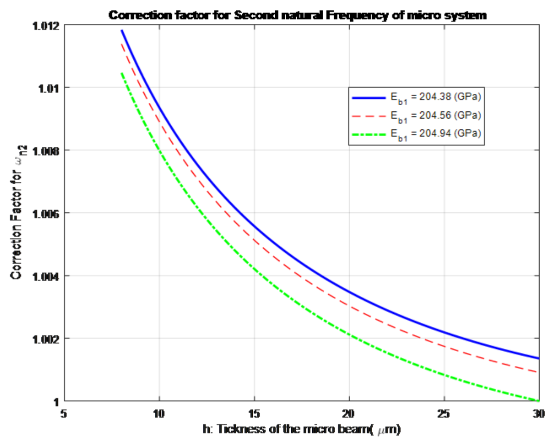

Fig. 11- Correction factor for first natural frequency of the microcantilever beam versus the thickness h for different values of the beam base Young's modulus E_b.

were derived using Hamilton's principle for a cantilever beam. Also, according to Ref. [5], the equation between the grain diameter and Young's modulus of HT-80 steel was obtained by using curve fitting. The analytical method was employed in the solution of the partial differential equations considering the effect of grain size on the young's modulus of the system. The first, second, and third natural frequencies of the system for various values of the grain diameter, beam thickness, and beam length was investigated. Also, correction factors for the first and second natural frequencies of a micro cantilever beam versus the micro beam thickness were obtained. The obtained results show that the grain diameter of the material and also the dimensions of the beam in the micro-scale have significant effects on the vibration behavior of the system.

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