

## Seismic Behavior Evaluation of Concrete Elevated Water Tanks

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**ABSTRACT:** Elevated tanks are important structures in storing vital products, such as petroleum products for cities and industrial facilities, as well as water storage. These structures have various types and are constructed in a way that a greater portion of their weight is concentrated at an elevation much about the base. Damage to these structures during strong ground motions may lead to fire or other hazardous events. In this research, a reinforced concrete elevated water tank, with 900 cubic meters capacity, exposed to three pairs of earthquake records was analyzed in time history using mechanical and finite-element modeling techniques. The liquid mass of the tank was modeled as lumped mass known as sloshing mass, or impulsive mass. The corresponding stiffness constants associated with the lumped mass were determined depending upon the properties of the tank wall and liquid mass. Tank responses including base shear, overturning moment, tank displacement, and sloshing displacement were also calculated. Obtained results revealed that the system responses are highly influenced by the structural parameters and the earthquake characteristics such as frequency content.

**Keywords:** Base Shear, Earthquake Characteristics, Fluid-Structure Interaction, Overturning Moment, Seismic Behavior, Sloshing Displacement.

### INTRODUCTION

Elevated tank structures are normally used to store water for domestic activities and also fire fighting purposes. Their safety performance is a critical concern during strong earthquakes. The failure of these structures may cause serious hazards for citizens due to the shortage of water or difficulty in putting out fires during earthquakes. Some elevated tanks have shown insufficient seismic resistance in past earthquakes which had prevented the fire

fighting process and other emergency response efforts (Barton and Parker, 1987). There have been several studies in which the dynamic behavior of liquid storage tanks have been analyzed, however most of them have focused on ground level cylindrical tanks, and very few of them have concentrated upon behavior of elevated tanks. They are heavy structures which a greater portion of their weight is concentrated at an elevation much about the base. Critical parts of the system are columns and braces through which the loads

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are transmitted to the foundation. Due to the high sensitivity of elevated water tanks to earthquake characteristics such as frequency contents, peak ground acceleration and effective duration of the earthquake records, it seems necessary to ponder the earthquake loading as a non-stationary random pattern.

## PAST EXPERIENCES

Some of recent and important studies on elevated liquid tanks are presented in this section. Haroun and Ellaithy (1985) developed a model for analyzing elevated rigid tanks exposed to shifting and rotation. Resheidat and Sunna (2001) investigated the behavior of a rectangular elevated tank considering the soil-foundation structure interaction during earthquakes. They neglected the sloshing effects on the seismic behavior of elevated tanks and the radiation damping effect of soil. Haroun and Temraz (2001) analyzed two-dimensional x-braced elevated tanks supported on the isolated footings in order to investigate the impact of dynamic interaction between the tower and the supporting soil-foundation system. In this study, they neglected the sloshing effects. Marashi and Shakib (2008) carried out an ambient vibration test for the evaluation of dynamic characteristics of elevated tanks. Dutta (1997) proposed alternate tank staging configurations for reduced torsional vulnerability. Dutta (2000) studied the supporting system of elevated tanks with reduced torsional vulnerability so that they suggested approximate empirical equations for the lateral, horizontal and torsional stiffness for different frame supporting systems. Dutta (2001) also investigated how the inelastic torsional behavior of a tank system with accidental eccentricity varies with the increasing number of panels. Subsequently,

Dutta (2002) showed that the soil-structure interaction could cause an increase in base shear particularly for elevated tanks with low structural periods. Livaoglu and Dogangun (2009) investigated the seismic behavior of fluid-elevated tank-foundation-soil systems in domain frequency. Livaoglu and Dogangun (2010) suggested a simple analytical procedure for seismic analysis of fluid-elevated tank-foundation-soil systems. Livaoglu (2011) conducted a comparative study on the seismic behavior of elevated tanks considering both fluid-structure and soil-structure interaction effects. Livaoglu and Dogangun (2008) studied the impact of foundation embedment on the seismic behavior of elevated tanks taking fluid-structure-soil interaction into account.

## ELEVATED TANK CHARACTERISTICS

In this research, a reinforced concrete elevated tank with support systems is considered. This elevated tank is placed on a framed structure, and the elevation and capacity of this tank is 32 meters and 900 cubic meters, respectively. Details of the elevated tank are shown in Figures 1 and 2. This sort of tanks and supporting systems are widely used in recent years worldwide (Barton, 1987). The specifications of this tank including mechanical properties considered for the steel, concrete and water are shown in Table 2.

## MODELING DETAIL

A Finite Element Model (FEM) is used to model the elevated tank system. To do this the Abacus software is utilized, which is a well-known and appropriate software. Columns and beams in the support system

are modeled as frame elements (with six degrees-of-freedom per node) and the truncated cone and container walls are modeled with quadrilateral shell elements (with four nodes and six degrees of freedom per node). Fluid-structure interaction problems can be investigated using different techniques such as added mass (AM) (Livaoglu, 2007) Lagrangian (LM) (1999), Eulerian (EM) (2002), and Lagrangian–Eulerian (L-E M) (2005) approaches in the FEM or by the analytical methods like Housner’s two-mass representation (1989) or multi-mass representations of Bauer (2006) and EC-8 (2003). In this research, Housner’s added mass approach is selected. In Housner’s analytical model of mass-spring (Housner, G.F., 1963), the fluid is modeled as a centered mass model and two impulsive and convective mass are used instead of the fluid. The parameters of the fluid are calculated using Housner’s relations, which are stated in Table 1. Three cases of completely filled, half filled, and empty tanks are considered in this study.

In the added mass method, the fluid mass being calculated through different methods, such as Housner or Bour methods, is added to the structure mass in the common level of the structure and fluid. For a system under earthquake motions, the equation of motion can be written as Eq. (1):

$$M\ddot{u} + C\dot{u} + Ku = -M\ddot{u}_g \tag{1}$$

where M, K, and C are the mass, stiffness and damping matrix respectively,  $\ddot{u}_g$  and  $u$  indicate the gravity acceleration and displacement varying with time, respectively. If the added mass approach is used, the above Eq. 1 can be written as Eq. (2):

$$M^*\ddot{u} + C\dot{u} + Ku = -M^*\ddot{u}_g \tag{2}$$

where  $M^*$  is the total mass matrix which includes M as the structure mass matrix and  $M_a$  as the added mass. In this method, it is assumed that  $M_a$  is vibrated simultaneously with the structure. Hence, M is added due to the fluid effects while C and K do not change significantly.

Performing the free vibration analysis, the tank dynamic properties, consisted of the period and modal partnership mass ratio, are obtained (see Table 3). Sum of the structure’s first six modes partnership is more than 90 percent. Considering the appropriate model of mass-spring that models the tank in two masses of impulsive and convective, there will be two different modes. The convective mass is joints to the container wall as a spring and the impulsive mass as a rigid.

**Table 1.** Mass-spring model parameters for filled and half filled cases.

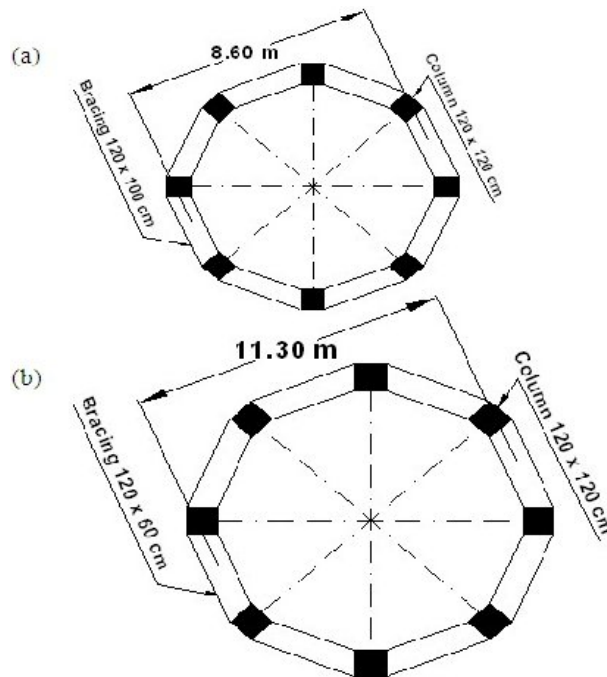
Parameters	Filled	Half-filled	Dimension
$m_i$	$6.30 \times 10^4$	$1.87 \times 10^4$	kgf
$m_c$	$2.79 \times 10^4$	$2.47 \times 10^4$	kgf
$h_i$	3.30	1.65	m
$h_c$	5.95	2.48	m
$k_c$	83.96	65.30	KN/m
H	8.80	4.40	m
R	6.0	6.0	m

**Table 2.** Tanks and material property.

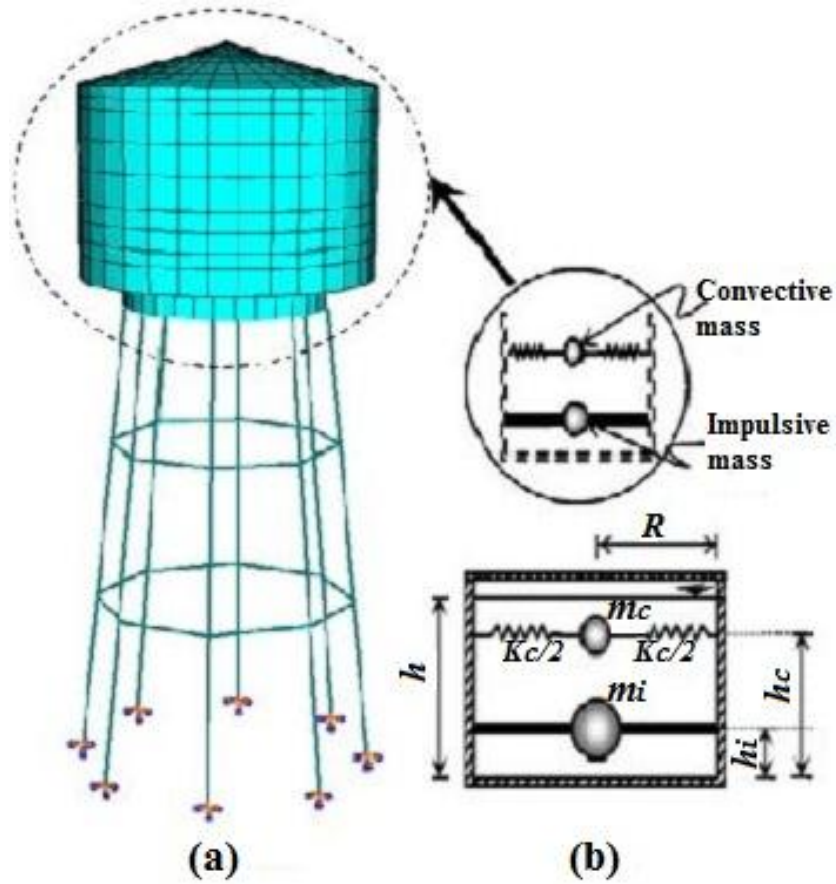
Tank vessel property (m)		Tank staging property (m)	
Geometric and section	Dimensions	Geometric and section	Dimensions
Inner diameter	12	Columns dimensions	1.20 × 1.20
Height	10.6	Columns height	7+7+6 = 20
Top Ring Beam	0.6 × 0.6	Staging inner diameter in top	8.60
Bottom Ring Beam	0.8 × 0.6	Staging inner diameter in bottom	12.75
Roof thickness	0.20	Beams dimensions in first floor	1.20 × 0.60
Vessel thickness	0.40	Beams dimension in second floor	1.20 × 0.60
Bottom slab thickness	0.50	Beams dimension in third floor	1.20 × 1.0

Material property			
	Concrete	Steel	Water
E (MPa)	$2.3 \times 10^4$	$2.1 \times 10^5$	-----
(MPa) $f'_c$	30	-----	-----
Weight of volume unit (KN/m <sup>3</sup> )	25	78.5	10
$F_y$ (MPa)	-----	240	-----



**Fig. 1.** (a) Arrangement of the columns and beams under the tank container;  
 (b) Arrangement of the columns and beam on the first storey.



**Fig. 2.** (a) Elevated tank modeling;  
(b) Housner's mass-spring model (Housner, 1963)

## GROUND MOTION

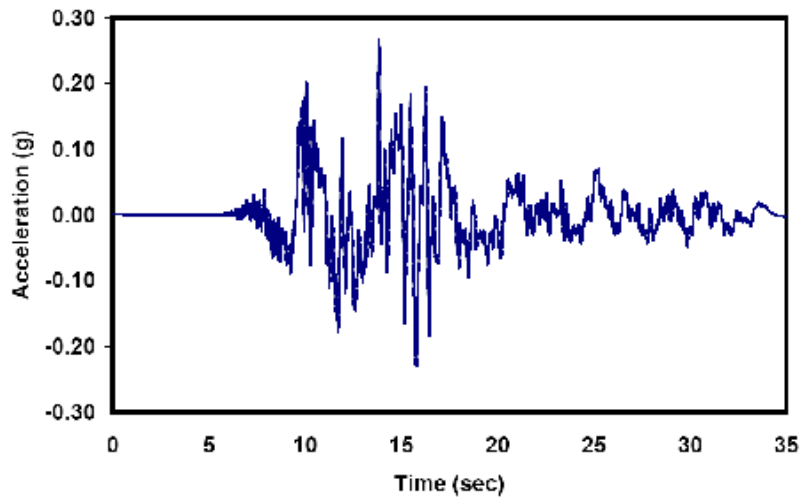
Three cases of filled, half filled, and empty are considered to assess the dynamic response of elevated tanks. Time history analysis has been undertaken using the above-mentioned equations. Rayleigh Damping is used in this analysis. In time history analysis, the tank is assumed to be in a C type soil according to the UBC-97 classification. Three pairs of earthquake records are used; the properties of the earthquake records are given in Table 4. The horizontal components of the Kocaeli earthquake acceleration are presented in

Figure 5 and the important values of response spectrum acceleration of the earthquakes are also given in Table 5. In accordance with Table 4, the maximum PGA on the basis of acceleration gravity for Kocaeli, Imperial Valley and Northridge records equal to 0.349, 0.485, and 0.843, respectively. The maximum PGV on the basis of cm/sec for the Kocaeli, Imperial Valley and Northridge records equal to 65.7, 76.6, 129.6 cm/sec, respectively. According to the UBC-97 code, the earthquake records should be scaled in 0.2T and 1.5T ranges considering the amount of natural frequency.

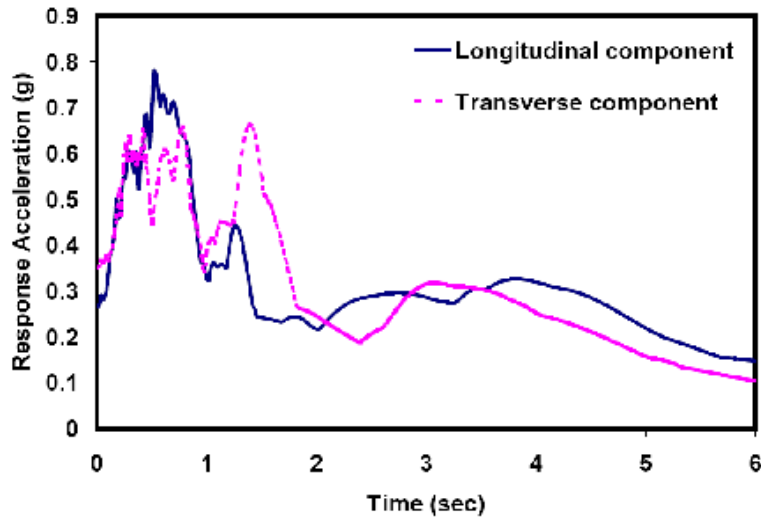
**Table 3.** Modal properties of the tank in filled, half filled, and empty cases.

State	Mode	1	2	3	4	5	6
Filled	T (sec)	3.68	1.03	0.73	0.20	0.13	0.12
	MPMR †	<b>8.60</b>	<b>83.90</b>	<b>0.00</b>	<b>2.90</b>	<b>3.60</b>	<b>0.00</b>
Half-filled	T (sec)	4.04	0.95	0.72	0.19	0.13	0.12
	MPMR	<b>8.11</b>	<b>83.42</b>	<b>0.00</b>	<b>3.55</b>	<b>3.83</b>	<b>0.00</b>
Empty	T (sec)	0.92	0.72	0.19	0.14	0.11	0.08
	MPMR	<b>90.4</b>	<b>0.00</b>	<b>3.51</b>	<b>4.80</b>	<b>0.00</b>	<b>0.03</b>

† Modal partnership mass ratio in percent



**Fig. 3.** Acceleration transverse component of Kocaeli earthquake.



**Fig. 4.** Response spectrum acceleration of Kocaeli earthquake in 5% damping.

**Table 4.** Used record properties.

Record		Imperial Valley (1979)		Record	Northridge (1994)		Record		Kocaeli, Turkey (1999)	
Station	El Centro	El Centro	Station	Sylmar - Olive	Sylmar - Olive	Station	Yarimca	Yarimca	YPT-	YPT-
Component	H-E04 -140	H-E04 -230	Component	SYL - 090	SYL - 360	Component	YPT- 060	YPT- 330	YPT-	YPT-
PGA (g)	0.485	0.36	PGA (g)	0.604	0.843	PGA (g)	0.268	0.349		
PGV (cm/s)	37.4	76.6	PGV (cm/s)	78.2	129.6	PGV (cm/s)	65.7	62.1		
PGD (cm)	20.23	59.02	PGD (cm)	16.05	32.68	PGD (cm)	57.01	50.97		
Duration (sec)	36.82	36.82	Duration (sec)	40	40	Duration (sec)	35	35		
M	6.5	6.5	M	6.7	6.7	M	7.4	7.4		

## RESULTS

The maximum responses are determined for different parameters of the elevated water tanks subjected to three pairs of the acceleration earthquake records. Table 5 shows the obtained maximum responses. These responses include base shear force, overturning moment, sloshing displacement, and roof displacement. As seen, the obtained maximum responses are different in three earthquake records. The maximum response in base shear is for the Northridge record in the half filled case; however, the maximum response in roof displacement is for the Northridge record in the filled case. Obtained time histories responses for each parameter are presented and their implications are studied.

## Base Shear Force

Figure 6 shows the variation of base shear forces against the percentage of capacity for the elevated water tanks in three earthquake records. The variation of base shear forces over the percentage of filling show that the maximum of base shear forces are happened for the half-full and full fillings. This may be due to the greater hydrodynamic pressures for half-full filling compared to full filled tanks. This pattern of variations is not the same for all the three earthquake records. Interestingly, the dynamic characteristics of the system and hydrodynamic influences considerably affect the amount of base shear forces. Also, the maximum time history of the base shear force for Northridge earthquake records in the half-filled case is presented in Figure 6.

**Table 5.** Seismic analysis results.

Parameter	Imperial Valley, (1979)		
Case of Filling	Full	Half Full	Empty
HW / HL †	1.00	0.50	0.00
Maximum Roof Displacement (cm)	20.33	17.96	16.99
Maximum Floor Container Displacement (cm)	24.19	21.58	23.29
Maximum Sloshing Displacement (cm)	101.20	67.50	0.00
Maximum Base shear (ton)	682.53	638.66	627.40
Maximum Overturning Moment (ton.m)	13300.81	12832.28	9510
Parameter	Northridge, (1994)		
Case of Filling	Full	Half Full	Empty
HW / HL †	1.00	0.50	0.00
Maximum Roof Displacement (cm)	17.79	20.33	19.58
Maximum Floor Container Displacement (cm)	21.11	24.94	23.29
Maximum Sloshing Displacement (cm)	121.7	54.81	0.00
Maximum Base shear (ton)	620.50	750.44	445.12
Maximum Overturning Moment (ton.m)	11821.73	11821.73	10270
Parameter	Kocaeli, Turkey, (1999)		
Case of Filling	Full	Half Full	Empty
HW / HL †	1.00	0.50	0.00
Maximum Roof Displacement (cm)	10.63	9.76	10.52
Maximum Floor Container Displacement (cm)	12.62	14.65	18.25
Maximum Sloshing Displacement (cm)	186.67	219.18	0.00
Maximum Base shear (ton)	564.50	474.65	480.83
Maximum Overturning Moment (ton.m)	7781.12	6375.54	5280.12

†  $H_w$ : Water height in vessel;  $H_L^T$ : Vessel Height

### Overturning Moment

The variation of maximum overturning moment against the percentage of tank capacity is presented in Figure 7. The maximum response happens in a case that the tank is full filled. An increase in the percentage of filling results in overturning

moment rising. The pattern of overturning moment variation is almost the same for the system with different earthquake records. Also, the maximum time history of overturning moment for Imperial Valley earthquake records in the full filled tank is presented in Figure 8.



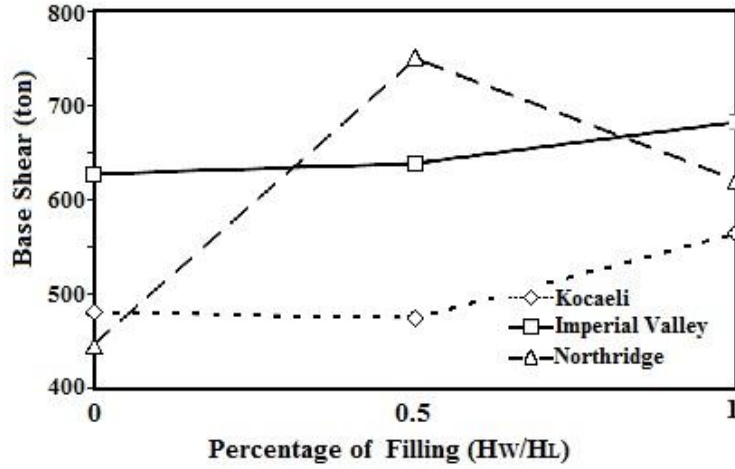


Fig. 5. Base shear variation based on the filling percent.

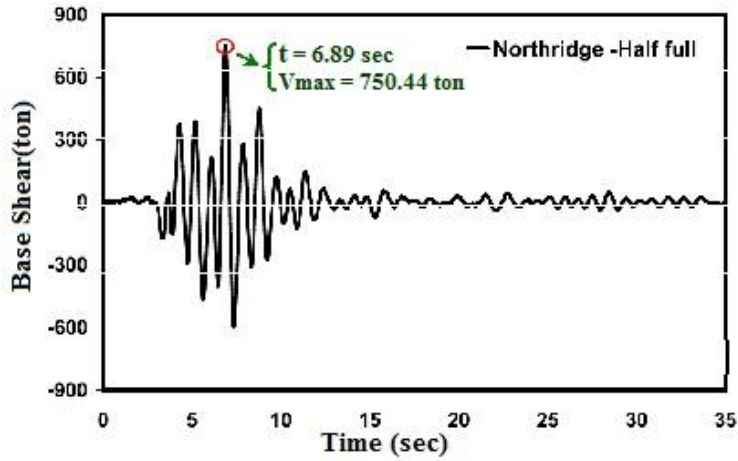


Fig. 6. Time history of base shear force under the Northridge earthquake in half full case.

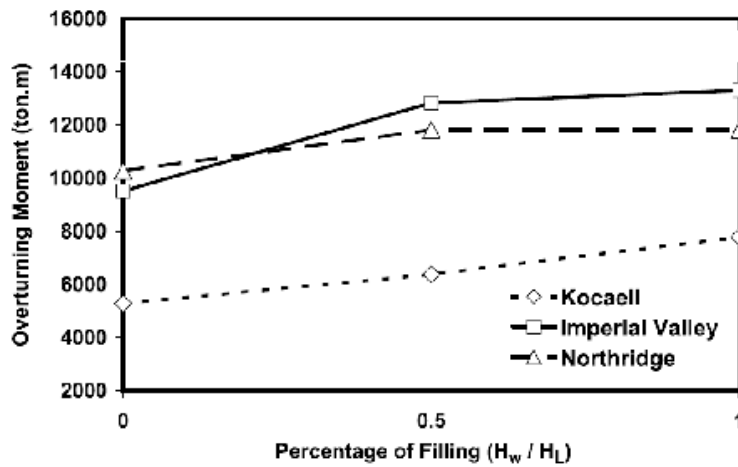


Fig. 7. Overturning moment variation based on the filling percent.

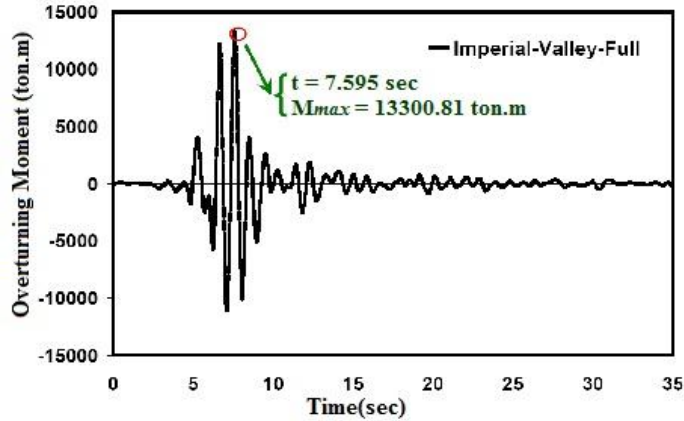


Fig. 8. Time history of overturning moment under the Imperial Valley earthquake in full case.

### Roof and Floor Displacements

Maximum displacements obtained along the height of elevated tank for three earthquake records are shown in Figure 9. The maximum displacements for three earthquake records occurred in the Northridge earthquake in all three cases (i.e. full, half full and empty). The results indicate that, in relatively stiff soils, the maximum displacement happened at the joint place of the columns and container. As Dogangun and Livaoglu (2007) observed, the maximum displacement occurred in the joint of column and container in a tank on stiffer soils, however, the maximum displacement in

tank systems on relatively softer soils would occur in the roof.

The variation of floor slab displacement against the percentage of tank capacity is presented in Figure 10. As seen in Figure 11, the floor displacement of the container would not always occur in the filled case. The container maximum floor displacement occurred in the Northridge record in the half filled case and in the Imperial Valley record on the empty case. The results are due to the earthquake properties and the given frequency content. The container floor displacement curve against the time of Northridge earthquake in the half full case is illustrated in Figure 11.

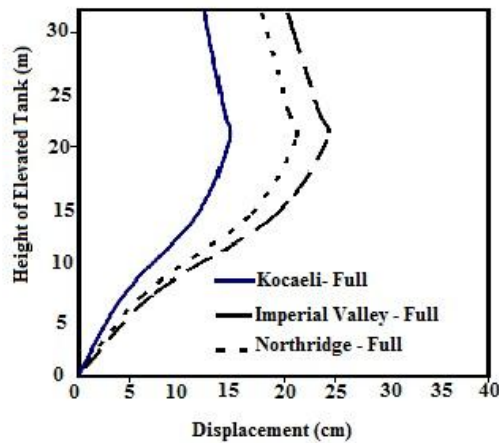


Fig. 9. Maximum displacements in full case.

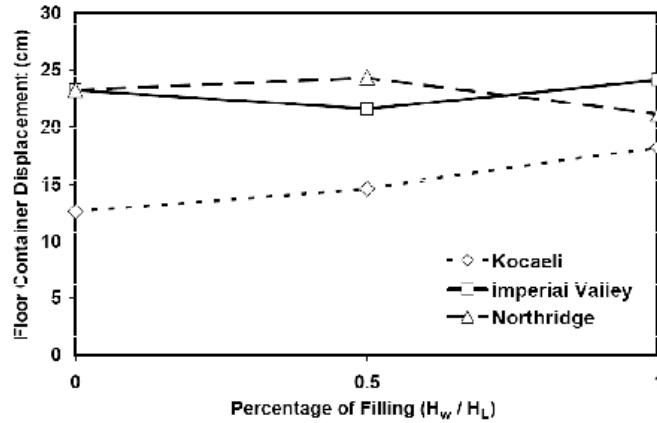


Fig. 10. Floor displacement variation based on filling percent.

### Sloshing Displacement

The variation of displacement due to sloshing versus the percentage of the tank filling is presented in Figure 12. The results show that the sloshing displacement would not always occur in the full tank. As seen in Table 6.1 and Figure 12, the pattern of variations of the sloshing displacement is not the same for the three earthquake records. Thus, in Northridge and Imperial Valley records, as the percentage of the tank fluid increases, the sloshing displacement increases. However, this relation is adverse in the Kocaeli record. Maximum sloshing displacement for three earthquake records

and three cases of filling occurred in the Kocaeli earthquake in the half-filled case. The time history of sloshing displacement under the Kocaeli earthquake in the half-filled case is illustrated in Figure 13. Also, the time history of sloshing displacement and roof for Northridge and Imperial Valley records are presented in Figures 14 and 15. As shown in Figures 15 through 15, the occurrence time of maximum roof and sloshing displacement are different for each earthquake record. The reason is related to different periods of impulsive and convective mass and also the frequency content of used records.

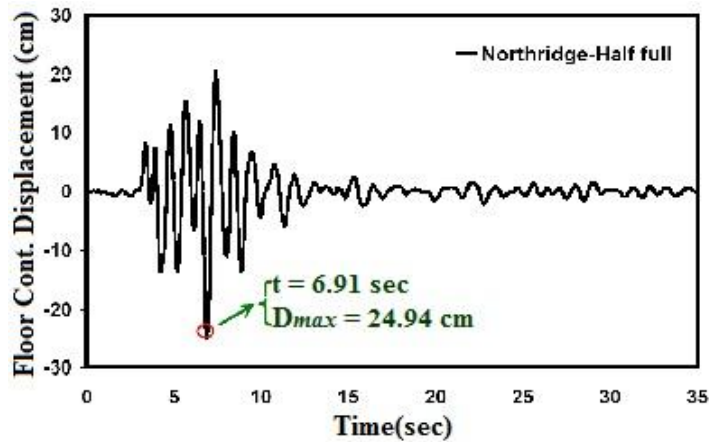


Fig. 11. Time history of floor displacement under the Northridge earthquake in half full case.

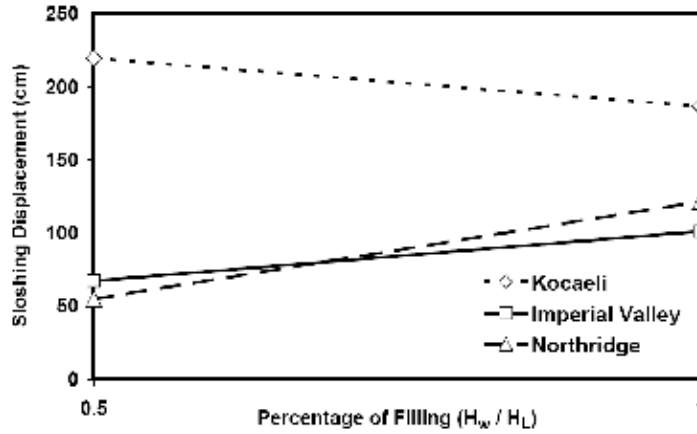


Fig. 12. Sloshing displacement variation based on filling percent.

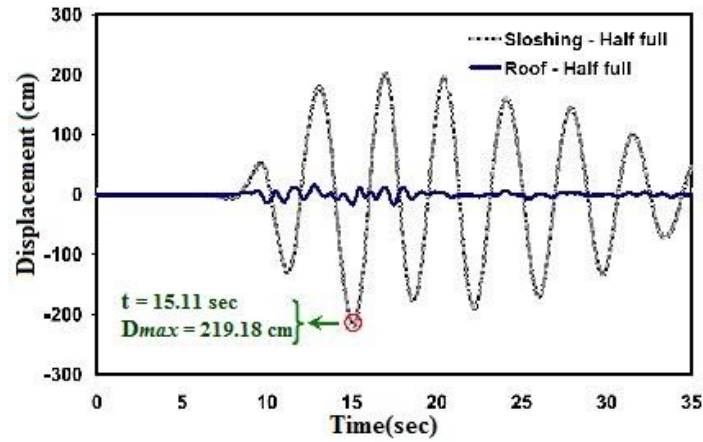


Fig. 13. Time history of roof and sloshing displacement under the Kocaeli earthquake in half full case.

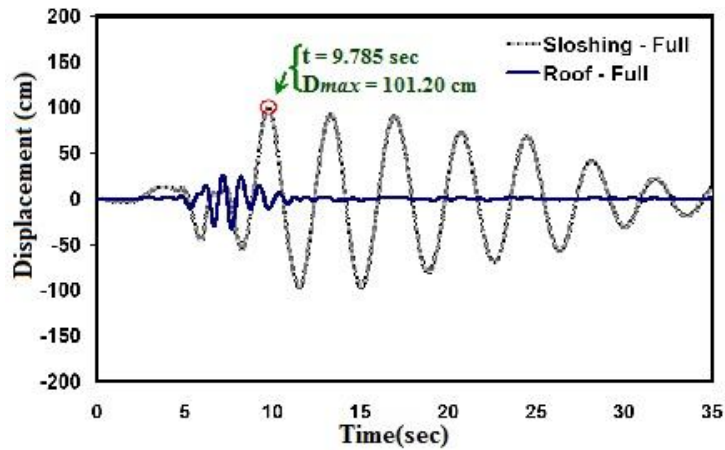


Fig. 14. Time history of roof and sloshing displacement under the Imperial Valley earthquake in full case.

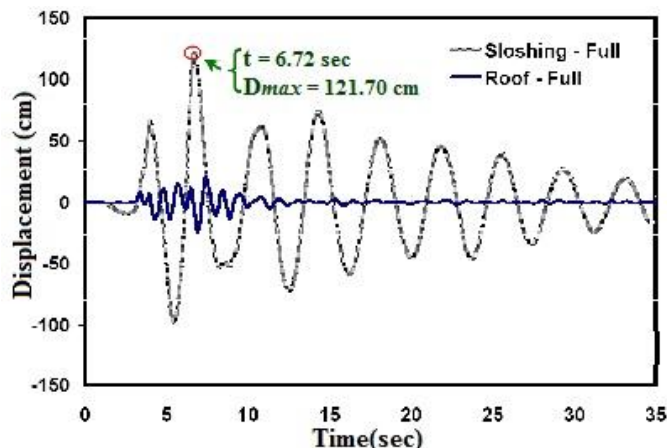


Fig. 15. Time history of roof and sloshing displacement under the Northridge earthquake in full case.

## CONCLUSIONS

In this study, an elevated  $900 \text{ m}^3$  water tank supported by a moment resisting frame was considered. Using Housner two-mass models, dynamic responses including base shear, overturning moment, roof and floor displacement, and sloshing displacement were assessed under three earthquake records. The dynamic responses of tank have been determined using time history analysis in three cases, i.e. empty, half-full and full. The obtained results are summarized as follows:

- The critical response of elevated tanks does not always occur in the full case of tanks and it may happen in the lower percentage of fluid, and even in the empty case of the tank depending on the earthquake characteristics.
- Frequency content and properties of the earthquake in ranges of natural frequency are the most important factors in reduction or intensity of tank responses.
- Freeboard considered for this tank (190 cm), the sloshing displacement obtained in the Kocaeli record was 219 cm which is more than the considered value. This point is confirmed in reference (Livaoglu and

Dogangun, 2005) where tanks were studied considering fluid–structure–soil interactions.

- The maximum displacement of the elevated tank, which is in a C type soil, according to the UBC-97 classification occurs in the support system joint with the container.
- Due to the difference between the impulsive and convective mass periods and also among the frequency contents and the properties of used earthquake records, the occurrence time of maximum roof and sloshing displacements are not the same, and depend on the aforementioned parameters.

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