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Nonlinearity in Exchange Rates and Forecasting

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Abstract

Exchange rates are subject to large and frequent fluctuations in mean and variances making it very difficult to model and forecast. In this paper, a series of tests for nonlinearity and chaos in exchange rates is conducted using the daily data on the market rates in Iran for the period 1991-2005. The tests for nonlinearity are BDS and ANN tests, and the tests for chaos are autocorrelation and Lyapunov exponents. The tests results suggest that the exchange rates and their rates of change follow complex nonlinear and stochastic processes. In the second part of the paper, an ANN model is designed to forecast the exchange rates. The results show that ANN outperforms both ARIMA and GARCH models in forecasting the exchange rates, but generates the same results as the alternative models in forecasting the rate of change of the exchange rates.

Keywords: Exchange rate, forecasting, nonlinear, chaos, BDS, Lyapunov exponents, ANN

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1- Introduction

Modeling and forecasting financial market time series have proved challenging since they are highly volatile and influenced by many factors which cannot be controlled for. Exchange rates are of special importance in financial markets because of their direct and indirect effects on international trade and capital flows among countries, foreign reserves, and risk and portfolio management. They are particularly vital for resource based countries like Iran whose major source of income is oil exports. Any change in the exchange rates has significant impacts on the government oil dependent revenue and therefore on the entire economy.

In general, there are three approaches to modeling and forecasting exchange rates: Structural models, time series models, and computational models. The structural models tend to identify the main derivers in the market and to examine the impacts of those factors on the exchange rates fluctuations. Purchasing power parity, uncovered interest rate parity, and open economy macro models are examples of the structural models in which the impacts of variables such as prices, interest rates, money supply, and balance of payments on the exchange rates are studied. The structural models measure the magnitude and the direction of the influence of each explanatory variable on the exchange rates and, therefore, are appropraite for policy evaluation. However, the forecasting performance of the structural models is not satisfactory. As Meese and Rogoff (1983a), Chinn and Meese (1995), Boughton (1987), McDonald and Taylor (1994), Berkowits and Giorgianni (2001), Mark and Sul (2001), Rapach and Wohar (2001) show, the structural models could not outperform the random walk model in forecasting exchange rates. The failure of these models to generate accurate forecasts can be ascribed to the following factors. First, there may be a simultaneity or even reverse causality between some of the explanatory variables and the exchange rates. Second, the forecasting ability of the model depends on the forecasting accuracy of the explanatory variables such as interest rates and prices. Third, these models assume the structure of the economy and the policies would remain the same in the future and, therefore, would result in poor forecasts in case of structural changes. Fourth, there is always a problem of misspecification due to the missing variables. And finally, the structural models focus only on the fundamental variables and are not able to capture the speculative moves in the market.

The time series models are developed mainly for dynamic analysis and forecasting. They usually outperform the structural models in forecasting since they use the lags of the dependent variables to generate the future values of the series. The underlying rational for this methodology is that the effects of other variables are embedded in, and reflected by, the actual behaviour of the exchange rates. Time series models, however, are often criticized because of their atheoretical approach in modeling economic series. Although this shortcoming should be admitted, the advantage of time series models in forecasting economic series over structural models should not be overlooked. This is particularly important when an economic series, such as exchange rates, is subject to various and frequent disturbances that cannot be controlled for by the structural models. Time series models can perform well when they are able to capture the data generating process. But, if the underlying data generating process of an economic series is chaotic, using traditional linear and nonlinear time series models to estimate and forecast the series would result in misleading outcomes. To forecast such complex series, we need to use flexible nonlinear time series such as Artificial Neural Networks (ANN) models. (Zhang et al, 1998, Swanson and White, 1999, Moshiri and Cameron, 2000, Moshiri and Foroutan 2006).

Since prices and the returns in stock markets swing frequently and sometimes in large magnitudes, time series models may not be able to capture all in irregularities. In fact, the efficient market hypothesis asserts that the financial market prices are random implying that their changes cannot be predicted. Chaos theory, however, affirms that a very complex behavior of economic series, which appears to be random, may be explained by a deterministic nonlinear system. Research on chaos and its applications in different disciplines has been growing remarkably and econometricians have been developing various tests for nonlinearity and chaos in economic series since the early 1990s.

This paper addresses two questions: First, does there exist chaos in the exchange rates market of Iran? Second, does ANN model forecast exchange rates more accurately than traditional linear and nonlinear time series models do? To answer these questions, the paper is organized as follows. In section

2, the data is described. In section 3, the results of different tests for chaos are presented. In section 4, the exchange rates and their growth rate are forecasted. Section 5 concludes the paper.

2- Description of the data

The exchange rate in Iran has been fluctuating considerably since 1979. Prior to the 1979 revolution, the exchange rates were stable, because the central bank relying on a high level of foreign reserves was able to keep the rate at a fixed level. However, during Iran-Iraq war in 1981-1985, the central bank could not respond to the market expectations and therefore, the local currency started to depreciate and the market exchange rates became volatile. Although the central bank continued with its fixed exchange rates policy in this period, it introduced the multi exchange rates regime to support exports and control the inflation rate. Since then, the parallel market was also developed and became very popular. After the war and the concurrent rise in oil prices, the foreign reserves increased and the exchange rate policy changed in line with the economic adjustment policy in favour of open market system. In the early 1993, the multi exchange rate policy was abandoned and the central bank narrowed down the spread between the official and nonofficial market rates. However in the second half of 1993, the exchange rates market went under severe fluctuations when oil prices in the global market started to fall, inflation went up, and the foreign debt problems and inability of the banks to pay them off became apparent. However, rising oil prices in 1996 along with implementing a series of policies including debt payment rescheduling and inflation control led the exchange rates market to enter into a period of stability and regularity. In the second half of 1997, beginning the East Asian financial crises and gradually decreasing the oil prices, the economy again faced with problems such as increasing inflation expectations, decreasing economic growth, and foreign debt settlement. As a result, the exchange market observed another significant exchange rate fluctuation in 1998. After establishment of a single exchange rate regime in the early 2002, banks were allowed to deal relatively freely with foreign exchange, i.e. to sell and buy foreign exchange rate at open market rates. It should be noted, however, that the central bank still plays a major role in the exchange rates relying on the government oil income revenues. The recent increase in oil prices and formation of the Foreign Reserve Fund have led the central bank to successfully control the fluctuations in exchange rate market and adopt a managed floating exchange rates regime. As a result, the trend has remained stable for these years. The Log of daily exchange rate (Iranian Rial / US Dollar) trend is shown in figure1.

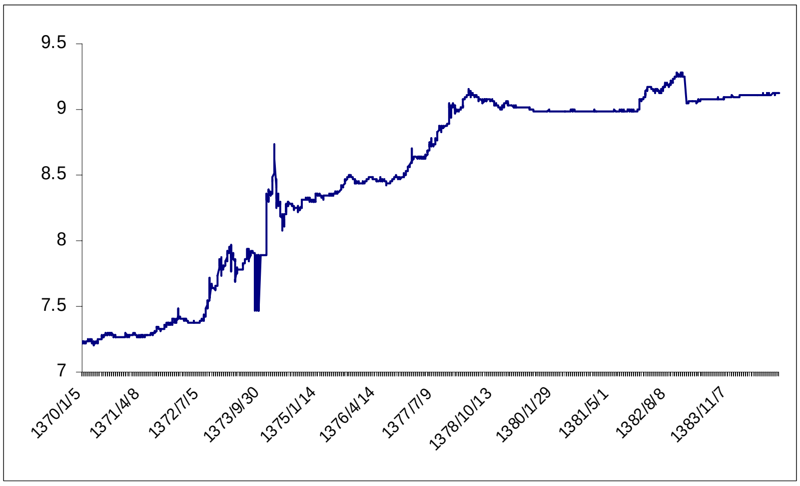

Figure1. The Log of Daily Exchange Rates (Iranian Rial / US Dollar)- 1991-2005

The data in this research are the daily exchange rates from April 5, 1991 to March 28, 2005, a sum of 4344 observations. Data are collected from the Iranian central bank. Table 1 shows the general statistics of daily exchange rates in about fourteen years. Since the returns on exchange rates are also of interest to market participants, we would also include them in our study. The rate of returns is defined as $R_t = \log\left(\frac{E_t}{E_{t-1}}\right)$, where E_t denotes the exchange rate at time t.

Table1. Statistical Properties of Daily Exchange Rate and its Returns in Log (1991-2005)

Statistical Properties	Exchange rate	Return's of Exchange rate
Mean	8.48	0.00043
Std.Dev	0.67	0.023
Skewness	-0.7	1.33
Kurtosis	0.041	277.2
Jarqe-Bera	550.53	1360
ADF	-1.71	-18.16
Peron's test	-5.63	-

The ADF test includes a constant term, 16 and 17 lags for exchange rate and its returns, respectively. The number of lags is chosen based on the SIC. The ADF critical values at the 1, 5 and 10 percent levels of significance are -3.43, -2.86 and -2.76, respectivly. The Peron's test with structural change includes a time trend, lag of exchange rates, and a dummy variable for 1996 shock. The peron critical value with $\lambda = 0.35$, at the 1, 5 and 10 percent levels are -4.39, -3.76 and -4.39, respectively.

The Skewness and Kurtosis values show that the distribution of exchange rate and their returns have fatter tails in comparison with normal distribution, and also the Jarqe-Bera test demonstrates that the exchange rates and their returns are not normal. From figure 1, two observations can be made. First, the variance varies in different periods, which indicates that there may be a nonlinear structure in the series. Second, there is an increasing trend in the exchange rate but at various rates. This fortifies the probability that a stochastic trend, rather than deterministic one, exists in the data. According to ADF test results, we cannot reject the null hypothesis of unit root at the 1 and 5 percent significance levels, suggesting that the exchange rate series is not stationary and needs to be differentiated. However, in order to ascertain that the 1994 shock was not the factor of inability to reject the null hypothesis, we re test the series for stationarity using the Perron test in the presence of structural break. The result of Perron test, when the 1994 shock is taken into account, indicates that the process is stationary and therefore, data transformation is not necessary.

The result of the ADF test for the exchange rate returns is presented in the last column of table 1. It shows that the null hypothesis of unit root can be rejected at the 1, 5 and 10 percent significance levels. Therefore, the exchange rate returns series is stationary and it is not necessary to be differentiated.

In the following section, we examine the underlying data generation process of exchange rate and its returns using various tests.

3- Choas in Exchange Rate Markets

A nonlinear deterministic but seemingly random process is called chaos. It can be generated when a nonlinear feedback process exists in a dynamic system. In the exchange rate market, speculation can generate chaos. For instance, when the exchange rate gets too high, the underlying currency is sold, pulling the exchange rate down, and when it is low, the currency is bought, leading to a reversion in the direction of the exchange rate. The feedback effect can be linear or nonlinear. Linear feedback implies a sequence of adjustments, which eventually forces the exchange rate to return to its mean. On the other hand, nonlinear feedback implies that corrections are not proportional and, therefore, may generate high level of fluctuations for a long period before returning to its long run state. One source of nonlinear reaction is market psychology that causes overreaction to bad news.

Empirical studies on chaos in exchange rate market can be grouped in two categories: Explaining chaos using economic theories and testing for chaos. The first group of studies develops models based on economics theories that could produce chaotic behavior under certain conditions. De Grauwe, Dewachter, and Embrechts (1993) apply Dornbusch model splitting expectations between charts- based and the fundamentals-based and find that the more speculator are involved in charting, the higher is the likelihood of chaos. Moreover, they do not find any relationship between the interventions of central bank, massive or limited, and chaotic behavior of exchange rates. However, Da Silva (2000) shows that massive intervention can remove chaos from the foreign exchange market. De Grauwe and Grimaldi (2001) develop a simple non-linear dynamic model and find that the size of shocks to the underlying fundamental exchange rate matters for the dynamics of the exchange rate. More specifically, they find that when these shocks are small relative to the size of the transactions cost band, exchange rate movements are complex, and can even be chaotic. They also argue that since in industrialized countries exchange rates deviate substantially from the underlying fundamentals and frequent structural breaks in the relationship between the exchange rates and fundamentals are observed, the movements of the exchange rates are likely subject to a non-linear speculative dynamics. In contrast, in high inflation countries, the movements of exchange rates can be explained much better by movements in underlying fundamental factors such as inflation differentials.

The second group of studies develops and applies tests for nonlinearity and chaos in the exchange rates market. The important tests developed for detecting nonlinearity and chaos are correlation dimension, BDS, Lyapunov exponent, neural network, R/S and Kolmogorov. It is important to note that not all these tests are necessarily tests for chaos. Chaos is nonlinear, but not all nonlinear processes are chaos. Therefore, if we reject linearity in favour of nonlinearity, or do not reject nonlinearity, we cannot necessarily imply chaos. For instance, the BDS and ANN tests are tests for nonlinearity, but the Lyapunov exponent is a test for chaos. Bask (2002) uses neural network and a blockwise bootstrap procedure to estimate the Lyapunov exponent in Swedish daily exchange rates against the Deutsche Mark, ECU, the US Dollar, and Yen from May 1991 to August 1995. The results show that, in most cases, the null hypothesis of zero Lyapunov exponent is rejected in favor of a positive exponent, which means that chaos exists. The exceptions are the Swedish Krona against the Deutsche Mark and the ECU exchange rates during the fixed period. However Jonsson (1997) examines daily data for the Swedish Krona against the US Dollar from 20 November 1992 to 30 December 1994, and finds that the largest Lyapunov exponent is negative. These conflicting results can be explained by the difference in length of the time series and also the methods used to estimate the largest Lyapunov exponents. Dechert and Gencay (1992) apply a multilayer feed forward neural network to calculate the Lyapunov exponents for the Canadian, German, Italian and Japanese monthly average spot exchange rates from 1960 to 1990. The results of this study show no evidence of chaos. Bajo-Rabio et al (1997) use the correlation dimension and the Lyapunov exponents tests to detect the presence of deterministic chaos in the Spanish peseta and the US dollar exchange rates. They show that chaos exists in the Spanish exchange rates market. In another study, Chappell and Eldridge

(1997) examine the behavior of the exchange rates between pound and the European currency unit (ECU) during the time before the UK joined the exchange rate mechanism of the European Monetary System and during the time of its membership. They find that during the membership period a GARCH (1,1) model fit the data very well, but during the period before membership there is evidence for significant nonlinearity and the possibility of chaotic process. Liao (1997) examines the exchange rates of five Asian countries. The results reveal that those exchange rates exhibit chaotic behavior. Richards (2000), shows that foreign exchange markets across a broad range of countries (Australia, Canada, UK, Japan etc.) are fractal. Cecen and Erkal. (1996) investigate the possibility of a low dimensional chaotic attractor in hourly returns of spot exchange rate in British Pound, Deutschmark, the Swiss France and the Japanese Yen. They find that correlation dimension estimates do not converge to a stable value, but the largest lyapunov exponents appear to be positive. They conclude that there is non-linear dependence in hourly exchange rate but the nature of this non-linear dependence is far from being deterministic. Hsiesh (1989) and Kugler and Lenz (1991) detect nonlinearities for several exchange rates but theses nonlinearities can be explained by a GARCH model. Rae Westron (2004) investigate correlation dimension of the ‘nil intervention’ NZ$/US$ exchange rate series and find that low dimensional deterministic structure exists in the system. They also compare the results with A$/US$ and C$/US$ exchange rate series where history of intervention is reported by respective central banks. The results show that there is no significant difference between NZ$/US$ series which had no intervention and the other two series in which there were a history of intervention.

3-1- BDS Test

The BDS test; named after Brock, Dechert, and Scheinkman (1988), is based on the concept of correlation dimension. The BDS tests for iid against a general nonlinearity in the series. The test statistic is calculated as

$$W_m(\varepsilon) = \frac{\sqrt{T}\left\{C_m(\varepsilon) - C_1(\varepsilon)^m\right\}}{\delta_m(\varepsilon)} \approx N(0,1)$$

Where T is the sample size, m is the embedding dimension, ε is a residual of the best linear fit to the data, $\delta_m(\varepsilon)$ is an estimation of the standard deviation under the null hypothesis and $C_m(\varepsilon)$ is the correlation integral. The test statistic is asymptotically distributed as N (0, 1). If W becomes large enough then the null of iid will be rejected in favor of non-linearity. To do the BDS test, we need first run an ARMA model in order to extract a linear structure from the data. We then apply the BDS test to the residuals to determine whether they are white noise or not. If the null of whit noise is rejected, it can be concluded that the data generating process of the original series is nonlinear.

In this study, we first apply the BDS test to the residuals of ARMA(15,25) and ARMA(15,2) for exchange rate and its returns, respectivly. The orders of the ARMAs are determined based on autocorrolation and partial autocorrolation functions. The results presented in table (2) and (3) demonstrate that the null hypothesis of iid for the residuls can be rejected at the 1 percent level of significant in different dimensions. This suggests that the exchange rates and their returns have a non-linear structure.

Table2: BDS test results for daily exchange rate (1991-2005)

$m\|\varepsilon/\sigma$	**0.5**	**1**	**1.5**	**2**
2	35.80	33.57	31.12	31.19
3	40.19	35.19	31.48	31.41
4	42.90	35.58	31.22	30.65
5	45.97	36.04	31.26	29.95

m is the embedding dimension. ε is equal to 0.5, 1 and 1.5 times the standard deviation. The critical values for the 10%, 5% and 1% signifivant levels are 1.64, 1.96 and 2.57, respectivly.

Table3. BDS test results for daily exchange rate returns (1991-2005)

$m \mid \varepsilon/\sigma$	0.5	1	1.5	2
2	35.18	35.32	30.15	30.55
3	38.53	33.48	29.87	30.15
4	39.95	33.72	29.98	29.84
5	41.19	33.64	29.87	29.19

m is the embedding dimension. ε is equal to 0.5, 1 and 1.5 times the standard deviation. The critical values for the 10%, 5% and 1% signifivant levels are 1.64, 1.96 and 2.57, respectivly.

3-2- The Artificial Neural Network Test

An augmented feed forward neural network model can be used for detecting linearity versus nonlinearity (Lee et al., 1991, and Kuan and White, 1994). The ANN model can be viewed as a standard linear regression model augmented with nonlinear terms.

$$y = \beta_0 + X\delta + \sum_{i}^{q} G(X\gamma_j)\beta_j + \varepsilon \qquad , \qquad j = 1,......, \quad q$$

Where y is the network's final output, G is a transformation function, ε is an iid residual, $X = [1, X_1,....X_r]$ is a matrix of input vectors, γ is a matrix of weights between input layer and intermediate layer, and β is a matrix of weights between intermediate layer and output layer. In this equation y and x are related to each other through a linear ($X\delta$) and nonlinear ($\sum_{j=1}^{q} G(X \ \gamma_j)\beta_j$) terms. When the series is nonlinear, there should not be any correlation between the residuals obtained from the linear model and regression process. So the null hypothesis can be defined as $E(e_t G_t) = 0$, where e_t is the residual obtained from the linear model, and G is the output vector of transformation function in the intermediate layer of ANN. Under the null hypothesis, the test statistic can be defined as $TR^2 \to \chi^2(q)$, where T is the number of observations and R^2 is the squared multiple correlation coefficient from a standard linear regression of e_t on x_t and principal components of G which are not correlated with x_t. If the statistic above becomes greater than the critical value, this implies that a dynamic nonlinear structure exists in the series, otherwise, the underlying data generating process is linear.

The results of the tests are presented in table 4. As it is shown the test results are greater than the critical value at 5 percent level of significance. $(\chi_5^2 = 7.5)$ .Thus, it can be concluded that the processes underlying the exchange rate and its returns are nonlinear, which is consistent with the BDS test results.

Table 4: The ANN test results for daily exchange rate and its returns (1991-2005)

$TR^2 \rightarrow \chi^2(q)$	**Models**
31.89	**Exchnage rate: ARMA(15,25) residuals**
42.32	**Eexchnage rate returns: ARMA(15,2) residua**

3-3- Correlation Dimension Test

The correlation dimension test, which is developed by Grassberger and Procaccia(1983), is based on the significant differences in the behavior of a chaotic process and a stochastic process. The correlation dimension is calculated by constructing *m* histories of filtered data. An *m* history is a point in *m*-dimensional space where *m* is called the 'embedding dimension'. The correlation integral C^m (ε) is then calculated as follows.

$$C_m(\varepsilon) = \lim_{N \rightarrow \infty} \frac{2}{N_m(N_m - 1)} \sum_{t<s} I_\varepsilon(X_t^m, X_s^m)$$

Where ε is a predefined small value, and $I_\varepsilon(X_t^m, X_s^m)$ is an index function defined as:

$$I_\varepsilon(X_t^m, X_s^m) \cong \begin{cases} 0 & ; \|X_t^m - X_s^m\| > \varepsilon \\ 1 & ; \|X_t^m - X_s^m\| \leq \varepsilon \end{cases}$$

The correlation dimension is the slope of the curve obtained by plotting log C^m (ε) on log (ε). For small values of ε, the slope is calculated as:

$$D_m = \lim_{\substack{\varepsilon \to 0 \\ N \to \infty}} \frac{\log C_m(\varepsilon)}{\log(\varepsilon)}$$

If D_m does not increase with *m*, the series is nonlinear and possibly chaotic, while for a random process the correlation dimension is infinity. The results are presented for exchnage rate and its returns in table (5) and (6), respectivly.

Teble 5: The correlation demension test results (D_m) for daily exchange rate (1991-2005)

M	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Exchange rate	0.538	0.542	0.546	0.548	0.551	0.553	0.555	0.557	0.559
HP filterred	0.541	0.542	0.543	0.544	0.545	0.546	0.547	0.548	0.50

$\varepsilon = 0.3237$.

Teble 6: The correlation demension test results (D_m) for daily exchange rate returns (1991-2005)

M	2	3	4	5	6	7	8	9	10
Exchnage rate retur	0.024	0.031	0.037	0.042	0.047	0.052	0.056	0.061	0.065
HP filterred	0.134	0.138	0.142	0.146	0.151	0.155	0.159	0.163	0.166

$\varepsilon = 0.3237$.

Since the correlation dimension test is sensitive to noisy data (Harrison et al, 1999), we recalculate the embedding dimension with the HP filtered data. The results in tables 5 and 6 show that the embedding dimension continues to rise with *m* and do not reach to a finite saturation limit. So it can be concluded that the series of exchange rate and its returns do not follow a chaotic process. The correlation dimension test is not a statistical test for chaos, and therefore, its results may not be generalized. We will now carry out a more rigorous test to examine the behavior of the series.

3-4- Lyapunov Exponents (LE) Test

The Laypunov exponent test is a direct test for chaos which is based on the sensitivity to the initial condition property of a chaotic series. In general, there are *n* Lyapunov exponents in an *n* dimensional system and if the largest Lyapunov exponent is positive, the data generating process is chaotic. There are two methods for calculating Lyapunov exponents: First, the direct method and second, the Jacobian method. In the direct method proposed by Wolf et al.(1985), Lyapunov exponent is measured by calculating the degree of divergence of nearby trajectories in the phase space as follows:

$$LE\ (m,n) = \lim \frac{1}{T-n}\sum_{n} \log \ d_n(m;i,j)$$

Where d_n is the ratio of the distance between pairs of X , n periods ahead ($r_n = \left|X_{i+n} - X_{g+n}\right|$) and the distance between pairs at the initial time ($r_0 = |X_i - Xj| \langle e$), e is a small number, T is the length of the original time series, and *m* is the embedding dimension. Since LE can obtain a positive value for nonlinear random processes (Fernandez-Rodrigues et al 2005), a positive value of LE obtained by the direct method can be viewed as a necessary but not a sufficient condition for chaos. Moreover, this method is very sensitive to noise and the number of observations (Brock and Sayers, 1988, Dechert and Gencay, 1992).

The Jacobian method uses the Takens (1981) theorem to estimate the Jacobian matrix and its eigenvalues. Dechert and Gencay (1992) have demonstrated that the largest Lyapunov exponents of a reconstructed dynamic system are the lyapunov exponents of the unknown system. There are different methods for calculating the Jacobian matrix such as Taylor series (Brown et al, 1991), nonparametric regression (McCaffery et al, 1992), and Feed forward neural networks (Dechert and Gencay, 1992). In this study, we apply the Dechert and Gencay algorithm to estimate the Lyapunov exponents.

We first reconstruct a dynamic system in an *m* dimensional space, and then estimate a model using a three- layer feedforward network with a sigmoid function in an intermediate layer. The Lyapunov exponents are then obtained from the derivative matrices of estimated networks model. The results for exchange rate and its returns are presented in tables (7) and (8), respectively.

Table 7: Lyapunov Exponent Test Results for Daily Exchange Rates (1991-2005)

m	Exchange rate	HP filterred
2	-0.0051	-0.0049
3	-0.0015	-0.0061
4	-0.0018	-0.013

Table 8: Lyapunov Exponent Test Results for Daily Exchange Rates (1991-2005)

m	**Exchange rate returns**	**HP filterred**
2	-0.98	-0.0107
3	-0.5	-0.000022
4	-0.43	-0.140

The results indicate that the underlying processes of exchange rate and its returns are not chaotic. The test restuls above would lead us to conclude that the Iranian exchange rate and its returns follow a nonlinear but stochastic process. The tests, however, do not help us find out the specification of the underlying data generating process, which is needed for forecasting. The specification of a process becomes very difficult when the series is generated from a nonlinear process. Contrary to the linear processes, there are many different specifications and there are no statistical methods to choose the best nonlinear specification. Thus, since the underlying data generating process of the exchange rate and its returns are nonlinear, but unknown, it seems that the flexible nonlinear models such as the Artificial

Neural Networks (ANN) can be used for forecasting the series. In the following section, we will develop an ANN model to forecast the nonlinear exchange rate series and compare the results with linear and nonlinear models.

4- Forecasting Exchange rates

In this section, we forecast the Iranian exchange rates by using a number of linear and nonlinear models. We use 75 percent of the total observations for estimation and 25 percent for forecasting and forecast evaluation. Our method of forecast is dynamic, i.e., forecasting exchange rate for each period would be based on the forecasted rates at the previous periods, and therefore, the forecast errors will be accumulated. We first use the best linear ARMA model to forecast the exchange rate and its returns. The autocorrelation and partial autocorrelation functions along with the Bayesian Schwartz criteria show that ARMA (15,25) and ARMA(15,2) are the best linear models to fit the exchange rate and its returns, respectively. The results presented in tables (9) and (10).

Table 9: ARMA Estimates for the Iranian Exchange rate (1991-2005)

$$P_t = \alpha_0 + \alpha_1 DU + \alpha_2 D + \alpha_3 t + \varphi_1 P_{t-1} + \varphi_3 P_{t-3} + \varphi_5 P_{t-5} + \varphi_8 P_{t-6} + \varphi_{12} P_{t-12} + \varphi_{15} P_{t-15} + \theta_1 \varepsilon_{t-1} + \theta_7 \varepsilon_{t-7} + \theta_8 \varepsilon_{t-8} + \theta_9 \varepsilon_{t-9} + \theta_{11} \varepsilon_{t-11} + \theta_{14} \varepsilon_{t-14} + \theta_{15} \varepsilon_{t-15} + \theta_{16} \varepsilon_{t-16} + \theta_{22} \varepsilon_{t-22} + \theta_{24} \varepsilon_{t-24} + \theta_{25} \varepsilon_{t-25} + \theta_{21} \varepsilon_{t-21} + \zeta_t$$

α_0	α_1	α_2	α_3	φ_1	φ_3	φ_5	φ_6	φ_{12}	φ_{15}	θ_1
0.039 (2.3)	0.049 (24)	0.0022 (1.5)	0.000002 (2)	0.89 (46)	0.10 (5.3)	-0.11 (-5.8)	0.11 (6.3)	0.02 (2)	-0.03 (-2.9)	-0.29 (-11)
θ_7	θ_8	θ_9	θ_{11}	θ_{14}	θ_{15}	θ_{16}	θ_{22}	θ_{24}	θ_{25}	θ_{21}
0.07 (3.7)	-0.03 (-1.9)	0.03 (1.7)	-0.19 (-11)	-0.09 (-4.9)	0.17 (9.2)	-0.10 (-6.2)	0.08 (5.07)	0.07 (4.1)	-0.11 (-6.6)	0.09 (5.4)
ADF-TEST -8.06 (0.000)			ARCH-LM 132.87 (0.000)							

Values in parentheses are t-statistic in the first row and P-values in the second row. D and DU are dommy variables for 11994 shock.

Table 10: ARMA Estimates for Exchange rate returns (1991-2005)

$$P_t = \alpha_0 + \varphi_1 P_{t-1} + \varphi_5 P_{t-5} + \varphi_7 P_{t-7} + \varphi_9 P_{t-9} + \varphi_{11} P_{t-11} + \varphi_{15} P_{t-15} + \theta_1 \varepsilon_{t-2} + \zeta_t$$

α_0	φ_1	φ_5	φ_7	φ_9	φ_{11}	φ_{15}	θ_1
0.0007	-0.35	-0.11	0.09	0.04	-0.14	0.13	0.19
(2.10)	(-21.1)	(-7.19)	(6.06)	(2.94)	(-8.82)	(8.24)	(-10.77)
ADF-test		ARCH-LM					
-56.87		84.03					
(0.00)		(0.00)					

Values in parentheses are t-statistic in the first row and P-values in the second row.

The ADF test on residuals confim that the models are valid. The ARCH-LM test results, however, show that the residual variances are not constant. To capture the non regular fluctuations in the data, we re estimate the model by a GARCH model. The GARCH (1,1) and GARCH(0,1) are selected as the best models for exchange rate and its returns, respectively. The results are presented in tables (11) and (12).

Table 11: The GARCH Estimates for Exchange rate (1991-2005)

Mean		
C	AR(1)	AR(2)
0.0007 (2.94)	0.71 (5.88)	0.28 (2.32)
$\sigma_t^2 = w + \beta_0 \sigma_{t-1}^2 + \alpha_0 \varepsilon_{t-1}^2 + \zeta_t$		
w	β_0	α_0
0.000005 (1.95)	0.732 (4.34)	0.16 (1.34)
ARCH-LM 0.22 (0.99)		

Values in parentheses are t-statistic in the first row and P-values in the second row

Table 12: The GARCH Estimates for Exchange rate returns (1991-2005)

Mean		
C	AR(1)	AR(11)
0 (2.65)	-0.30 (-3.67)	-0.162 (-1.52)
$\sigma_t^2 = w + \alpha_0 \varepsilon_{t-1}^2 + \zeta_t$		
w		α_0
0.0002 (1.90)		0.97 (1.93)
ARCH-LM 0.027 (1)		

Values in parentheses are t-statistic in the first row and P-values in the second row

The ARCH-LM test results show that our GARCH models have been able to remove the heteroskedasticity from the data. In the next step, we apply a feedforward ANN model to estimate and forecast the exchange rate and its returns. The ANN models are essentially a collection of interconnected units (neurons) grouped in layers that send information to each other. They tend to mimic the information processing system in human brian. The ANN models, in general, consist of three input, intermediate, and output layers. The layers are connected with each others with transfer functions and weights. The structure of the network used for estimation and forecasting the exchange rate and its returns is presented in table (13).

Table 13: Neural network structures for estimation and forecasting exchange rate and its returns

	ANN Model	
	Exchange rate	**Exchange rate returns**
Input	1 – 3 lags	1 – 3 lags
Output	Exchange rates	Exchange rates returns
Intermediate layers numbers	4	5
Transfer function in intermediate layer	Hyperbolic tangent	Hyperbolic tangent
Transfer function in out put layer	Identical linear function	Identical linear function

Hyperbolic tanget function is a smoothing function which returns values between -1 and 1.

We use the three estimation models, that is, ARIMA, GARCH, and ANN, to forecast the exchange rate and its returns. To compare the forecasting performance of these models, we use RMSE, MAE, and U-stat criteria. The results are presented in table 14 and 15.

Table 14: Forecasting Results for exchange rate (1991-2005)

Model	RMSE	MAE	U-stat	Diebold
ARMA	0.16	0.13	0.008	32.06
GARCH	0.078	0.070	0.0042	5.04
ANN	0.068	0.046	0.0037	-

As the results show, the ANN model is more accurate than ARMA and GARCH models by all three criteria. We also test for statistical significance of difference between two alternative forecasts using the Diebold-Mariano (1995) test. The Diebold Mariano test statistic has a normal distribution under the null hypothesis that there is no difference between the forecasting performances by two models. According to the test results shown in column 4 of table 14, the null hypotheses are rejected for the ANN and the other two ARMA and GARCH models.

Table 15: Forecasting Results for exchange rate returns (1991-2005)

Model	RMSE	MAE	U-stat	Diebold
ARMA	0.0072	0.00189	0.9309	32.06
GARCH	0.0072	0.0021	0.8743	2.88
ANN	0.0071	0.00182	0.97	-

Table 15 presents the forecasting performance results by the three models for the exchange rate returns. As the results show, the forecasting error functions by all three models are close to each other. However, the results of the Diebold-Mariano test indicates that there is a significant statistical difference between the ANN and the GARCH models, but no significant differences between the ANN and ARMA models.

5. Conclusions

Exchange rates play a significant role in the national and international economies, yet its modeling and, in particular, its forecasting remains a challenging area in Economics. The financial markets are very sensitive to various changes in the economy so that the structural models have not been able to explain the high level of volatility which exists in these markets. Even if the models focusing on market fundamentals are able to provide good explanations of the movements in the pastt, they usually fail in forecasting. The time series models, however, have been able to capture the dynamics of the financial market series and therefore to forecast them more accurately. The recent development of nonlinear models and computational techniques to model complex series has helped this line of research advance considerably. Seemingly random variables can be generated by a chaotic process and its future values can be forecasted by nonlinear neural networks models.

In this paper, we investigate the nonlinearity and chaos in the Iranian daily exchange rate series and its returns during the period 1991 to 2005. We first test for nonlinearity which is a necessary condition for chaos using ANN and BDS tests. We then apply the Lyapunov exponents and correlation dimension tests to detect chaos in daily exchange rate market of Iran. The results indicate that the exchange rates and their rate of returns follow a complex nonlinear but stochastic system. As Da Silva (2000) argues, the non chaotic behavior of exchange rate series can be explained by the massive and long lasting intervention of central bank.

Forecasting nonlinear exchange rates is very challenging, since, contrary to the linear model, there are many nonlinear model specifications to choose from. When the series is nonlinear, but the specification is not known, the use of nonlinear flexible models such as ANN models would be appropriate. We apply a feed forward ANN model to estimate and forecast nonlinear exchange rates and compare the results with the best alternative linear and nonlinear models such as ARIMA and GARCH models. Our results show that the ANN models are able to generate more accurate forecasts in the case of exchange rates, and as good forecasts as the other linear and nonlinear models statistically in the case of the exchange rate returns.

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