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Spontaneous Emission Spectrum from a Driven Three-Level Atom in a Double-Band Photonic Crystal

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Abstract

The spontaneous emission spectrum from a driven three-level atom placed inside a double-band photonic crystal has been investigated. We use the model which assumes the upper levels of the atomic transition are coupled via a classical driving field. The transition from one of the upper levels to lower level couples to the modes of the modified reservoir, and the transition from the other upper level to lower level interactwith the free vacuum modes. The effect of classical driving field on the spontaneous emission spectrum of this latter transition is investigated in detail. Most interestingly it is shown that only in the presence of the classical driving field; there is a black dark line in the spontaneous emission spectrum of free-space transition when the modified reservoir is of the photonic band gap type. This dark line is not seen in the case where the modified reservoir is of the free space type for relatively weak driving field.

Keywords: Spontaneous emission; Photonic crystal; Autler-Townes splitting; Non-Markovian Reservoir; Quantum interference

1. Introduction

It is well known that the spontaneous emission and probe absorption of an atom depends not only on the properties of the excited atomic system but also on the nature of the surrounding environment [1,2]. From the point of view of the surrounding environment of atoms, photonic band gap (PBG) structures have been shown to have different density of states (DOS) compared with a free-space vacuum field [3-5]. The study of quantum and nonlinear optical phenomena, in atoms (impurities) embedded in such structures, have led to the prediction of many interesting effects [6]. As examples, the localization of light and the formation of photon-atom bound states [7,8], suppression and even complete cancellation of spontaneous emission [9,10], population trapping in two-atom systems [11], the phase dependent behavior of the population dynamics [12,13] and other phenomena [14-16] can be mentioned.

On the other hand, driving a multi-level atom with a sufficiently strong resonant field alters the radiative dynamics in a fundamental way, even in the ordinary vacuum. It leads to such interesting effects as the enhancement of the index of refraction with greatly reduced absorption, electromagnetically induced transparency and optical amplification without population inversion. In view of these results, it would be interesting to investigate the combined effects of coherent control by an external driving field and photon localization facilitated by a PBG on spontaneous

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emission from a three-level atom embedded in a PBG material. This is precisely what is done in this paper.

While the spontaneous emission spectrum near the photonic band edge has been mentioned in a few papers [17,18], the discussions are limited to population involution and distribution of the upper levels based on a single-band PBG reservoir. In contrast, in this paper we focus on the spontaneous emission spectrum, rather than the population involution and distribution, and we consider a three-level atom embedded in a double-band photonic crystal described by both isotropic and anisotropic dispersion relations at the band edges.

In the present work we consider the model which consists of two upper levels resonantly driven by a laser radiation where either of levels may decay to a lower level. One of the transitions interacts with the free vacuum modes, and the other transition couples to the modes of a) the isotropic photonic band gap (PBG), b) the anisotropic PBG and c) free vacuum respectively. Most interestingly we show that when one of the transitions couples to the modes of the photonic band gap, the spontaneous emission spectrum of the other transition can exhibit ”dark lines” (zeros in the spectrum at certain values of the emitted photon frequency). This effect is due to the combined effects of applied driving field and DOS of modified reservoir, and it is not seen for relatively weak driving field, if the first transition couples to the modes of free space vacuum.

This paper is organized as follows. In Section 2 we investigate the coherent control of spontaneous emission for a three-level atom located within a perfect PBG structure. We apply the time-dependent Schrödinger equation to describe the interaction of our system with the modified vacuum and calculate the spontaneous emission spectrum in the free-space reservoir. The general calculated results and their analysis are presented in Section 3. The major conclusions are summarized in Section 4.

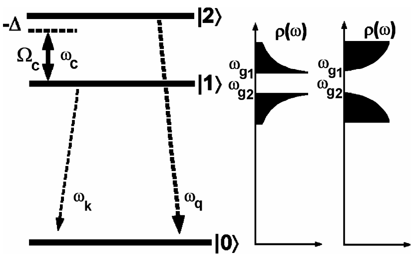

Figure 1. Schematic diagram of a three-level driven atomic system. The solid two arrow line denotes the coupling laser, the thin dashed arrow denotes the coupling to the modified reservoir (PBG) and the thick dashed arrow denotes the coupling to the free space reservoir.

2. Equations for the Spontaneous Emission Spectrum

Consider a single three-level atom placed inside a PBG material which is then driven by a laser field. We label the ground state of the atom by $|0\rangle$, and the two excited states by $|1\rangle$ and $|2\rangle$, as shown in Figure 1. The transition $|1\rangle \to |0\rangle$ is taken to be near resonant with a modified reservoir (this will later be referred to as the non-Markovian reservoir), while the transition $|2\rangle \to |0\rangle$ is assumed to be occurring in free space (this will later be referred to as the Markovian reservoir). The spectrum of this latter transition is of central interest in this section. We assume that spontaneous emission on the transitions $|2\rangle \to |1\rangle$ is inhibited by symmetry considerations. In the configuration shown in Figure 1, the upper levels $|1\rangle$ and $|2\rangle$ are of the same symmetry so that the external control laser field of frequency ω_c which couples levels $|2\rangle$ and $|1\rangle$ drives a two-photon transition $(2\omega_c = \omega_{21})$. The single-photon spontaneous emission $|2\rangle \to |1\rangle$ is not dipole allowed, since the levels are of the same symmetry. Two-photon spontaneous emission is considered to be negligible compared to the two-photon stimulated emission on the transitions $|2\rangle \to |1\rangle$, induced by the classical control laser field.

We assume the following initial configuration for the model system. At $t = 0$, the radiation-field reservoir is initially in the vacuum state (no photon in the system), and the atom is prepared in a coherent superposition of its two upper levels $|2\rangle$ and $|1\rangle$ in the form:

$$|\Psi(t = 0)\rangle = e^{i\phi_p} \sin\theta |1,\{0\}\rangle + \cos\theta |2,\{0\}\rangle \quad (1)$$

The parameter θ measures the degree of superposition of $|2\rangle$ and $|1\rangle$. A value of $\theta = 0$ means that the atom is initially prepared on the upper level $|2\rangle$, whereas $\theta = \pi/4$ means that the atom is initially prepared as an equal superposition of the upper levels $|2\rangle$ and $|1\rangle$. The factor $e^{i\phi_p}$ gives the relative phase between the expansion coefficients of $|2\rangle$ and $|1\rangle$. The coherent superposition state (1) can be prepared by an ultrashort pumping laser pulse of appropriate pulse area. At time $t = 0$ this atom starts to interact with a laser field of frequency ω_c and phase ϕ_c that couples the two upper levels. The dynamics of the system can be described using the Schrödinger equation. Then the wave function of the system at time t can be expressed in terms of the state vectors as:

$$\begin{aligned}|\psi(t)\rangle &= a_1(t)|1,\{0\}\rangle + a_2(t)|2,\{0\}\rangle \\ &+\sum_{ke} a_{ke}(t)|0,\{\vec{k}e\}\rangle + \sum_{qe} a_{qe}(t)|0,\{\vec{q}e\}\rangle\end{aligned} \tag{2}$$

where $\vec{k}$ and $\vec{q}$ denote the momentum vectors of the emitted photons and e denote the polarization of the emitted photons. The function $a_j(t)$ gives the probability amplitude to find the atom in the excited state $|j\rangle$ and the photon reservoir in the vacuum state. On the other hand, $a_{\lambda e}(t)$ gives the probability amplitude to find the atom on the ground state $|0\rangle$ and a single photon of wave vector λ and polarization e in the photon reservoir.

The Hamiltonian describing the dynamics of this system in the interaction picture and the rotating wave approximation can be written as:

$$\begin{aligned}H_I &= \hbar(\Omega e^{i\Delta t + i\phi_c}|1\rangle\langle 2| + \sum_{ke} g^{20}_{ke} e^{-i\delta_k t}|2\rangle\langle 0|\hat{b}_{ke} \\ &+\sum_{qe} g^{10}_{qe} e^{-i\delta_q t}|1\rangle\langle 0|\hat{b}_{qe} + H.C)\end{aligned} \tag{3}$$

Here $\Delta = \omega_c - \omega_{21}$, $\delta_k = \omega_k - \omega_{20}$, $\delta_q = \omega_q - \omega_{10}$ and Ω is the Rabi frequency, which is considered real for convenience in our problem.

The external control laser field of frequency ω_c which couples levels $|2\rangle$ and $|1\rangle$ drives a two-photon transition $(2\omega_c = \omega_{21})$. In this case, the Rabi frequency Ω is obtained from second order perturbation theory [19]

$$\hbar\Omega = \sum_i \frac{(\vec{d}_{2i}.\vec{E}_0)(\vec{d}_{1i}.\vec{E}_0)}{\hbar(\omega_c - \omega_{i1})} \tag{4}$$

Here the summation is over all intermediate states $|i\rangle$ of the atom. $\Delta = 2\omega_c - \omega_{21}$ represents the laser field detuning, and $\delta_\lambda = \omega_\lambda - \omega_{ij}$ represents the detuning of the radiation mode frequency ω_λ from the atomic transition frequency ω_{ij}. $g^{ij}_{\lambda e}$ is the frequency-dependent coupling constant between the atomic transition $|i\rangle \to |j\rangle$ and the mode $\{\lambda e\}$ of the radiation field. More precisely:

$$g^{ij}_{\lambda e} = \frac{\omega_{ij} d_{ij}}{\hbar}\left(\frac{\hbar}{2\varepsilon_0\omega_\lambda V}\right)^{1/2}\vec{e}_{\lambda e}\cdot\vec{d}_{ij} \tag{5}$$

Here $\vec{d}_{ij}$ is the atomic dipole moment unit vector for the transition $|i\rangle \to |j\rangle$, $\vec{e}_{\lambda e}$ is the polarization unit vector of the radiation fields V is the sample volume and ε_0 is the permittivity of free space.

We substitute this Hamiltonian into the Schrödinger equation and obtain the following set of equations:

$$i\dot{a}_1(t) = \Omega e^{i\Delta t + i\phi_c} a_2(t) + \sum_{qe} g^{10}_{qe} e^{-i\delta_q t} a_{qe}(t) \tag{6}$$

$$i\dot{a}_2(t) = \Omega e^{-i\Delta t - i\phi_c} a_1(t) + \sum_{ke} g^{20}_{ke} e^{-i\delta_k t} a_{ke}(t) \tag{7}$$

$$i\dot{a}_{qe}(t) = g^{01}_{qe} e^{i\delta_q t} a_1(t) \tag{8}$$

$$i\dot{a}_{ke}(t) = g^{02}_{ke} e^{i\delta_k t} a_2(t) \tag{9}$$

By formal time integration of Equation (8) and Equation (9) and eliminating $a_q(t)$ from Equation (6) and $a_k(t)$ from Equation (7) we get:

$$\dot{a}_1(t) = -i\Omega e^{i\Delta t + i\phi_c} a_2(t) - \int_0^t a_1(t')K_{11}(t-t')\,dt' \tag{10}$$

$$\dot{a}_2(t) = -i\Omega e^{-i\Delta t - i\phi_c} a_1(t) - \int_0^t a_2(t')K_{22}(t-t')\,dt' \tag{11}$$

where

$$K_{11}(t-t') = \sum_{qe}\left|g^{10}_{qe}\right|^2 e^{-i\delta_q(t-t')} \tag{12}$$

$$K_{22}(t-t') = \sum_{ke}\left|g^{20}_{ke}\right|^2 e^{-i\delta_k(t-t')} \tag{13}$$

are the retarded Green functions. The resulting Green function depends very strongly on the photon density of states of the relevant photon reservoir. Because the reservoir with modes k is assumed to be Markovian, we can apply the usual Weisskopf-Wigner result [4], and obtain:

$$K_{22}(t-t') = \frac{1}{2}\gamma_{20}\delta(t-t') \tag{14}$$

For the summation in Equation (12), the one associated with the modified reservoir modes, we have [2]:

$$K_{11}(t-t') = \frac{1}{2}\beta_{10}^{3/2}\left\{\frac{e^{i[\delta_{10g2}(t-t')-\pi/4]}}{\sqrt{\pi(t-t')}} + \frac{e^{i[\delta_{10g1}(t-t')+\pi/4]}}{\sqrt{\pi(t-t')}}\right\} \tag{15.a}$$

$$K_{11}(t-t')=\frac{1}{2}\alpha_{10}^{2}\left\{\frac{e^{i[\delta_{10g2}(t-t')+\pi/4]}}{\sqrt{4\pi(t-t')^{3}}}+\frac{e^{i[\delta_{10g1}(t-t')-\pi/4]}}{\sqrt{4\pi(t-t')^{3}}}\right\} \tag{15.b}$$

$$K_{11}(t-t')=\frac{1}{2}\gamma_{10}\delta(t-t') \tag{15.c}$$

for isotropic PBG, anisotropic PBG and free space reservoirs respectively, with $\delta_{10gi}=\omega_{10}-\omega_{gi}\quad(i=1,2)$. The definitions of the parameters are:

$$\beta_{10}^{3/2}=\frac{1}{2\pi\varepsilon_0}\frac{\omega_{10}^{2}d_{10}^{2}\omega_{g2}^{3/2}}{3\hbar c^{3}} \tag{16.a}$$

$$\alpha_{10}=\frac{1}{2\pi\varepsilon_0}\frac{\omega_{10}^{2}d_{10}^{2}\omega_{g2}^{1/2}}{3\hbar c^{3}} \tag{16.b}$$

$$\gamma_{ij}=\frac{2}{\pi\varepsilon_0}\frac{\omega_{ij}^{3}d_{ij}^{2}}{3\hbar c^{3}} \tag{16.c}$$

The spontaneous spectrum $S(\omega_\lambda)$ for the λ mode of spontaneous emission field is the Fourier transform of (see reference [20])

$$\left\langle E^{-}(t+\tau)E^{+}(t)\right\rangle_{t\to\infty}=\left\langle\psi(t)\right|\sum_{\lambda\lambda'}\hat{a}_{\lambda e}^{+}\hat{a}_{\lambda' e}e^{i\omega_\lambda(t+\tau)}e^{-i\omega_{\lambda'}t}\left|\psi(t)\right\rangle_{t\to\infty} \tag{17}$$

By introducing Equation (2) into Equation (17), we have:

$$\left\langle E^{-}(t+\tau)E^{+}(t)\right\rangle_{t\to\infty}=\int_{-\infty}^{+\infty}e^{i\omega_k\tau}d\,\omega_k D(\omega_k)\int\sum_{e=1}^{2}[a_{ke}^{*}(\infty)a_{ke}(\infty)]d\,\Omega \tag{18}$$

$$\left\langle E^{-}(t+\tau)E^{+}(t)\right\rangle_{t\to\infty}=\int_{-\infty}^{+\infty}e^{i\omega_q\tau}d\,\omega_q D(\omega_q)\int\sum_{e=1}^{2}[a_{qe}^{*}(\infty)a_{qe}(\infty)]d\,\Omega \tag{19}$$

for the spontaneous emissions along the transitions from the level $\left|2\right\rangle$ to the level $\left|0\right\rangle$, and from the level $\left|1\right\rangle$ to the level $\left|0\right\rangle$, respectively. Here $D(\omega_k)$, $(D(\omega_q))$ is the DOS of the radiation field, which can be derived as [2]

$$D(\omega_q)\sim\frac{1.0}{\sqrt{\omega_q-\omega_{g2}}}\theta(\omega_q-\omega_{g2})+\frac{1.0}{\sqrt{\omega_{g1}-\omega_q}}\theta(\omega_{g1}-\omega_q) \tag{20.a}$$

$$D(\omega_q)\sim\sqrt{\omega_q-\omega_{g2}}\,\theta(\omega_q-\omega_{g2})+\sqrt{\omega_{g1}-\omega_q}\,\theta(\omega_{g1}-\omega_q) \tag{20.b}$$

$$D(\omega_q)\sim 1.0 \tag{20.c}$$

for isotropic PBG, anisotropic PBG and free space reservoirs respectively, with θ being the Heaviside step function. From Equations (8-11), and Equations (18, 19), we have:

$$S(\omega_q)=D(\omega_q)[P_{10}\tilde{a}_1^{*}(s\to -i\,\delta_q)\tilde{a}_1(s\to -i\,\delta_q)] \tag{21}$$

$$S(\omega_k)=D(\omega_k)[P_{20}\tilde{a}_2^{*}(s\to -i\,\delta_k)\tilde{a}_2(s\to -i\,\delta_k)] \tag{22}$$

for the spontaneous emission spectra along the transitions from the level $\left|2\right\rangle$ to the level $\left|0\right\rangle$, and from the level $\left|1\right\rangle$ to the level $\left|0\right\rangle$, respectively. Here the definitions of P_{ij} are:

$$P_{ij}=\beta_{ij}^{3/2} \tag{23.a}$$

$$P_{ij}=\alpha_{ij} \tag{23.b}$$

$$P_{ij}=\gamma_{ij} \tag{23.c}$$

for isotropic PBG, anisotropic PBG and free space reservoirs respectively, and $\tilde{a}_1(s)$, $(\tilde{a}_2(s))$ is the Laplace transform of $a_1(t)$, $(a_2(t))$. Taking the Laplace transform of Equations (10, 11), we obtain

$$\tilde{a}_1(s)=\frac{(s+\gamma_{20}/2)a_1(0)-i\,\Omega e^{i\phi_c}a_2(0)}{(s+\gamma_{20}/2)(s+\tilde{K}_{11}(s))+|\Omega|^2} \tag{24}$$

$$\tilde{a}_2(s)=\frac{-i\,\Omega e^{-i\phi_c}a_1(0)+(s+i\,\Delta+\tilde{K}_{11}(s+i\,\Delta))a_2(0)}{(s+\gamma_{20}/2)(s+i\,\Delta+\tilde{K}_{11}'(s+i\,\Delta))+|\Omega|^2} \tag{25}$$

here, $\tilde{K}_{11}(s)$ is the Laplace transform of the retarded Green function in Equation (12), which can be derived in the following forms:

$$\tilde{K}_{11}(s)=\frac{1}{2}\beta_{10}^{3/2}\left(\frac{i}{\sqrt{is+\delta_{10g1}}}+\frac{1}{\sqrt{is+\delta_{10g2}}}\right) \tag{26.a}$$

$$\tilde{K}_{11}(s) = \frac{1}{2}\alpha_{10}\left(-i\sqrt{is+\delta_{10g1}} + \sqrt{is+\delta_{130g2}}\right) \qquad (26.b)$$

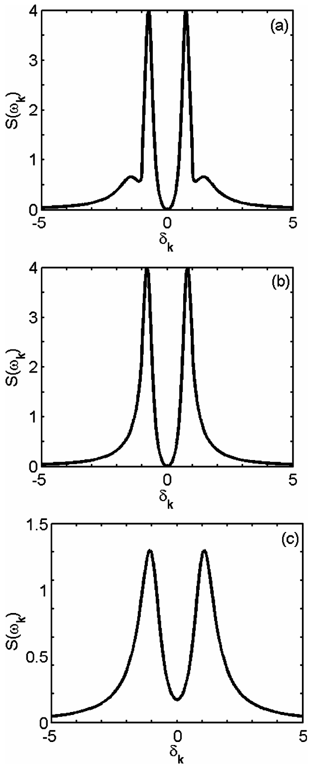

Figure 2. The spontaneous emission spectrum $S(\omega_k)$ (in arbitrary units) for transition from the upper level $|2\rangle$ to the lower level $|0\rangle$ as a function of detuning δ_k for different reservoirs. a) Double-band isotropic PBG reservoir and $\beta_{10}=1.0$, b) double-band anisotropic PBG reservoir and $\alpha_{10}=1.0$, c) free vacuum reservoir and $\gamma_{10}=1.0$. The other parameters used are $\theta=0.0$, $\delta\phi=0.0$ (the relative phase between the pump and coupling fields), $\Omega=1.0$, $\delta_{10g1}=-\delta_{10g2}=1.0$. All parameters are in unit of γ_{20}.

$$\tilde{K}_{11}(s) = \frac{1}{2}\gamma_{10} \qquad (26.c)$$

for isotropic PBG, anisotropic PBG and free space reservoirs respectively, with $\delta_{10gi}=\omega_{10}-\omega_{gi}$ (i=1,2). We use the formulas obtained above, and calculate the spontaneous emission for several parameters of the system.

3. Results and discussion

We study first the case where an atom is initially pumped to the upper level $|2\rangle$. In Figures 2 (a-c), the spontaneous emission spectra ($S(\omega_k)$) are shown for

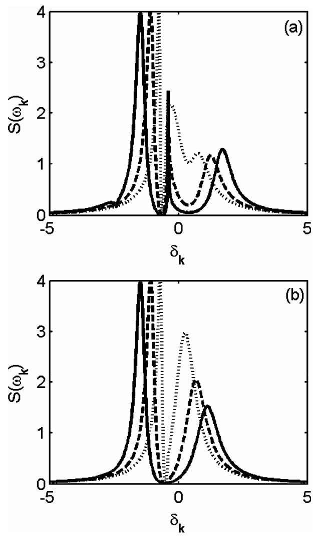

Figure 3. The spontaneous emission spectrum $S(\omega_k)$ (in arbitrary units) for transition from the upper level $|2\rangle$ to the lower level $|0\rangle$ as a function of detuning for different reservoirs. a) Double-band isotropic PBG reservoir, b) double-band anisotropic PBG reservoir. $\delta_{10g1}=2.5$, $\delta_{10g2}=0.5$, $\Omega=1.5$ (solid lines), $\Omega=1.0$ (dashed lines) and $\Omega=0.5$ (dotted lines). The other parameters used are the same as those in Figure 2.

the cases of isotropic PBG reservoir, anisotropic PBG reservoir, and free-space vacuum reservoir, respectively. Here, symmetric values of parameters for the transition are employed (*i.e.*, $\delta_{10g1}=-\delta_{10g2}=1.0$, $\beta_{10}=1.0$, $\alpha_{10}=1.0$, $\gamma_{10}=1.0$).

Figure 2 shows that there is a black dark line, $S(\omega_k)=0$ in the spontaneous emission spectrum in the case of PBG reservoir. On the other hand, no dark line exists in the case of free space reservoir, and no $S(\omega_k)=0$, can be found. We can show from Equation 22 that in the case of PBG reservoir as the atom is initially in state $|2\rangle$ ($\cos\theta=1$) and $\delta_{10g2}=-\delta_{10g1}$, $S(\omega_k)$ will be equal to zero at the frequency $\delta_k=\Delta$. Here we would like to emphasize that the center of dark line is absolutely black, and that the existence of a dark line in the spectrum is independent of the Rabi frequency and detuning. This dark line is the consequence of DOS of modified reservoir and applying driving field, which can be seen from Equations (22, 25, 26). In fact level $|2\rangle$ is split into two dressed states.

This dressed-state splitting is the effect of the Autler-Townes splitting [21] by the external field. So that the

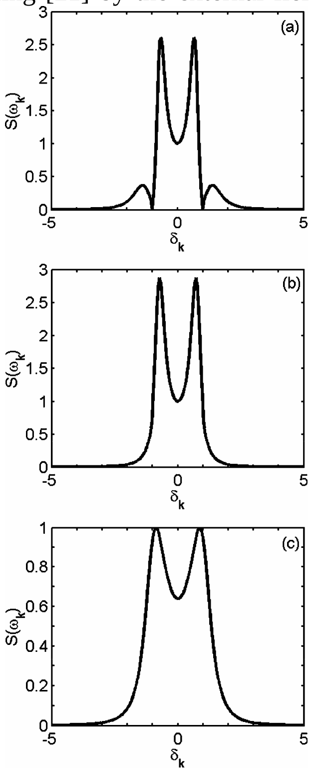

Figure 4. The spontaneous emission spectrum $S(\omega_k)$ (in arbitrary units) for transition from the upper level $|2\rangle$ to the lower level $|0\rangle$ as a function of detuning δ_k for different reservoirs. a) Double-band isotropic PBG reservoir and $\beta_{10}=1.0$, b) double-band anisotropic PBG reservoir and $\alpha_{10}=1.0$, c) free vacuum reservoir and $\gamma_{10}=1.0$. Here $\theta=\pi/2$ and the other parameters used are the same as those in Figure 2.

spontaneous emission spectrum $S(\omega_k)$ consists of two components, the quantum interference of which in the case of PBG reservoir leads to a dark line at $\delta_k = \Delta$. On the other hand, in the case of free space reservoir this quantum interference is not completely destructive, so that no dark line exists in the case of free space reservoir.

Since this dark line is the consequence of combined effects of classical driving field and DOS of modified reservoir, so the existence of the dark line in the spontaneous emission spectrum is independent of the position of ω_{10} (the frequency of transition in modified reservoir) within the PBG. For showing this, the spontaneous emission spectrum is plotted in Figure 3, in the case where ω_{10} is on the upper band of PBG (not within the PBG). The used parameter are $\delta_{10g1}=2.5$,

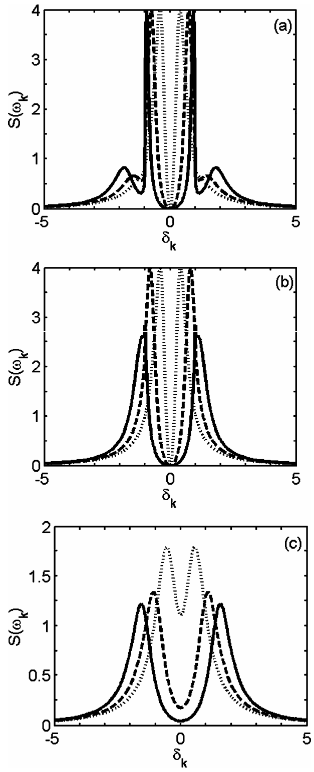

Figure 5. The spontaneous emission spectrum $S(\omega_k)$ (in arbitrary units) for transition from the upper level $|2\rangle$ to the lower level $|0\rangle$ as a function of detuning δ_k for different reservoirs. a) Double-band isotropic PBG reservoir, b) double-band anisotropic PBG reservoir, c) free vacuum reservoir. $\Omega=1.5$ (solid lines), $\Omega=1.0$ (dashed lines), $\Omega=0.5$ (dotted lines). The other parameters used are the same as those in Figure 2.

$\delta_{10g2}=0.5$, $\beta_{10}=1.0$, $\alpha_{10}=1.0$, $\Omega=1.5$ (solid lines), $\Omega=1.0$ (dashed lines) and $\Omega=0.5$ (dotted lines). It is seen that, this dark line present even for relatively weak driving field, but in the absence of the driving field there is no dark line in the spontaneous emission spectrum.

In the case where an atom is pumped to the upper level $|1\rangle$ $(\sin\theta=1)$ only for the double-band isotropic PBG reservoir two black dark lines exist in the spontaneous emission spectrum (Fig. 4). The origin for this feature can be traced back to Equations (22, 25, 26). From these equations, the forms of Laplace transform of the delayed Green function (in isotropic PBG reservoir) involved in the spontaneous emission spectrum include the following items:

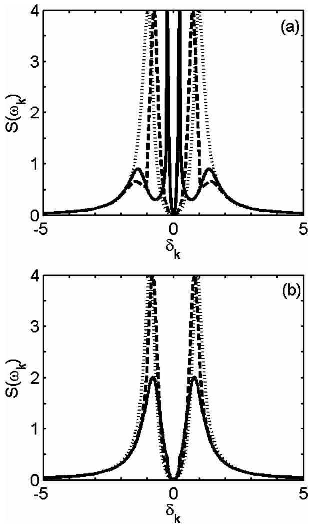

Figure 6. The spontaneous emission spectrum $S(\omega_k)$ (in arbitrary units) for transition from the upper level $|2\rangle$ to the lower level $|0\rangle$ as a function of detuning for different reservoirs. a) Double-band isotropic PBG reservoir, b) double-band anisotropic PBG reservoir. $\delta_{10g1} = -\delta_{10g2} = 0.25$ (solid lines), $\delta_{10g1}=-\delta_{10g2}=1.0$ (dashed lines) and $\delta_{10g1}=-\delta_{10g2}=5.0$ (dotted lines). The other parameters used are the same as those in Figure 2.

$$\left(\frac{i}{\sqrt{\delta_k + \Delta + \delta_{10g1}}} + \frac{1}{\sqrt{\delta_k + \Delta + \delta_{10g2}}}\right) \quad (27)$$

The above items show clearly that the singularities of the Laplace transform of the delayed Green function for isotropic PBG modes are the origin of dark lines.

In order to investigate the effects of driving field on the spontaneous emission spectra in the PBG and free vacuum reservoirs, we plot the spontaneous emission spectra as functions of detuning δ_k in three cases, as shown in Figure 5. From Figure 5 (a) we see that there are two pronounced peaks at $\delta_k = \delta_{10g1}$ and $\delta_k = \delta_{10g2}$, and two lower peaks in the sides, in the case of double band isotropic PBG reservoir. The increasing of the Rabi frequency of driving field reduces the width of the pronounced peaks and increases the intensity of side peaks. On the other hand, we see that in the cases of double-band anisotropic PBG reservoir and free space reservoir, (Figures 5 (b),(c)) the increasing of the Rabi frequency of driving field reduces the intensity of two side peaks.

The effect of the width of PBG on the spontaneous emission spectrum for transition from the upper level $|2\rangle$ to the lower level $|0\rangle$ (for $\theta = 0.0$) is displayed in Figure 6. From Figure 6(a) we see that in the case of double-band isotropic PBG reservoir, the increasing of the width of PBG reduces the intensity of two side peaks.

Also Figure 6(b) shows that in the case of double-band anisotropic PBG reservoir the increasing of the width of PBG increases the intensity of the peaks. In both cases by increasing the width of PBG, the spontaneous emission spectrum goes to the Autler–Townes spectrum. This is because the transition from the upper level $|1\rangle$ to the lower level $|0\rangle$ is forbidden for frequency within PBG. This situation is similar to free space case with $\gamma_{10} = 0$.

4. Conclusion

The spontaneous emission spectra of a driven three-level atom embedded in a double-band photonic crystal have been investigated. A transition from one of the upper levels to one of the lower levels was assumed to interact with free vacuum modes, and the transitions from the other upper level to the other lower level were assumed to interact separately with isotropic PBG modes, anisotropic PBG modes and free vacuum modes. The spontaneous emission spectra for the transition coupled to the free vacuum modes are studied. Most interestingly it is shown that only in the presence of the classical driving field; there is a black dark line in the spontaneous emission spectrum of free space transition, when the modified reservoir is of the PBG type. On the other hand, this dark line is not seen in the case where the modified reservoir is of the free space type for relatively weak driving field. It is shown that in the case of double-band isotropic PBG reservoir there are two kinds of dark lines. These dark lines are the results of the quantum interference between two Autler-Townes components of the spontaneous emission spectra, and the singularities of the Laplace transform of the delayed Green function for isotropic PBG modes.

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