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HELICITY AND PLANAR AMPLITUDES IN PION-PROTON SCATTERING AT 6.0 GeV/c

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Abstract

In addition to optimal conditions, invariant laws of Lorentz, parity and time reversal are imposed to find the relation between observables (spin rotation parameters) and bilinear combination of helicity amplitudes in pion-proton elastic scattering at 6.0 GeV/c. By normalizing the differential cross-section to unity, the magnitudes of helicity amplitudes and the angle between them are determined. Planar amplitudes are found in terms of the helicity amplitudes and the angle between momentum vector and quantization direction in the reaction plane.

Introduction

In pion-proton reaction, determination of amplitudes in the transversity frame is done easily [1] because there is no phase between them. In the helicity frame, the quantization direction of particles is in the scattering plane (characterized by incident and scattered momenta). The angle between the helicity amplitudes is also involved. There are only two independent complex spin amplitudes in the helicity frame, namely, single-flip and non-flip amplitudes. Only these two amplitudes survive after the imposition of symmetry laws such as rotational, time reversal and parity invariance to the optimal form [2] of the overall reaction matrix (relating experimental observables and bilinear combination of helicity amplitudes). Spin rotation parameters (R and A) are given in terms of the helicity amplitudes and laboratory recoil angle, θ [3]. Therefore, relativistic kinematics is used to get the recoil angle in terms of momentum transfer squared t and square of C.M. energy S [4].

In case the quantization direction of the target particle, which is in the reaction plane, is not in the direction of the momentum and makes and angle β with it, then the corresponding amplitudes are planar. Planar amplitudes are given in terms of the helicity amplitudes and angle β [5].

Keywords: Spin; Amplitude; Polarization; Scattering; Helicity

Helicity Amplitudes and Phase Determination

Let's denote the two helicity amplitudes by D_h^{++} and D_h^{+-}. The spin rotation parameters R and A the polarization parameter P and the differential cross-section are given in terms of the helicity channel amplitudes (3) as

$$\left|D_h^{++}\right|^2+\left|D_h^{+-}\right|^2=\frac{q^2}{\pi}\frac{d\sigma}{dl} \quad (1)$$

$$P=-2I_m D_h^{++}D_h^{+-*} \quad (2)$$

$$R=-\left(\left|D_h^{++}\right|^2-\left|D_h^{+-}\right|^2\right)cos\theta-2ReD_h^{++}D_h^{+-*}.sin\theta \quad (3)$$

$$A=-\left(\left|D_h^{++}\right|^2-\left|D_h^{+-}\right|^2\right)sin\theta-2ReD_h^{++}D_h^{+-*}cos\theta \quad (4)$$

Writing each amplitude in terms of its magnitude and phase and defining the phase difference

Table 1. The magnitudes of the helicity amplitudes and the angle between them α for different values of spin rotation parameters at various recoil angles θ for the π^+p at 6 (GeV/c)

t	θ	R	A	P	$\|D_h^{++}\|$	$\|D_h^{+-}\|$	α
.229	73.391	.100	.971	.217	.975	.221	330.181
.265	72.180	-.060	.982	.179	.988	.153	324.343
.298	71.148	-.200	.961	.191	.993	.114	303.584
.391	70.236	-.200	.970	.138	.995	.099	316.667
.365	69.236	-.110	.982	.154	.989	.146	328.044
.392	63.523	-.030	.988	.152	.982	.187	336.411
.444	67.227	-.020	.993	.116	.981	.196	343.646
.499	65.949	-.130	.990	.055	.989	.147	350.887

Table 2. The magnitudes of the helicity amplitudes and the angle between them α for different values of spin rotation parameters at various recoil angles θ for the π^-p at 6 (GeV/c).

t	θ	R	A	P	$\|D_h^{++}\|$	$\|D_h^{+-}\|$	α
.227	73.461	-.210	.969	.130	.997	.075	300.185
.260	72.342	-.300	.947	.115	.998	.058	272.993
.293	71.300	-.150	.985	.085	.995	.097	334.146
.329	70.236	-.200	.976	.086	.997	.083	329.276
.356	69.481	-.420	.902	.100	.998	.063	308.342
.392	68.523	-.060	.997	.049	.987	.159	352.986
.444	67.227	-.330	.943	.043	.999	.037	325.282
.497	65.994	-.070	.997	.033	.985	.174	355.522

$$\alpha=\theta_1-\theta_2 \tag{5}$$

then

$$D_h^{++}\cdot D_h^{+-*}=\left|D_h^{+-}\right|\left|D_h^{++}\right|e^{i\alpha} \tag{6}$$

Normalizing the differential cross-section to unity and imposing the following constraint on observables, [3] i.e.,

$$P^2+R^2+A^2=1 \tag{7}$$

we can determine magnitudes of the amplitudes and their phases in terms of observables and recoil angle θ as

$$\left|D_h^{++}\right|^2=\frac{1+A\sin\theta-R\cos\theta}{2} \tag{8}$$

$$\left|D_h^{+-}\right|^2=1-\left|D_h^{++}\right|^2 \tag{9}$$

and

$$\sin\alpha=\frac{-p}{2\left|D_h^{++}\right|\left|D_h^{+-}\right|} \tag{10}$$

for different values of t, the corresponding lab recoil angle are given by [4]

$$tg\,\theta=\frac{2m_2\left\{\left(S-(m_2+m_1)^2\right)\left(S-(m_2-m_1)^2\right)+St\right\}^{\frac{1}{2}}}{\left(S-m_1^2+m_2^2\right)\sqrt{-t}} \tag{11}$$

where

$$S=m_1^2+m_2^2+2m_2\sqrt{m^2{}_1+p_1^2} \tag{12}$$

$$p_1=6GeV/c \tag{13}$$

$$m_1=0.140GeV \tag{14}$$

$$m_2=0.938GeV \tag{15}$$

Experimental values of R and A are given for different t-values [3]. Helicity amplitudes and their phases are completely determined as a function of t and are given in Figures 1-4 and tabulated in Tables 1 and 2 for π^+p and π^-p reactions. Determination of the helicity amplitudes enable us to test the validity of the dynamical models [6,7].

Determination of Planar Amplitudes

The relations between the helicity channel amplitudes and the planar amplitudes are given as [2]

$$D_p^{++}(\beta)=D_h^{++}\cos\beta-D_h^{+-}\sin\beta \tag{16}$$

$$D_p^{+-}(\beta)=D_h^{++}\sin\beta+D_h^{+-}\cos\beta \tag{17}$$

where D_p^{++} and D_p^{+-} stand for the planar amplitudes and β was defined previously. If D_h^{++} is chosen to be all real (since the overall phase in arbitrary), then equations 16 and 17 are simplified as

$$ReD_p^{++}(\beta)=D_h{++}\cos\beta-ReD_h^{+-}\sin\beta \tag{18}$$

$$ImD_p^{++}(\beta)=-ImD_h^{+-}\sin\beta \tag{19}$$

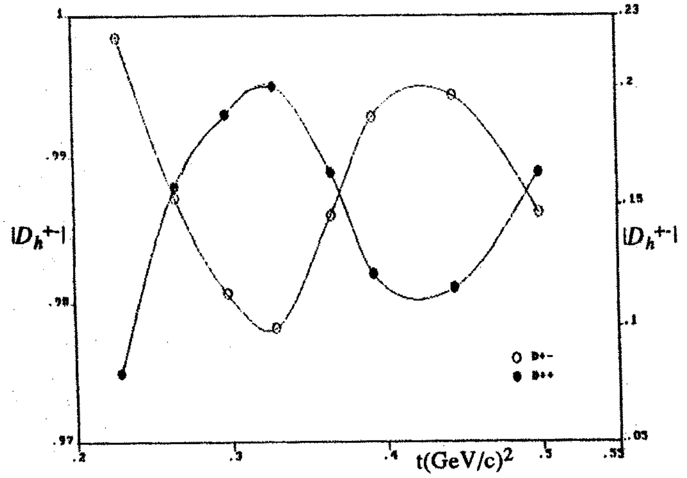

Figure 1. The magnitudes of the helicity spin flip and non-flip amplitudes in π^+p scattering at 6 GeV/c

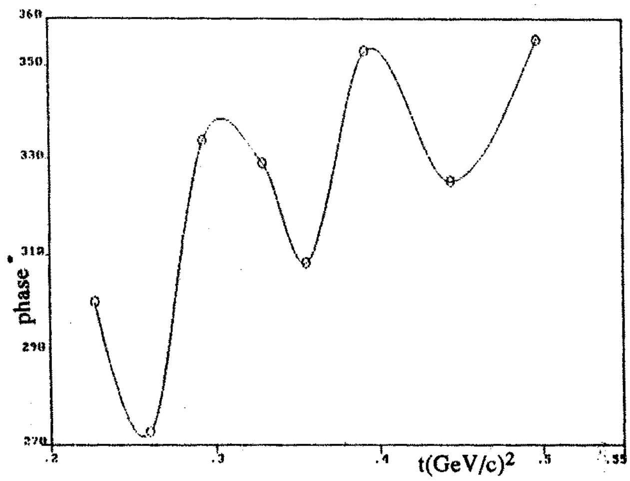

Figure 4. The phase angle between the helicity amplitudes in π^-p scattering for different t-values at 6 GeV/c

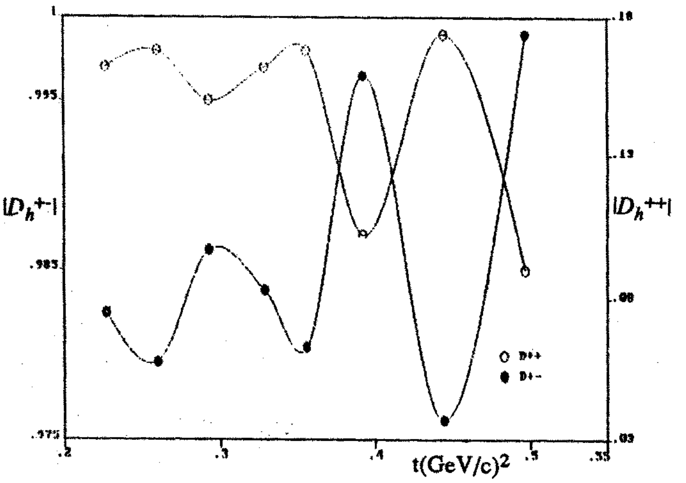

Figure 2. The magnitudes of the helicity amplitudes in π^-p elastic scattering at 6 GeV/c

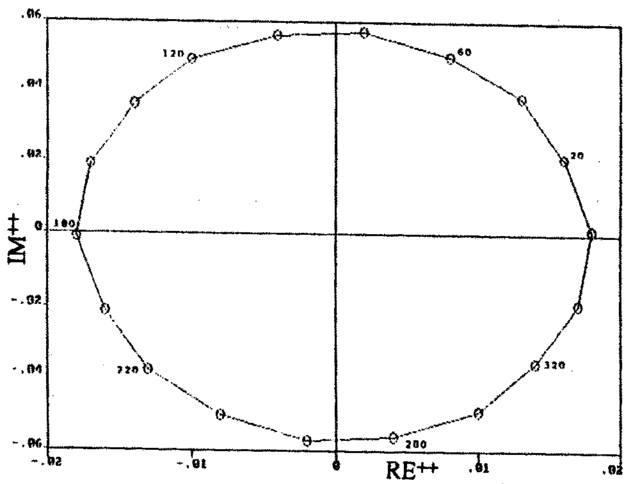

Figure 5. The planar amplitudes $D_p^{++}(\beta)$ in π^-p scattering for $|D_h^{++}| = .998$ and $|D_h^{+-}| = .058$ at t = .260 (GeV/C)^2 for β = 0 - 360°

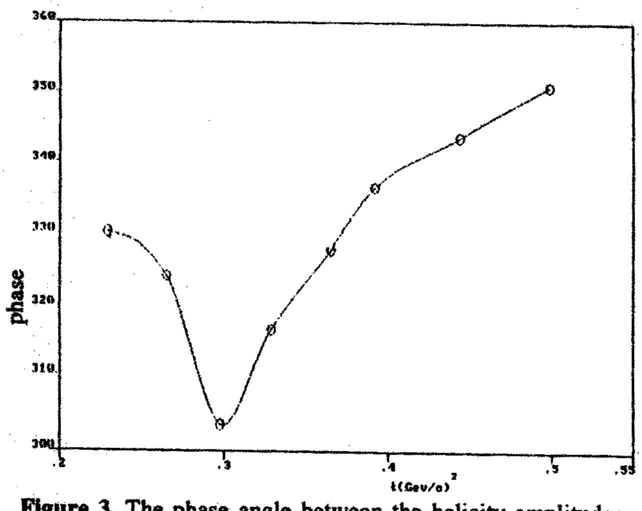

Figure 3. The phase angle between the helicity amplitudes in π^+p scattering for different t-values at 6 GeV/c

$$ReD_p^{+-}(\beta) = D_h^{++} \sin\beta - ReD_h^{+-} \cos\beta \qquad (20)$$

$$ImD_p^{+-}(\beta) = ImD_h^{+-} \cos\beta \qquad (21)$$

As β changes from 0-360° degrees, the corresponding planar amplitudes form an ellipse in complex plane. A sample of such ellipses is shown in Figure 5.

Conclusion

Non-dynamical analysis of spin amplitudes in optimal form enable us to find complex amplitudes in the transversity, helicity and planar frames. In the transversity frame, only one parameter, namely p_0, determines the magnitudes of amplitudes, but in the helicity frame, due to phase involvement, additional spin rotational parameters, namely R and A, must be measured for a complete determination of amplitudes. There are only two independent amplitudes, i.e. a single-flip and a non-flip in the helicity frame, after imposing invariant laws.

Planar amplitudes are calculated using the helicity amplitudes and angle β between quantization axis and momentum of particle and their corresponding ellipses in complex plane are found. Numerical calculations indicate that while there exist fluctuations in the relative magnitude of both amplitudes as t increases, the magnitude of the non-flip amplitudes remain larger (five up to twenty-seven times) than single-flip amplitudes in both π^+p and π^-p reactions.

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References

1. Ghahramany, N. *J. Science, I.R. Iran*, **3**, (3-4), (1992).
2. Goldstein, G. R. and Moravcsik, M. J. *Ann. Phys.*, **142**, 219, (1982).
3. Delesquen, A. *et al. Phys. Lett.*, **40B**, 277, (1972).
4. Hagedorn, R. *Relativistic kinematics*. Benjamin Inc., N. Y. 57 (1963).
5. Ghahramany, N., Goldstein, G. R. and Moravcsik, M. J. *Phys. Rev.*, **D 28**, 1086, (1983).
6. Goldstein, G. R, Moravcsik, M. J. and Ghahramany, N. *Phys. Lett.*, **152B**, 265, (1985).
7. Ghahramany, N., Moravcsik, M. J. and Goldstein, G. R. *Phys. Rev.*, **D31**, 195, (1985).