J. Sci. I. R. Iran

Vol.7 No.4
Autumn 1996

A MODIFIED THEORY OF NEWTONIAN MECHANICS

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Abstract

A specific form of the inertial law is presented by which we can gain a deeper insight into the essence of mass and inertia. In this modified theory, there is no need to keep the concept of absolute space and Newton's third law as principles. By introducing a convenient form for gravitational law the coupling constant G becomes a function of inertial parameters of the universe.

According to Newton's second law of motion, the relation $\vec{F} = m\vec{a}$ holds between applied force on a particle and its acceleration in absolute space and any other frames which are related to it by Galilean transformations. These reference frames are called inertial frames and m is inertial mass of the particle. The relation betweeen coordinates of two inertial systems S and S' which are moving with constant velocity $\vec{V}$ with respect to each other are:

$$\begin{cases} t' = t \\ \vec{a}' = \vec{a} \\ \vec{u}' = \vec{u} - \vec{V} \\ \vec{x}' = \vec{x} - \vec{V}t + \vec{x}_0 \end{cases} \tag{1}$$

Despite its simple appearance and its practical applications, Newton's concept of absolute space, which is the basis of Newtonian mechanics (NM) and in turn the basis of whole physics through introducing the concept of mass and energy, is not well defined and has been criticized. Cogent arguments against absolute space are [1]:

(a) There is no unique way of locating Newton's absolute space within the infinite class of inertial frames.
(b) It conflicts with one's understanding to conceive of a thing which acts but cannot be acted upon.
(c) It is purely *ad hoc*.

Inertial frames play a crucial role in Newtonian mechanics but the latter can not offer an explanation for their existence.

According to Mach's principle, inertia is due to an interaction with average mass of the universe [2]. There have been many attempts to find a physical description for the source of inertia. Einstein's main aim in the theory of general relativity was to solve this problem through giving an equation for gravitation and inertia together. But it came out that the theory shows some non-Machian aspects, and does not fulfill this requirement. For instance it has a solution for empty space. The Brans-Dicke theory is another attempt [3,4,5,6,7].

The general theory of relativity, which is accepted universally as the standard theory of gravitation, suffers some singular problems and the source of inertia too is not well defined. It seems with closer scrutiny of second law we may find our way to solving these problems. We propose that inertial effect is a mutual interaction between two particles which in any non-

Keywords: Absolute Space; Cosmology; Inertia

rotating arbitrary reference frame S are proportional to the difference of their accelerations with respect to S and to the inertial charge of each particle as follows:

$$\vec{F}_i=\mu . c_1 . c_2(\vec{a}_1-\vec{a}_2) \tag{2}$$

where $\vec{a}_1$ and $\vec{a}_2$ are accelerations of particles 1 and 2 with respect to S, c_1 and c_2 are their inertial charges respectively, and μ is inertial coupling constant. This can be easily extended to systems consisting of N particles. Again in any non-rotating arbitrary reference frame S we have:

$$\vec{F}_i=\mu . c_i \sum_{j=1}^{N} c_j(\vec{a}_i-\vec{a}_j) \tag{3}$$

where $\vec{F}_i$ is the applied force on particle i, c_i is its inertial charge and $\vec{a}_i$ is its acceleration with respect to S. Summation is done over all particles.

In the real world the inertial charge and inertial mass of a particle are related as follows:

$$m_i=\mu . c_i \sum_{j=1}^{al} c_i \text{ (over all particles in the universe)} \tag{4}$$

Since local inhomogeneities have no observed effects on the inertial mass then it is well accepted that the inertial mass is determined by the global structure of the universe and the relation (4) for the inertial mass is in accordance with this general belief. In terms of inertial mass, equation (3) can be rewritten in the following form:

$$\vec{F}_i=m_i\left(\vec{a}_i-\frac{\sum_{j=1}^{al} m_j \vec{a}_j}{\sum_{j=1}^{all} m_j}\right) \tag{5}$$

These new forms of the second law i.e. equations (3) and (5) are invariant in any other reference frame S′ which is related to S by the following transformations:

$$\begin{cases} t'=t \\ \vec{a}'=\vec{a}-\vec{b} \\ \vec{u}'=\vec{u}-\vec{b}t-\vec{v} \\ \vec{x}'=\vec{x}-\frac{1}{2}\vec{b}t^2-\vec{v}t+\vec{x}_0 \end{cases} \tag{6}$$

where $\vec{b}$ is a constant acceleration of S′ with respect to S, $\vec{v}$ is the velocity of S′ with respect to S at t = 0, and $\vec{x}_0$ is the position of S′ with respect to S at t = 0.

Within the set of reference frames S′ with different values of $\vec{b}$ and $\vec{v}$ there is a unique subset called S″ which is moving with acceleration $\vec{b}_0$

$$\vec{b}_0=\frac{\sum_{j=1}^{al} m_j \vec{a}_j}{\sum_{j=1}^{al} m_j} \tag{7}$$

with respect to S. In S″ the form of inertial law is reduced to the Newtonian one: $\vec{F}_i = m\vec{a}_i''$. Therefore, inertial frames are a unique set of frames which are moving with acceleration b_0 with respect to S and $\vec{v}$ and $\vec{x}_0$ can have any arbitrary values. Since b_0 is determined globally then local inhomogeneities have a negligible effect on it and inertial frames.

It is seen that equations (3) and (5) also satisfy Newton's third law. For a two particle system we have:

$$\vec{F}_1=-\vec{F}_2=\mu . c_1 . c_2(\vec{a}_1-\vec{a}_2) \tag{8}$$

and for a system with N particle they make:

$$\sum_{i=1}^{N} \vec{F}_i=0 \tag{9}$$

So there is no need to introduce it as an extra principle.

We can extend this model to Newton's law of gravity. Equivalence principle here means that the source of inertia and gravitation is the same. Then if we define the gravitational force between two particles of inertial charges c_1 and c_2 as:

$$|\vec{F}_G|=\frac{\mu^2 . c_1 . c_2}{|\vec{r}_{12}|^2} \tag{10}$$

where μ is the inertial coupling constant and $|\vec{r}_{12}|=|\vec{r}_1-\vec{r}_2|$ is the relative separation between particles 1 and 2, then we can express gravitational constant G in terms of inertial charges c_is;

$$G=\left(\sum_{j=1}^{\text{all}} c_j\right)^{-2} \tag{11}$$

Since the inertial mass and the gravitational constant are finite quantities, this means that $\left(\sum_{j=1}^{\text{all}} c_j\right)$ is finite too, and the universe can not be infinitely extended.

The Lagrangean form of the mechanics based on this model of inertial law is the same as the one for NM except that the kinetic energy of system T which is equal to $\sum_i \frac{1}{2} m_i v_i^2$ in NM is replaced by:

$$T=\sum_i \frac{1}{2} m_i v_i^2 - \frac{\left[\sum_i m_i \vec{v}_i\right]^2}{2\sum_i m_i} \tag{12}$$

$$=\frac{1}{4}\sum_i \sum_j m_i m_j \frac{(\vec{v}_i-\vec{v}_j)^2}{\left(\sum_j m_j\right)}$$

This is an invariant scalar. The total energy of the system which is the sum of kinetic energy T and potential energy $V(r_{ij})$, a function of relative separation $r_{ij}=\vec{r}_i-\vec{r}_j$, is invariant in all non-rotating reference frames.

Remarks

(i) It may seem that choosing non-rotating frames is some kind of restriction which reduces the generality of the chosen frames. This is however, not the case, since rota[illegible]ames with respect to the whole universe are distinguishable and can be fixed.

(ii) The standard models of cosmology and stellar structure which are based upon the general theory of relativity (GR) are beset with serious problems which call for a modification of the theory. In deriving GR equation we use the same concepts of gravitational coupling constant, inertial mass and energy-stress tensor which NM introduces us to.

Our results show that G and m are not constants of nature and may change anytime the total inertial charge of the universe changes significantly. This can happen in the early stage of evolution of the universe, i.e. in epochs of pair production when the kind of dependence can be discovered from given results.

(iii) The concept of energy which this new theory introduces us too, is independent of the reference frame. This is an important point which should be taken into account when writing new energy-stress tensor.

(iv) There is no inertial structure for empty space in this theory.

This shows us how some things should change in a modified theory of general relativity.

References

1. Rindler, W. *Essential relativity*, (2nd ed). Springer-Verlag, (1977).
2. Weinberg, S. *Gravitation and cosmology*. John Wiley, (1972).
3. Joshi, P.S. *Global aspects in gravitation and cosmology*. (1993).
4. Raychaudhuri, A.K. *Theoretical cosmology*. (1974).
5. Milgron, M. *Annals of Phys.*, **229**, 383-415, (1994).
6. Graneau, P. *Phys. Essays*, **8**, (3), 376-83, (Sept. 1995).
7. Kolich, S.J. *Annals of Phys.*, **246**, 121-132, (1996).