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Competitive supply of durable goods under stochastic fluctuation in stock

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ABSTRACT

This paper presents a theoretical model in which the stock growth rate of durable goods has stochastic fluctuation over time. It concludes that a social planner increases the expected percentage rate of production since uncertainty increases the user cost from consumer's pointview.

I. Introduction

In 1972, Coase in a durable good model suggested that since the monopolist were unable to precommit to a future sales path, its market power would varish "in the twinkling an of ege". This suggestion has been known as a Coase conjecture in the literature. It is analyzed in many stationary and non-stationary equilibrium models of consumer's expectations. In stationary models, the expected future levels of the stock are assumed to be a continuous function of the current stock. Its functional form is identical in each period of time. These issues were developed formally by Bulow (1982), Stokey (1981) and Gul etal (1986). Furthermore, with stationary rational expectations, Coase conjecture has been verified by Kahn (1986) with increasing marginal cost and by Bond and Samuelson (1984) with non-zero depreciation. considering both increasing marginal cost and depreciation, Driskill (1997) found that the monoply supply would be lower than the competitive levels of steady state output. In non-stationary equilibrium models, consumer's expectations act like trigger strategies in which consumers will punish the monopolist if it deviates its supply from the competitive level.

This argument has been found by Bond and Samuelson (1987) with depreciation and constant marginal cost and has been proved by Ausubel and Deneckere (1989) and by Sobel (1991) with constant marginal cost[(1)].

In contrast to these models, this paper has developed a theoretical model under uncertainty condition in which the stock of durable good changes randomly over time.

A social planner under competitive conditions determines the expected percentage change of the supply in order to maximize the expected sum of the discounted total net benefit over the planning horizon. The paper concludes that uncertainty increases the implicit shadow price of the stock relative to the marginal value of the services demanded by consumers for using durable goods, which it causes an increase in the expected rate of change in the supply.

In the next section, a theoretical model is developed under stochastic fluctuation in the stock of durable goods. In the subsequent section, the final result of the model is used to derive the dynamics of the production in order to analyze the factors affecting the expected supply path. The final section contains the main conclusion.

II. Theoretical Model

The stock of durable good is used by consumers and provides services demanded by consumers over its lifetime. The value of services for using durable goods is a function of the stock of durable goods held by consumers. In any point of time, this function is given by $S = S(X)$, where X is the stock of durable goods, and $S_x > 0$ with $S_{xx} < 0$.

S_x is the derivative of S with respect to X at time t. It represents the marginal value of total services. S_{xx} is the derivative of S_x with respect to X and shows the negative slope of the demand schedule. Letting b denote the rate of depreciation and then bx is the amount of the stock to be decayied at time t. Thus, the stochastic changes of the stock of durable goods over

1. The credibility of the Coas conjecture has been investigated in the models where a unit of exhaustible resource can be used to produce a one unit of durable good. These models has been developed by Karp (1993) and by Malueg and solow (1990).

time is written as:

$$dx = (q - bx)dt + \sigma(x)dz \tag{1}$$

Where q is the rate of production and dz follows a wiener process with $dz \sim N(0, dt)$ [1]. Equation (1) shows that the stock fluctuates randomly over time with $E_t[dx] = (q - bx)dt$ and $Var[dx] = \sigma^2(x)dt$ [2]. $\sigma(x)$ is the stock variance and its derivative with respect to x is positive, that is, $\sigma' = \frac{d\sigma}{dx} > 0$.

It is assumed that a social planner controls the production of durable goods in a competitive market. The net benefit function is defined by $B = S(x) - C(q)$, where C(q) is production cost function with $C_q = \frac{dc}{dq} > 0$ and $C_{qq} = \frac{d^2c}{dq^2} > 0$. C_q is the increasing marginal cost function. It is assumed that a social planner maximizes the expected sum of the discounted net benefits over time, to this end, the objective function can be written as:

$$E_t \left[\int_t^T B(x, q) e^{-r(u-t)} du \right] \tag{2}$$

The planner's problem is to maximize (2) with respect to (1). Thus, its solusion provides the optimal value function as follows:

$$J(x, t) = \underset{q}{Max} \, E_t \left[\int_t^T B_d(x, q) du \right] \tag{3}$$

Where, $J(x,t) = V(x)e^{-rt}$, and $B_d(x,q) = B(x,q)e^{-ru}$. Using stochastic dynamic programming, the fundamental equation of optimality is obtained as:

$$0 = \underset{q}{Max} \left[B_d(x, q) + (\frac{1}{dt}) E_t (d(j)) \right] \tag{4}$$

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1. *Since* $Z(t) \sim N(0, t)$ *then* $dz \sim N(0, dt)$. *On the other hand,* $Var(dz) = dt$ *and then* $E(dz^2) = dt$. $Var(dz^2) = 0$ *because* $dt^n \rightarrow 0$ *for* $n > 1$ *and therefore* $E(dz^2) = dz^2 = dt$. *Also,* $Var(dz\,dt) = 0$ *and then* $E[dz\,dt] = dz\,dt = 0$.

2. *From equation (1),* dx^2 *is obtained as:*

$$dx^2 = (q - bx)^2 dt^2 + \sigma^2 dt^2 + 2(q - bx)\sigma\, dz\, dt \tag{i}$$

Taking expectation from both sides of (i), the result will be:

$$E(dx^2) = \sigma^2 dt + O(dt)$$

Where $O(dt)$ *represents terms that vanish as* $dt \rightarrow 0$. *Since* $Var(dx^2) = 0$, *then* $E(dx^2) = dx^2 = \sigma^2 dt$. *Also,* $Var(dx\,dt) = 0$ *and then* $E(dx\,dt) = dx\,dt = 0$.

Equation (4) determines the expected dynamics of production at the optimal level which maximizes the sum of the discounted net benefit and the expected rate of value function. Since $J(x,t)$ is a function of the stochastic process x, its Ito's differential lemma can be applied as[1]:

$$(\frac{1}{dt})E_t(d(j)) = \left[V_x(q-bx) - rV(x) + \frac{1}{2}V_{xx}\,\sigma^2(x)\right]e^{-rt} \quad (5)$$

$V(x)$ is a function of stochastic process x with $V_x = \frac{dV}{dx} > 0$ and $V_{xx} = \frac{d^2V}{dx^2} < 0$. V_{xx} has a negative sign because B_d is a concave function and so that $V(x)$ as a indirect objective function is also concave. Substituting (5) and $B_d(x,q)$ into (4), the final optimality equation can be obtained as:

$$rV(x) = \underset{q}{Max}\left[B(x,q) + V_x(q-bx) + \frac{1}{2}V_{xx}\,\sigma^2\right] \quad (6)$$

In condition (6), $rV(x)$ is the rate of return on the durable good stock has two component, the net benefit as a cash flow $B(x,q)$, and the expected capital gain $(\frac{1}{dt})E_t(dV)$[2].

In order to determine, the expected rate of change of production, the derivatives of (6) with respect to q and x are obtained as follows:

$$C_q = V_x \quad (7)$$

$$(r+b)V_x - \sigma'\sigma V_{xx} - S_x = (\frac{1}{dt})E_t(d(V_x)) \quad (8)$$

Where $(\frac{1}{dt})E_t(d(V_x)) = V_{xx}(q-bx) + \frac{1}{2}V_{xxx}\,\sigma^2$.

Equation (7) is an equilibrium condition for the planner in which V_x

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1. $J(x,t) = V(x)e^{-rt}$ *is used to apply Ito's expansion. That is:*

$$dJ = J_x dx + J_t dt + \frac{1}{2}J_{xx}\sigma^2 dt \quad (ii)$$

Substituting $J_x = V_x e^{-rt}$, $J_{xx} = V_{xx}e^{-rt}$ *and* $J_t = -rVe^{-rt}$ *into (ii), the outcome will be as:*

$$dJ = (V_x dx - rVdt + \frac{1}{2}V_{xx}\sigma^2 dt)\,e^{-rt} \quad (iii)$$

Applying Ito's differential operator for (iii), equation (5) can be obtiained.

2. $V = V(x)$ *is the stochastic function and its expansion by Ito's lemma can be written as:*

$$dV = V_x dx + \frac{1}{2}V_{xx}\sigma^2 dt \quad (iv)$$

Applying Ito's rule for (iv), the result will be as:

$$(\frac{1}{dt})E_t(dV) = V_x(q-bx) + \frac{1}{2}V_{xx}\sigma^2$$

measures the implicit shadow value of a per unit of the stock. This shadow price is equal to the incremental cost that is obtained by selling an additional unit. In equation (8), $(r + b)V_x - \sigma'\sigma V_{xx}$ is the opportunity cost adjusted to the stock uncertainty. Its difference with the marginal value of the services S_x is called the adjusted user costs. At the optimal level of production, consumers equate the expected rate of the shadow price of the stock with the adjusted user cost as shown by condition (8).

Since both sides of equation (7) is a function of the stochastic process, its time derivative does not exist. Therefore, the Ito's lemma can be applied to both sides of equation (7) and the following result can be obtained:

$$(\frac{1}{dt})\, E_t\,(d(V_x)) = C_{qq}\,(\frac{1}{dt})\,E_t\,(dq) \tag{9}$$

Equation (8) and (9) can be combined to eliminate V_x and obtain the following equation:

$$(r + b)\,V_x - \sigma'\sigma V_{xx} - S_x = C_{qq}\,(\frac{1}{dt})\,E_t\,(dq) \tag{10}$$

In equation (10), the stock uncertainty can affect the expected dynamics of production in the competitive market.

III. The expected dynamics of supply

With increasing marginal cost, the expected growth rate of the durable good supply is derived by equation (10), That is:

$$(\frac{1}{dt})\, E_t\,(dq) = \frac{1}{C_{qq}}\left[(r + b + \sigma'\sigma L(x))\,V_x - S_x\right] \tag{11}$$

Where $L(x) = -\frac{V_{xx}}{V_x}$ with $V_{xx} < 0$ and so that $L(x) > 0$. $L(x)$ can be thought of an index of the implicit absolute risk aversion. It measures the amount of risk premium that the planner would like to pay in order to eliminate any stock uncertainty. Since $\sigma' > 0$, with an increase in x, the stock variance rises by $\sigma\sigma'$, and that its multiplication with L(x), that is $\sigma\sigma' L(x)$, will yield the amount of risk premium. The planner needs to pay that premium to offset the stochastic fluctuation for increasing production rate. Due to (11), $(r + b + \sigma'\sigma L(x))\,V_x$ can be interpreted as an opportunity cost from consumer's viewpoint. Its difference with S_x is called user cost adjusted by the stock uncertainty. The user cost is positive due to equation (8), and so that an increase in the stock variance leads to increase in the user cost which in turn will cause an increase in the expected rate of change of the supply.

If $\sigma' = 0$, the stock variance will be constant over time. In this case the expected production rate is unaffected by the stock uncertainty and will be the same as in the deterministic case. If $\sigma = 0$, the result will become as a certainty case. If b = 0, the equation (11) will yield the expected dynamics of perfect durable good supply.

IV. Conclusion

This paper presents a theoretical model to determine the expected dynamics of the durable good supply produced by a social planner. The conclusion is that since uncertainty increases the stock variance, the planner should increase the expected rate of production by paying risk premium. In addition, uncertainty increases total user cost from consumer's viewpoint which in turn causes the social planner increases the expected rate of supply.

If monopolist produces a durable good and considers the services function instead of the gross income, the expected rate of production would be determined at the competitive level.

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