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Relaxation and Thermal Conductivity of Hot and Thermal Quasiparticles and Fermi Liquid Interactions in d-wave Superconductors

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Abstract

In this paper, we calculate theoretically the relaxation rates of hot, thermal and antinodal quasiparticles by using Fermi's golden rule. The transition probabilities at low temperatures express in terms of Bogoliubov coefficients, singlet and triplet scattering amplitudes and dimensionless Landau parameters. The values of Landau parameters may be determined by comparison with the experimental data. Furthermore, we calculate thermal conductivity coefficients of YBCO in terms of antinodal quasiparticle relaxation rates which are in accordance with experimental results.

Keywords: d-wave superconductor; Fermi liquid; Relaxation rate; Landau parameter; Thermal conductivity

Introduction

Experiments such as penetration depth, transport and angle resolved photoemission support the well-defined quasiparticle excitations in the high-T_c superconductors at low temperatures [1]. There are also considerable experimental evidences that the high-T_c cuprates exhibit the simple power law dependencies at low temperatures. Superconducting state properties of the cuprates thus appear to be consistence with d-wave BCS theory with nodal quasiparticle excitations. Furthermore, the normal state properties of the high-T_c superconductors, especially, optimal and underdoped ones, cannot be considered in terms of well-defined quasiparticles [2]. Fermi liquid theory for the normal Fermi systems is based on the existence of well-defined quasiparticle excitations. Nevertheless, one can use this theory for high-T_c superconductors at low temperatures since sharp quasiparticle peaks do appear all over the Fermi surface in the superconducting state at $T<T_c$ [3]. The superfluid Fermi liquid theory is at least consistent with the data in the overdoped and optimally doped regimes, but dubious in the underdoped regime [4].

Larkin [5] and Leggett [6] in their classic papers could extend the Landau theory of Fermi liquids to s-wave superfluids. Leggett's microscopic theory of a superfluid Fermi liquid determined the static properties of this liquid in terms of normal phase Landau parameters. Gross et al. [7], Xu et al. [8], Walker [1], Paramekanti and Randeria [9] generalized the theory of Landau to superconductors with anisotropic Fermi surface, especially, to d-wave superconductors.

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Motivated by pump-probe experiments of Segre et al. [10-12] and Gedik et al. [13] that in the limit of strong intensity the quasiparticle relaxation rates depend on T^3 and $T^2\exp(\Delta(T)/T)$ at low temperatures, we are going to calculate the relaxation rates of quasiparticles in a d-wave superconductor in the presence of Fermi liquid interactions. Howell et al. [14] by presenting a theory on which hot quasiparticles scatter through the antinodal bottleneck before recombination and within a model where quasiparticles are scattered by a simple local interaction, could show that at high temperatures there is a fast relaxation due to umklapp scattering. They supposed that below some crossover temperature relaxation rate is dominated by diffusion in momentum space along the Fermi surface from the antinodes to the nodes.

In this paper, we use the phenomenological theory of Fermi liquid interaction of Paramekanti and Randeria [9] for a two-dimensional d-wave superconductor on a square lattice to calculate the transition probabilities of different processes in terms of the Landau parameters. Then by writing the Fermi's golden rule for the quasiparticle relaxation rate, $1/\tau$, in terms of the transition probabilities, we determine the temperature dependence of the quasiparticle relaxation rate in terms of the Landau parameters. Finally, we calculate thermal conductivity coefficients of YBCO by using Khalatnikov et al.'s work [15] and compare our results with the experimental data [16,17].

The paper is organized as follows. In the next section we calculate the transition probabilities for processes in which two quasiparticles emerge into two, one quasiparticle into three; and three quasiparticles coalescence into one in terms of symmetric and anti-symmetric amplitudes. Then we write the scattering amplitudes in terms of Landau parameters. We present calculations of hot and thermal quasiparticles relaxation rates at low temperatures in the section of results. We also calculate thermal conductivity coefficients of YBCO and compare the result with experiment. Finally, in the discussion section, we give some remarks on the determination of the values of Landau parameters in high-T_c superconductors. The analytical results for the transition probabilities in terms of T-matrix and Bogoliubov coefficients come in the Appendix.

Materials and Methods

1. Transition Probabilities

To obtain the transition probabilities, at low temperatures in which two-particle process dominate, we start with the interaction term in the Hamiltonian as

$$H = \frac{1}{4}\sum_{1,2,3,4} \langle 3,4 \,|\, \mathrm{T} \,|\, 1,2\rangle a_4^\dagger a_3^\dagger a_2 a_1, \tag{1}$$

where $i = 1,2,3,4$ stands for both momentum $(\vec{P}_i)$ and spin (σ_i) variables. As we have mentioned in the spirit of a Fermi liquid approach, we assume the existence of well-defined quasiparticles. By using Bogoliubov transformations, the particle creation $a^\dagger_{\vec{p}\sigma}$ and annihilation $a_{\vec{p}\sigma}$ operators may be replaced by the quasiparticle creation $b^\dagger_{\vec{p}\sigma}$ and annihilation $b_{\vec{p}\sigma}$ operators as [18]

$$\begin{aligned} b_{\vec{p}\sigma} &= u_{\vec{p},\sigma\sigma'} a_{\vec{p}\sigma'} - v_{\vec{p},\sigma\sigma'} a^\dagger_{-\vec{p}\sigma'}, \\ b^\dagger_{\vec{p}\sigma} &= v^*_{\vec{p},\sigma\sigma'} a_{\vec{p}\sigma'} + u^*_{\vec{p},\sigma\sigma'} a^\dagger_{-\vec{p}\sigma'}, \end{aligned} \tag{2}$$

where the matrix elements $u_{\vec{p},\sigma\sigma'}$ and $v_{\vec{p},\sigma\sigma'}$ can be chosen for d-wave superconductors as

$$u_{\vec{p},\sigma\sigma'} = \sqrt{\frac{1}{2} + \frac{\varepsilon_p}{2E_p}}\delta_{\sigma\sigma'}, \; ; \; v_{\vec{p},\sigma\sigma'} = \sqrt{\frac{1}{2} - \frac{\varepsilon_p}{2E_p}}\delta_{\sigma\sigma'}, \tag{3}$$

with $E_p = \sqrt{\varepsilon_p^2 + \Delta_p^2}$ and assume a d-wave gap parameter $\Delta_p = \Delta_0 \cos 2\theta_p$ over a Fermi surface. Here, θ_p is the angle between $\vec{p}$ and say the p_y -axis in the 2D momentum space and Δ_0 is the maximum of the gap. For the non-unitary state of d-wave superconductor, there are the following properties between u and v

$$u_{-\vec{p},\sigma\sigma'} = u_{\vec{p},\sigma\sigma'}, \; ; \; v_{-\vec{p},\sigma\sigma'} = v_{\vec{p},\sigma\sigma'}. \tag{4}$$

By using of Eqs. (2) in Eq. (1), we obtain H in terms of Bogoliubov quasiparticle creation and annihilation operators

$$\begin{aligned} H = \frac{1}{4}\sum_{1,2,3,4} \\ \{[\langle 3\uparrow 4\uparrow|\mathrm{T}|1\uparrow 2\uparrow\rangle (u_4 b^\dagger_{4\uparrow} - v_4 b_{-4\downarrow}) \\ (u_3 b^\dagger_{3\uparrow} - v_3 b_{-3\downarrow})(u_1 b_{1\uparrow} - v_1 b^\dagger_{-1\downarrow})(u_2 b_{2\uparrow} - v_2 b^\dagger_{-2\downarrow})] \\ +[\langle 3\downarrow 4\uparrow|\mathrm{T}|1\uparrow 2\downarrow\rangle (u_4 b^\dagger_{4\uparrow} - v_4 b_{-4\downarrow}) \\ (v_3 b_{-3\uparrow} + u_3 b^\dagger_{3\downarrow})(u_1 b_{1\uparrow} - v_1 b^\dagger_{-1\downarrow})(u_2 b_{2\downarrow} - v_1 b^\dagger_{-2\downarrow})] \\ +[\langle 3\uparrow 4\downarrow|\mathrm{T}|1\uparrow 2\downarrow\rangle (v_4 b_{-4\uparrow} + u_4 b^\dagger_{4\downarrow}) \\ (u_3 b^\dagger_{3\uparrow} - v_3 b_{-3\downarrow})(u_1 b_{1\uparrow} - v_1 b^\dagger_{-1\downarrow})(u_2 b_{2\downarrow} + v_2 b^\dagger_{-2\uparrow})] \\ +[\langle 3\downarrow 4\uparrow|\mathrm{T}|1\downarrow 2\uparrow\rangle (u_4 b^\dagger_{4\uparrow} - v_4 b_{-4\downarrow}) \end{aligned}$$

$$(v_3 b_{-3\uparrow} + u_3 b_{3\downarrow}^{\dagger})(u_1 b_{1\downarrow} + v_1 b_{-1\uparrow}^{\dagger})(u_2 b_{2\uparrow} - v_2 b_{-2\downarrow}^{\dagger})]$$
$$+[\langle 3\uparrow 4\downarrow |\mathrm{T}| 1\downarrow 2\uparrow\rangle (v_4 b_{-4\uparrow} + u_4 b_{4\downarrow}^{\dagger})$$
$$(u_3 b_{3\uparrow}^{\dagger} - v_3 b_{-3\downarrow})(u_1 b_{1\downarrow} + v_1 b_{-1\uparrow}^{\dagger})(u_2 b_{2\uparrow} - v_2 b_{-2\downarrow}^{\dagger})] \quad (5)$$
$$+[\langle 3\downarrow 4\downarrow |\mathrm{T}| 1\downarrow 2\downarrow\rangle (v_4 b_{-4\uparrow} + u_4 b_{4\downarrow}^{\dagger})$$
$$(v_3 b_{-3\uparrow} + u_3 b_{3\downarrow}^{\dagger})(u_1 b_{1\downarrow} + v_1 b_{-1\uparrow}^{\dagger})(u_2 b_{2\downarrow} + v_2 b_{-2\uparrow}^{\dagger})]\},$$

where i=1,2,3,4 stands for momentum $\vec{P}_i$. This Hamiltonian contains the terms $b_4^{\dagger}b_3^{\dagger}b_{-2}^{\dagger}b_1$, $b_4^{\dagger}b_3^{\dagger}b_2 b_1$, $b_4^{\dagger}b_3 b_2 b_1$, $b_4^{\dagger}b_3^{\dagger}b_{-2}^{\dagger}b_{-1}^{\dagger}$, and $b_{-4}b_{-3}b_2 b_1$. These terms decay one quasiparticle into three, convert two quasiparticles into two, coalescence three quasiparticles into one, create four quasiparticles from the condensate, and scatter four quasiparticles into the condensate, respectively. The last two processes are not allowed, because in each process the total energy should be conserved.

One may define the transition probability for two-particle process with parallel spin as

$$W_{22}(\uparrow\uparrow) = 2\pi \left| \langle 3'\uparrow 4'\uparrow |H| 1'\uparrow 2'\uparrow\rangle \right|^2, \tag{6}$$

where subscripts on W indicate binary processes in which two quasiparticles with spin up scatter to two quasiparticles with spin up. For the antiparallel spin case we have

$$W_{22}(\uparrow\downarrow) = 2\pi \left| \langle 3'\uparrow, 4'\downarrow |H| 1'\uparrow, 2'\downarrow\rangle \right|^2 + 2\pi \left| \langle 3'\downarrow, 4'\uparrow |H| 1'\uparrow, 2'\downarrow\rangle \right|^2. \tag{7}$$

The transition probability for coalesced processes, are

$$W_{31}(\uparrow\uparrow) = 2\pi \left| \langle 4'\uparrow |H| 1'\uparrow, 2'\uparrow, -3'\uparrow\rangle \right|^2, \tag{8}$$

$$W_{31}(\uparrow\downarrow) = 2\pi \left| \langle 4'\uparrow |H| 1'\uparrow, 2'\downarrow, -3'\uparrow\rangle \right|^2. \tag{9}$$

Similarly the transition probabilities for decay processes as

$$W_{13}(\uparrow\downarrow) = 2\pi \left| \langle 3'\uparrow, 4'\downarrow, -2'\uparrow |H| 1'\uparrow\rangle \right|^2, \tag{10}$$

$$W_{13}(\uparrow\uparrow) = 2\pi \left| \langle 3'\uparrow, 4'\uparrow, -2'\uparrow |H| 1'\uparrow\rangle \right|^2. \tag{11}$$

In the Appendix, we present the above transition probabilities in terms of the T-matrix elements. These elements may be expressed in terms of the scattering amplitudes for pairs of quasiparticles in singlet and triplet states, T_s and T_t, respectively. By considering properties of T_s and T_t, [18,19], we have

$$\begin{aligned}
&\langle -2\uparrow, 3\uparrow |\mathrm{T}| -4\uparrow, 1\uparrow\rangle \\
&\quad = \langle -2\downarrow, 3\downarrow |\mathrm{T}| -4\downarrow, 1\downarrow\rangle \equiv \mathrm{T}_{t\mathrm{I}}, \\
&\langle 4\uparrow, 3\uparrow |\mathrm{T}| 1\uparrow, 2\uparrow\rangle \equiv \mathrm{T}_{t\mathrm{II}}, \\
&\langle -1\uparrow, 3\uparrow |\mathrm{T}| -4\uparrow, 2\uparrow\rangle \equiv \mathrm{T}_{t\mathrm{III}}, \\
&\langle -2\uparrow, 3\downarrow |\mathrm{T}| -4\downarrow, 1\uparrow\rangle \\
&\quad = \langle -2\downarrow, 3\uparrow |\mathrm{T}| -4\uparrow, 1\downarrow\rangle \equiv \tfrac{1}{2}(\mathrm{T}_{t\mathrm{I}} - \mathrm{T}_{s\mathrm{I}}), \\
&\langle 4\uparrow, 3\downarrow |\mathrm{T}| 1\downarrow, 2\uparrow\rangle \\
&\quad = \langle 4\uparrow, 3\downarrow |\mathrm{T}| 1\downarrow, 2\uparrow\rangle \equiv \tfrac{1}{2}(\mathrm{T}_{t\mathrm{II}} - \mathrm{T}_{s\mathrm{II}}), \\
&\langle -1\uparrow, 3\downarrow |\mathrm{T}| -4\downarrow, 2\uparrow\rangle \\
&\quad = \langle -1\downarrow, 3\uparrow |\mathrm{T}| -4\uparrow, 2\downarrow\rangle \equiv \tfrac{1}{2}(\mathrm{T}_{t\mathrm{III}} - \mathrm{T}_{s\mathrm{III}}), \\
&\langle -2\uparrow, 3\downarrow |\mathrm{T}| -4\uparrow, 1\downarrow\rangle \\
&\quad = \langle -2\uparrow, 3\downarrow |\mathrm{T}| -4\downarrow, 1\uparrow\rangle \equiv \tfrac{1}{2}(\mathrm{T}_{t\mathrm{I}} + \mathrm{T}_{s\mathrm{I}}), \\
&\langle 4\uparrow, 3\downarrow |\mathrm{T}| 1\uparrow, 2\downarrow\rangle \\
&\quad = \langle 4\downarrow, 3\uparrow |\mathrm{T}| 1\uparrow, 2\downarrow\rangle \equiv \tfrac{1}{2}(\mathrm{T}_{t\mathrm{II}} + \mathrm{T}_{s\mathrm{II}}), \\
&\langle -3\uparrow, 1\downarrow |\mathrm{T}| -4\uparrow, 2\downarrow\rangle \\
&\quad = \langle -3\downarrow, 1\uparrow |\mathrm{T}| -4\uparrow, 2\downarrow\rangle \equiv \tfrac{1}{2}(\mathrm{T}_{t\mathrm{III}} + \mathrm{T}_{s\mathrm{III}}).
\end{aligned} \tag{12}$$

We can express the singlet and triplet scattering amplitudes in terms of symmetric amplitude T^s and anti-symmetric amplitude T^a,

$$\mathrm{T}_s = \mathrm{T}^s - 3\mathrm{T}^a, \;, \; \mathrm{T}_t = \mathrm{T}^s + \mathrm{T}^a. \tag{13}$$

The transition probabilities for quasiparticles which are located near the nodes and the antinodes can be simplified by using the following approximation for the Bogoliubov coefficients: near the nodes one may write $u_p^2 \approx 1$, $v_p^2 \approx 0$; and $u_p^2 \approx v_p^2 \cong 1/2$ for antinodales.

By substituting the values of u_p and v_p in the equations of transition probabilities [Eqs. (A1-A7)], we finally obtain the following results for transition probabilities in the case of thermal quasiparticles scattering as

$$
\begin{aligned}
&W_{22}(\uparrow\uparrow)=2\pi\left|\mathrm{T}_{tII}\right|^2,\\
&W_{22}(\uparrow\downarrow)=\frac{2\pi}{4}\left(\left|\mathrm{T}_{tII}-\mathrm{T}_{sII}\right|^2+\left|\mathrm{T}_{tII}+\mathrm{T}_{sII}\right|^2\right),
\end{aligned}
\tag{14}
$$

and the other transition probabilities have nearly zero value:

$$
W_{13}(\uparrow\uparrow)=W_{13}(\uparrow\downarrow)=W_{31}(\uparrow\uparrow)=W_{31}(\uparrow\downarrow)\simeq 0. \tag{15}
$$

At low temperatures only binary processes are dominated for scattering of thermal quasiparticles with each other. In the next section we consider this problem extensively.

By substituting the values of u_p and v_p for thermal and hot quasiparticles in the transition probabilities in appendix (A1-A7), we have

$$
\begin{aligned}
W_{22}(\uparrow\uparrow)=&\frac{2\pi}{4}\{\frac{1}{4}\left|\mathrm{T}_{tII}\right|^2+\frac{1}{16}\left|\mathrm{T}_{tI}+\mathrm{T}_{sI}\right|^2\\
&+\frac{1}{8}\left(\mathrm{T}_{tII}(\mathrm{T}_{tI}^*+\mathrm{T}_{sI}^*)+\mathrm{T}_{tII}^*\left(\mathrm{T}_{tI}+\mathrm{T}_{sI}\right)\right)\},\\
W_{22}(\uparrow\downarrow)=&\frac{2\pi}{4}\{\frac{1}{16}\left(\left|\mathrm{T}_{tII}-\mathrm{T}_{sII}\right|^2+\left|\mathrm{T}_{tII}+\mathrm{T}_{sII}\right|^2\right)\\
&+\frac{1}{4}\left(\left|\mathrm{T}_{tI}\right|^2+\frac{1}{4}\left|\mathrm{T}_{tI}-\mathrm{T}_{sI}\right|^2\right)\\
&+\frac{1}{4}(-\frac{1}{4}(\mathrm{T}_{tII}+\mathrm{T}_{sII})(\mathrm{T}_{tI}^*-\mathrm{T}_{sI}^*)\\
&-\frac{1}{4}(\mathrm{T}_{tI}-\mathrm{T}_{sI})(\mathrm{T}_{tII}^*+\mathrm{T}_{sII}^*)\\
&+\frac{1}{2}\mathrm{T}_{tI}(\mathrm{T}_{tII}^*-\mathrm{T}_{sII}^*)+\frac{1}{2}\mathrm{T}_{tI}^*(\mathrm{T}_{tII}-\mathrm{T}_{sII}))\},
\end{aligned}
\tag{16}
$$

and

$$
\begin{aligned}
&W_{13}(\uparrow\uparrow)=2\pi\left|\frac{1}{4}(\mathrm{T}_{tIII}-\mathrm{T}_{sIII})\right|^2,\\
&W_{31}(\uparrow\downarrow)=2\pi\left|\frac{1}{4}(\mathrm{T}_{tIII}+\mathrm{T}_{sIII})\right|^2,\\
&W_{31}(\uparrow\uparrow)=2\pi\left|-\frac{1}{2}\mathrm{T}_{tII}+\frac{1}{4}(\mathrm{T}_{tI}+\mathrm{T}_{sI})\right|^2,\\
&W_{13}(\uparrow\downarrow)=2\pi\left|-\frac{1}{4}(\mathrm{T}_{tI}-\mathrm{T}_{sI})-\frac{1}{4}(\mathrm{T}_{tII}+\mathrm{T}_{sII})\right|^2.
\end{aligned}
\tag{17}
$$

For the scattering relaxation rate of thermal quasiparticles with antinodal quasiparticles, we obtain

$$
\begin{aligned}
W_{22}(\uparrow\uparrow)=&2\pi\{\frac{1}{4}\left|\mathrm{T}_{tI}\right|^2+\frac{1}{16}\left|\mathrm{T}_{tI}+\mathrm{T}_{sI}\right|^2\\
&+\frac{1}{8}\left[\mathrm{T}_{tII}(\mathrm{T}_{tI}^*+\mathrm{T}_{sI}^*)+T_{tII}^*(\mathrm{T}_{tI}+\mathrm{T}_{sI})\right]\},\\
W_{22}(\uparrow\downarrow)=&2\pi\{\left\{\frac{1}{4}\left|\mathrm{T}_{tI}\right|^2+\frac{1}{4}\left|\mathrm{T}_{tI}-\mathrm{T}_{sI}\right|^2\right.\\
&+\frac{1}{16}\left[(\left|\mathrm{T}_{tII}-\mathrm{T}_{sII}\right|^2+\left|\mathrm{T}_{tII}+\mathrm{T}_{sII}\right|^2)\right.\\
&+\frac{1}{4}[-\frac{1}{4}(\mathrm{T}_{tII}+\mathrm{T}_{sII})(\mathrm{T}_{tI}^*-\mathrm{T}_{sI}^*)\\
&-\frac{1}{4}(\mathrm{T}_{tI}-\mathrm{T}_{sI})(\mathrm{T}_{tII}^*-\mathrm{T}_{sII}^*)\\
&+\frac{1}{2}\mathrm{T}_{tI}(\mathrm{T}_{tII}^*-\mathrm{T}_{sII}^*)+\frac{1}{2}\mathrm{T}_{tI}^*(\mathrm{T}_{tII}-\mathrm{T}_{sII})\left.\right]\},\\
W_{31}(\uparrow\uparrow)=&\frac{\pi}{8}\left|(\mathrm{T}_{tIII}-\mathrm{T}_{sIII})\right|^2,\\
W_{31}(\uparrow\downarrow)=&\frac{\pi}{2}\left|\mathrm{T}_{tIII}\right|^2,\\
W_{13}(\uparrow\uparrow)=&\frac{\pi}{2}\left|\mathrm{T}_{tI}+\frac{1}{2}(\mathrm{T}_{sII}-\mathrm{T}_{tII})\right|^2,\\
W_{13}(\uparrow\downarrow)=&\frac{\pi}{2}\left|\mathrm{T}_{tII}-\frac{1}{2}(\mathrm{T}_{tI}+\mathrm{T}_{sI})\right|^2.
\end{aligned}
\tag{18}
$$

As it is obvious, the decay and coalescence processes transition probabilities are not zero and we see in the next section that they change the temperature dependence of the relaxation rate of the thermal quasiparticles with antinodal quasiparticles

2. Scattering Amplitudes

The scattering amplitudes can be written in terms of Landau interaction function, $f^{s,a}_{\vec{p},\vec{p}'}$ as [20]

$$
\mathrm{T}^{s,a}_{\vec{p},\vec{p}'}=f^{s,a}_{\vec{p},\vec{p}'}(q)+2\alpha^{s,a}(q)\sum_{\vec{p}''}f^{s,a}_{\vec{p},\vec{p}''}(q)\chi_{\vec{p}''}\mathrm{T}^{s,a}_{\vec{p},\vec{p}''}, \tag{19}
$$

where

$$
\chi_{\vec{p}''}=\frac{n^0_{\vec{p}''+\vec{q}/2}-n^0_{\vec{p}''-\vec{q}/2}}{\varepsilon^0_{\vec{p}''+\vec{q}/2}-\varepsilon^0_{\vec{p}''-\vec{q}/2}}. \tag{20}
$$

The value of $\alpha^{s,a}(q)$ for $q\simeq 0$ equals to one and for $q>p_F$ nearly equals to zero [20]. Furthermore, we can express the Landau interaction function for a two dimensional d-wave superconductor as the following [9]

$$f(\theta,\theta')=\sum_{l\le m}F_{l,m}\left[\cos(l\theta+m\theta')+\cos(l\theta'+m\theta)\right], \quad (21)$$

where $l+m=4n$ and n can be positive or negative integer. There are many more Landau parameters $F_{l,m}$ on the lattice, labeled by two parameters l and m. Here for simplicity and for tackling the problem under our consideration, we keep only Landau parameter $F_{1,-1}$ and $F_{0,0}$, and set all other $F_{l,m}=0$, hence Eq. (21) gives

$$f(\theta,\theta')=2F_{0,0}+2F_{1,-1}\cos(\theta,\theta'). \quad (22)$$

We define θ_{I}, θ_{II}, θ_{III}, the angles between the momenta of the quasiparticles 2, 3 and 4 with the momentum of the quasiparticle 1 as following

$$\theta_{\mathrm{I}}=\theta_{12}, \quad ; \quad \theta_{\mathrm{II}}=\theta_{14}, \quad ; \quad \theta_{\mathrm{III}}=\theta_{13}, \quad (23)$$

There are three cases which are relevant to the existence of experimental data. Following, we first consider the scattering amplitudes for the relaxation rate of nodal (thermal) quasiparticles which are near the nodes of the gap parameter, then for the case of the scattering of antinodal quasiparticles with thermal quasiparticles and finally for the case of the scattering of thermal quasiparticles with antinodal quasiparticles:

2.1. First Case: Scattering of Thermal Quasiparticles with Each Other

Since the thermal quasiparticles are located near the nodes of the gap parameter at low temperatures, the angles between the momentum $\vec{p}_1$ and momenta $\vec{p}_2$, $\vec{p}_3$ and $\vec{p}_4$ are small. Therefore, if we take the angle between $\vec{p}_1$ and $\vec{p}_2$ equal to θ, then we have

$$\theta_{\mathrm{I}}=\theta_{\mathrm{II}}\simeq\theta, \text{ and } \theta_{\mathrm{III}}\simeq 0. \quad (24)$$

It can be shown on the basis of the gap parameter function, energy and relaxation time that the maximum value of θ is $\pi T/\Delta_0$ [16]. At low temperatures and near the nodes Eq. (19) can be written as

$$\mathrm{T}_{\mathrm{II}}=2F_{0,0}+2F_{1,1}+O(T), \quad (25)$$

where $O(T)$ stands for the contribution of the second term in Eq. (19) which is proportional to T and may be compared to the other terms, i.e. at low temperatures the relation between T_{II} and scattering amplitudes of Landau parameters is linear. Finally, by substituting Eqs. (24) and (25) in Eq. (14), we find

$$W_{22}(\uparrow\uparrow)=W_{22}(\uparrow\downarrow)=8\pi(f_{0,0}+f_{1,-1})^2. \quad (26)$$

2.2. Second Case: Scattering of Antinodal Quasiparticles with Thermal Quasiparticles

In this case, the gap parameter for the umklapp processes is nearly independent of temperature, i.e., $\Delta_U=$ constant. On the other hand, the gap parameter of thermal quasiparticles is temperature dependent and is proportional to T [19]. By substituting the values of u_p and v_p for the thermal or antinodal quasiparticles into Eqs. (16) and (17), after doing some algebra, we finally get

$$\begin{aligned}
&W_{22}(\uparrow\uparrow)=2\pi(f_{0,0}^2+0.134f_{1,-1}^2-1.46f_{0,0}f_{1,-1}),\\
&W_{22}(\uparrow\downarrow)=2\pi(2f_{0,0}^2+f_{1,-1}^2-(1-\sqrt{3})f_{0,0}f_{1,-1}),\\
&W_{13}(\uparrow\uparrow)=2\pi(2f_{0,0}^2+0.75f_{1,-1}^2+\sqrt{3}f_{0,0}f_{1,-1}),\\
&W_{13}(\uparrow\downarrow)=0,\\
&W_{31}(\uparrow\uparrow)=2\pi(2f_{0,0}^2+0.13f_{1,-1}^2+1.46f_{0,0}f_{1,-1}),\\
&W_{31}(\uparrow\downarrow)=2\pi(2f_{0,0}^2+0.75f_{1,-1}^2-\sqrt{3}f_{0,0}f_{1,-1}).
\end{aligned} \quad (27)$$

2.3. Third Case: Scattering of Nodal Quasiparticles with Antinodal Quasiparticles

Following the similar approach, Eq. (18) finally gives

$$\begin{aligned}
&W_{22}(\uparrow\uparrow)=2\pi(4f_{0,0}^2+0.134f_{1,-1}^2-1.46f_{0,0}f_{1,-1}),\\
&W_{22}(\uparrow\downarrow)=2\pi(2f_{0,0}^2+f_{1,-1}^2-0.732f_{0,0}f_{1,-1}),\\
&W_{13}(\uparrow\uparrow)=2\pi(0.134f_{1,-1}^2),\\
&W_{13}(\uparrow\downarrow)=2\pi(f_{0,0}^2+0.25f_{1,-1}^2+f_{0,0}f_{1,-1}),\\
&W_{31}(\uparrow\uparrow)=2\pi f_{0,0}^2,\\
&W_{31}(\uparrow\downarrow)=2\pi(2f_{0,0}^2+0.25f_{1,-1}^2+f_{0,0}f_{1,-1}).
\end{aligned} \quad (28)$$

We note that in this and second case, the order of magnitude of the transition probabilities for decay and coalescence processes are not the same as transition probabilities for two quasiparticle processes.

Results

1. Calculation of Relaxation

Using the Fermi's golden rule we can evaluate the relaxation rate, $1/\tau_p$, or quasiparticle lifetime τ_p in the limit of the strong intensity and for the general case under our consideration as

$$\frac{1}{\tau_{p_1}} = 2\pi \sum_{2,3,4} \Big\{ \Big[W_{22}(\uparrow\downarrow) + \frac{1}{2} W_{22}(\uparrow\uparrow) \Big] \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \delta(E_1 + E_2 - E_3 - E_4) \times \delta_{\sigma_1+\sigma_2, \sigma_3+\sigma_4} n_2 (1-n_3)(1-n_4)$$
$$+ \Big[W_{13}(\uparrow\downarrow) + \frac{1}{2} W_{13}(\uparrow\uparrow) \Big] \delta(\vec{p}_1 - \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \delta(E_1 - E_2 - E_3 - E_4) \times \delta_{\sigma_1+\sigma_2, \sigma_3+\sigma_4} (1-n_2)(1-n_3)(1-n_4) \quad (29)$$
$$+ \Big[W_{31}(\uparrow\downarrow) + \frac{1}{2} W_{31}(\uparrow\uparrow) \Big] \delta(\vec{p}_1 + \vec{p}_2 + \vec{p}_3 - \vec{p}_4) \delta(E_1 + E_2 + E_3 - E_4) \times \delta_{\sigma_1+\sigma_2, \sigma_3+\sigma_4} n_2 n_3 (1-n_4) \Big\}.$$

where n_i is Fermi-Dirac distribution function for *i*th quasiparticle. Throughout the paper we set $\hbar = k_B = c = 1$. It is obvious from Eqs. (26-28) that the Fermi liquid corrections renormalize the relaxation rates through the transition probabilities. The effects of backflow terms, i.e., the dependence of energies on quasiparticle occupations do not affect the results of the relaxation rates.

The relaxation rate of thermal quasiparticles which scatter with each other in a two dimensional d-wave superconductor for the first case of the section 2.1 is

$$\frac{1}{\tau_{p_1}} = 2\pi \int \frac{d^2p_2 d^2p_3 d^2p_4}{(2\pi)^6} \Big[W_{22}(\uparrow\downarrow) + \frac{1}{2} W_{22}(\uparrow\uparrow) \Big] \times \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \delta(E_1 + E_2 - E_3 - E_4) n_2 (1-n_3)(1-n_4), \quad (30)$$

where the transition probabilities are given in Eq. (26). After the integrations of $\delta(\vec{p}_{1x} + \vec{p}_{2x} - \vec{p}_{3x} - \vec{p}_{4x})$ and $\delta(\vec{p}_{1y} + \vec{p}_{2y} - \vec{p}_{3y} - \vec{p}_{4y})$ as well as using the relation $d\varepsilon_i = p_i dp_i / m^*$, we have

$$\frac{1}{\tau_{p_1}} = 2\pi \frac{m^*}{(2\pi)^4} \int d\varepsilon_2 d\theta_2 d\varepsilon_3 d\theta_3 \Big[W_{22}(\uparrow\downarrow) + \frac{1}{2} W_{22}(\uparrow\uparrow) \Big] \quad (31)$$
$$\times \delta(E_1 + E_2 - E_3 - E_4) n_2 (1-n_3)(1-n_4) \Big|_{\vec{p}_4 = \vec{p}_1 + \vec{p}_2 - \vec{p}_3}.$$

Now we can use the above equation for the calculation of the relaxation rate of thermal quasiparticles which scatter with each other. In this case, by using the approximation $u_p^2 \approx 1$, $v_p^2 \approx 0$ and transition probabilities from Eq. (26) and the integrations with respect to energies and angles we finally get

$$\frac{1}{\tau_{p_1}} = \frac{9T^3}{2\Delta_0^2} (F_{0,0} + F_{1,-1})^2, \quad (32)$$

where $F_{l,m} = f_{l,m} m^* / \pi$ are dimensionless Landau parameters. As it is obvious from Eq. (32) the relaxation rate of thermal quasiparticles which scatter with each other obtain in terms of Landau parameters $F_{0,0}$, $F_{1,-1}$, Δ_0 and T. The T^3 dependence of the relaxation rate has been obtained on the basis of scaling argument by Walker-Smith [21].

To calculate the relaxation rate in umklapp process, for scattering antinodal quasiparticle with thermal quasiparticles one may write $u_p^2 \simeq v_p^2 \simeq 1/2$ and $u_p^2 \simeq 1$, $v_p^2 \simeq 0$, respectively. Moreover, the scattering angles in the hot quasiparticles are restricted to the region Δ_U and are nearly temperature independent, but the gap parameter for thermal quasiparticles is proportional to T [19]. Putting Eq. (27) for transition probabilities in Eq. (29) and doing the integrations with respect to energy and angle variables, we have

$$\frac{1}{\tau_{p_1}} = \frac{2T^2 \Delta_U}{\Delta_0 E_F} (8F_{0,0}^2 + 6.42F_{1,-1}^2 + 2.6F_{0,0}F_{1,-1})^2 [1 - n(\Delta_U)], \quad (33)$$

where we put $\theta_{U_{\max}} \simeq \Delta_U / E_F$.

For the third case in the section 2.3, the relaxation rate for scattering of nodal quasiparticle with antinodal quasiparticles may be written as

$$\frac{1}{\tau_{p_1}} = \frac{4T^2 \Delta_U}{\Delta_0 E_F} (11F_{0,0}^2 + 2.83F_{1,-1}^2 + 2.34F_{0,0}F_{1,-1})^2 n(\Delta_U)[1 - n(\Delta_U)], \quad (34)$$

For temperatures $T < \Delta_U$, we may simplify above equation as

$$\frac{1}{\tau_{p_1}} = \frac{4T^2 \Delta_U}{\Delta_0 E_F} (11F_{0,0}^2 + 2.83F_{1,-1}^2 - 4.34F_{0,0}F_{1,-1})^2 e^{-\Delta_U / \mathrm{T}}. \quad (35)$$

This relaxation rate has been calculated by Walker-Smith [21]. They obtain the above temperature dependence with undetermined coefficient. In their calculations, only the two quasiparticle' processes have been taken into account, whereas in our approach decay, two quasiparticle' and coalescence scattering processes take part in this case.

2. Calculation of Thermal Conductivity Coefficients

2.1. First Case: Calculation of Thermal Conductivity in Collision of Thermal Quasiparticles by Thermal Quasiparticles

Thermal conductivity is defined by [15]:

$$K_{ij}=\frac{1}{T}\int d\tau E\,\frac{\partial E}{\partial p_i}n^0(1-n^0)\frac{\partial E}{\partial p_j}\tau_{p_1}. \tag{36}$$

where n^0 is the Fermi-Dirac distribution function and T is the temperature. By substituting of τ_{p_1} from Eq. (32) in above relation, we get

$$K_{ij}=\frac{1}{T}\int\frac{pdpd\theta}{(2\pi)^2}\frac{e^t}{(1+e^t)^2}\; Ev_F^2\hat{p}_i\hat{p}_j\varepsilon^2E^{-2}\frac{2\Delta_0^2}{9T^3}(F_{0,0}+F_{1,-1})^{-2}. \tag{37}$$

and by explaining the momentum in terms of quasiparticles effective mass it gets the below form

$$K_{ij}=\frac{1}{T}v_F^2\frac{2\Delta_0^2m^*}{9T^3}\frac{(F_{0,0}+F_{1,-1})^{-2}}{(2\pi)^2}\int d\theta\hat{p}_i\hat{p}_j\int\frac{d\varepsilon}{E}\frac{e^t}{(1+e^t)^2}\varepsilon^2. \tag{38}$$

In the anisotropic case for thermal quasiparticle, γ changes from 0 to $\gamma_{\max}$ instead of 0 to 2π. Also around the nodes we define θ as $\gamma+\pi/4$, so we have

$$K_{xx}=K_{yy}=K_{xy}=K, \tag{39}$$

where

$$K=\frac{m^*\Delta_0^2v_F^2(F_{0,0}+F_{1,-1})^{-2}}{36\pi^2T^4}T^2\int_0^{\gamma_m}d\gamma\int_\delta^{t_m}dt\frac{e^t}{(1+e^t)^2}, \tag{40}$$

The solution of integral on *t* gives

$$K=\frac{m^*\Delta_0^2v_F^2(F_{0,0}+F_{1,-1})^{-2}}{36\pi^2T^2}\left(\int_0^{\gamma_m}\frac{d\gamma}{1+e^\delta}-\int_0^{\gamma_m}d\gamma\frac{1}{1+e^{t_m}}\right). \tag{41}$$

By defining $X=2\Delta_0\gamma/T$, we calculate K for two limits of $X\gg 1$ and $X\ll 1$. First for the $X\gg 1$ we obtain

$$K=\frac{m^*\Delta_0^2v_F^2(F_{0,0}+F_{1,-1})^{-2}}{36\pi^2T^2}\left(-\gamma_me^{-\frac{E_m}{T}}+\frac{T}{2\Delta_0}-\frac{T}{2\Delta_0}e^{-\frac{2\Delta_0\gamma_m}{T}}\right), \tag{42}$$

which means $K\propto 1/\mathrm{T}$. Next, for the $X\ll 1$, we obtain

$$K=\frac{m^*\Delta_0^2v_F^2(F_{0,0}+F_{1,-1})^{-2}}{36\pi^2T^2}\left(-\gamma_me^{-\frac{E_m}{T}}+\gamma_m\right), \tag{43}$$

which means $K\propto T^{-2}$.

2.2 Second Case: Calculation of Thermal Conductivity in Collision of Antinodal Quasiparticles by Antinodal Quasiparticles

By using Fermi's golden rule, we write relaxation rates for calculating thermal conductivity such as

$$\frac{1}{\tau}=2\pi\int\frac{d^2p_2d^2p_3d^2p_4}{(2\pi)^6}\left[W_{22}(\uparrow\uparrow)+\frac{1}{2}W_{22}(\uparrow\uparrow)\right]\times\delta(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4)\,\delta(E_1+E_2-E_3-E_4)n_2(1-n_3)(1-n_4). \tag{44}$$

The integration of the Delta function and using the $d\varepsilon_i=p_idp_i/m^*$, gives

$$\frac{1}{\tau}=2\pi\int d\varepsilon_2d\theta_2d\varepsilon_3d\theta_3\left[W_{22}(\uparrow\uparrow)+\frac{1}{2}W_{22}(\uparrow\uparrow)\right]\times\delta(E_1+E_2-E_3-E_4)\,n_2(1-n_3)(1-n_4)\big|_{\vec{p}_4=\vec{p}_1+\vec{p}_2-\vec{p}_3}. \tag{45}$$

Furthermore, by calculation this integral on angle elements and energies, we see that the relaxation rate is proportional to $\exp(-\Delta_U/T)$, where Δ_U is fraction of maximum superconductor gap around the antinodals. By using Eq. (36) and integration, we find K as

$$K\propto\frac{1}{T}\int_0^{E'}dE\,\varepsilon e^{-\Delta_U/T}e^{\Delta_U/T}=\frac{1}{T}\int_0^{E'}dE\sqrt{E^2-\Delta^2}\simeq\frac{1}{T}\int_0^{E'}dEE=\frac{1}{2T}E'=\frac{T}{2}. \tag{46}$$

From this equation we see that K is proportional to T, which is consistence with experimental results [16,17].

Discussion

This paper has developed a phenomenological approach to evaluation of the Landau parameters in d-wave superconductors with anisotropic Fermi surfaces and energy gaps. As we have emphasized, Fermi liquid theory cannot provide to give an adequate description of the normal state properties of the high-T_c superconductors. Thus, we restricted our calculations well below the critical temperature T_c that will be considered as being possibly explicable in terms of Fermi liquid theory [22]. The symmetry of the square lattice restricts the form of expansion of the interaction function to more Landau parameters. As the case for Fermi liquid in normal and superfluid ^{3}He [23,24], we keep only two Landau parameters and set all the other equal to zero in calculation of the relaxation rates.

The very strong interactions in the normal state of high-T_c superconductors suggest that electron-electron rather than electron-phonon scattering is the dominated relaxation mechanism [22]. In the pump-probe laser pulse experiments the exited quasiparticles decay to equilibrium over a series of time which have been considered extensively by Howell et al. [14]. In this paper, we concentrated on the relaxation rates that there are experimental data on them. In the experiment of Segre et al. [10] the extrapolated relaxation rate of quasiparticles of energy Δ_0 vanishes as $T\to 0$ and for low intensity of laser pulse behaves as T^3 law. While the later results [13], suggest activated behavior for the lowest measured temperature (down to 10 K). In the case of umklapp scattering the quasiparticles, the relaxation rate has been considered experimentally by Hosseini et al. [25] and theoretically by Walker-Smith [21] and Howell et al. [14]. The temperature dependence of this relaxation rate is proportional to $f(T)\exp(-\Delta_U/T)$, where at $T\to 0, f(T)\to T^2$.

In this paper, we calculated the normal and umklapp scattering processes with a powerfull approach in which we could take into account the Landau parameters in the relaxation rates. In the spirit of a Fermi liquid approach, we assume the existence of well-defined quasiparticles and write the transition probabilities in terms of Bogiliubov coefficients (u_i and v_i) and T-matrix elements, then by using appropriate approximations on u_i, v_i and angles between the momentums in the scattering processes, we could obtain simple forms for the transition probabilities. As we have mentioned already two dimensionless non-zero Landau parameters $F_{0,0}$ and $F_{1,-1}$ contribute in these transition probabilities. Finally, we calculate the relaxation rates by using the Fermi's golden rule for scattering processes like thermal quasiparticles with each other, hot quasiparticles with thermal quasiparticles and vice versa. We are now in a position to make some remarks.

1. Umklapp processes involving collisions with a quasiparticle with its momentum and energy fairly close to those of quasiparticles which their energy gaps occur in the neighborhood of Δ_U. In the obtaining of Eqs. (33-35), we suppose that $\theta_{U_{\max}} \simeq \Delta_U / E_F$. At $T \approx 8\,\mathrm{K}$ by assuming $\Delta_0 \approx 200\,\mathrm{K}$ [26], $\Delta_U = 2\Delta_0/3$ [21] and $E_F \simeq 8\Delta_0$, we have $\theta_{U_{\max}} \simeq 4.8^\circ$. The relaxation rates in Eqs. (33-35) probably contain this undetermined parameter. More experimental results on the relaxation rates or other thermodynamics quantities are needed to cancel this uncertainty. To estimate the value of $F_{0,0}$ approximately one may put $F_{1,-1}=0$ in Eq. (35), then Hosseini et al. data for $YBa_2Cu_3O_{6.99}$ $x = 0.99$ give the value for the Landau parameter $F_{0,0} = \pm 0.68$.

2. By comparing the coefficients of the relaxation rates in Eqs. (32) and (35) with the experimental results of Segre et al. [10] and Gedik et al. [13] respectively and assuming $\Delta_0 \sim 35\,\mathrm{K}$ and $\Delta_U = 29\,\mathrm{K}$ at $T = 10\,\mathrm{K}$ we get (i): $F_{0,0} = \pm 0.23$ and $F_{1,-1} = \pm 0.1$ or $F_{0,0} = \pm 0.05$ and (ii): $F_{1,-1} = \mp 0.37$ for $x = 0.5$.

3. Paramekanti and Randeria [9] have kept only a single Landau parameter $F_{1,-1} \neq 0$ in their calculation of the in-plane superfluid stiffness through the calculation of penetration depth of a d-wave superconductor. Furthermore, they assumed a doping dependence $F_{1,-1} = B + Cx$. With the help of Uemura et al. [27] data we get $F_{1,-1}(x=0.2) \simeq -0.7$ to -0.5. Here, we should mention that this Landau parameter in the work of Paramekanti and Randeria is the antisymmetric amplitudes, whereas our $F_{1,-1}$ is the symmetric amplitudes and presumably are different with each other.

4. Walker and Smith [21] within a constant could fit their theoretical results on umklapp processes with the data of Hosseini et al. [25]. Both theoretical results of Walker-Smith and us on the umklapp relaxation rate show that the relaxation rate is $cT^2\exp(-\Delta_U/T)$. We obtain the coefficient c in terms of Landau parameters $F_{0,0}$ and $F_{1,-1}$ [see Eqs. (34-35)]. $1/\tau \to 0$ as $T \to 0$. This is the case for weak intensity of laser pulses, more experimental results are needed in this area to extract the correct values of $F_{0,0}$ and $F_{1,-1}$.

5. The Landau parameters in the normal or superfluid ^{3}He are pressure dependence whereas here depend on the oxygen doping of the cuprate

superconductors [28], probably on the intensity of the laser pulses and impurities.

In conclusion, this paper gives a detailed description of the Landau parameter contribution to the relaxation rates of quasiparticles scattering in high-T_c superconductors, at low temperatures. There are many more Landau parameters $F_{l,m}$ on the lattice, that we suppose only $F_{0,0}$ and $F_{1,-1}$ have the non-zero values in the calculation of the relaxation rates of the thermal and hot quasiparticle' scattering. In remark 1 we mentioned more experimental results on the relaxation rates or other thermodynamic quantities of high-T_c superconductor are needed to determine exactly the values of $F_{0,0}$ and $F_{1,-1}$. We have shown that for vanishing laser intensity there are three relaxations mechanism for quasiparticles at low temperatures: thermal quasiparticles scatter with each other; Eq. (32), with a T^3 law, the other is for the scattering of antinodal quasiparticle with thermal quasiparticles, Eq. (33) and finally scattering of nodal quasiparticle with antinodal quasiparticles, Eq. (35). The lifetime of antinodal quasiparticle with thermal quasiparticles is proportional to T^2 at low temperatures whereas as the lifetime of thermal quasiparticles with each other is proportional to T^3. Hence at low temperatures we can ignore the scattering of antinodal quasiparticles with nodal ones. Furthermore, thermal conductivity in collision of thermal quasiparticles with each other do not correspond to experimental results, but thermal conductivity in collision of antinodal quasiparticles with each other is in good agreement with experimental results. This shows that in d-wave superconductors in addition of existence electronic and phononic mechanisms, another mechanism influences on the mechanism of thermal conductivity.

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Appendix

Transition Probabilities in Terms of **T**-*matrix*

In this appendix we present the transition probabilities in terms of matrix elements of T and Bogoliubov coefficients. Using the Hamiltonian in Eq. (5) with the transition probability for two-particles with parallel spin process, Eq. (6), and writing the Dirac bracket in terms of vacuum state with help of Wick's theorem and carrying out a lengthy algebra we find

$$
\begin{aligned}
W_{22}(\uparrow\uparrow) = 2\pi\Big\{ & \Big(|u_2|^2|u_3|^2|u_1|^2|u_4|^2+|v_1|^2|v_2|^2|v_3|^2|v_4|^2+2u_1v_1u_2v_2u_3v_3u_4v_4\Big)\Big|\langle 4\uparrow 3\uparrow|\mathrm{T}|1\uparrow 2\uparrow\rangle\Big|^2 \\
& -\Big(|u_3|^2|u_2|^2u_1v_1u_4v_4+|u_1|^2|u_4|^2u_2v_2u_2u_3v_3+|v_1|^2|v_4|^2u_2v_2u_3v_3+|v_2|^2|v_3|^2u_1u_1u_4v_4\Big) \\
& \times\Big(\langle 4\uparrow 3\uparrow|\mathrm{T}|1\uparrow 2\uparrow\rangle^*\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\uparrow 2\downarrow\rangle+\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\uparrow 2\downarrow\rangle^*\langle 4\uparrow 3\uparrow|\mathrm{T}|1\uparrow 2\uparrow\rangle\Big) \\
& +\Big(|u_3|^2|u_1|^2u_2v_2u_4v_4+|u_4|^2|u_2|^2u_1v_1u_3v_3+|v_2|^2|v_4|^2u_1v_1u_3v_3+|v_1|^2|v_3|^2u_2v_2u_4v_4\Big) \\
& \times\Big(\langle 4\uparrow 3\uparrow|\mathrm{T}|1\uparrow 2\uparrow\rangle^*\langle 2\downarrow 3\uparrow|\mathrm{T}|1\uparrow -4\downarrow\rangle+\langle 3\uparrow -2\downarrow|\mathrm{T}|-4\downarrow 1\uparrow\rangle^*\langle 4\uparrow 3\uparrow|\mathrm{T}|1\uparrow 2\uparrow\rangle\Big) \qquad \text{(A1)} \\
& +\Big(|u_3|^2|v_1|^2|u_2|^2|v_4|^2+|u_4|^2|v_2|^2+|u_1|^2|v_3|^2+2u_2v_3u_1u_2v_2u_4v_4\Big)\Big|\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\uparrow 2\downarrow\rangle\Big|^2 \\
& -\Big(|u_3|^2|v_4|^2u_1v_1u_2v_2+|u_2|^2|v_1|^2u_3v_3u_4v_4+|u_1|^2|v_2|^2u_3v_3u_4v_4+|u_4|^2|v_3|^2u_1v_1u_2v_2\Big) \\
& \times\Big(\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\uparrow 2\downarrow\rangle\langle -2\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 1\uparrow\rangle^*+\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\uparrow 2\downarrow\rangle^*\langle -2\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 1\uparrow\rangle\Big) \\
& +\Big(|u_3|^2|v_2|^2|u_1|^2|v_4|^2+|u_4|^2|v_1|^2|u_2|^2|v_3|^2+2u_4u_3v_4v_3u_1v_1u_2v_2\Big)\Big|\langle -2\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 1\uparrow\rangle\Big|^2\Big\},
\end{aligned}
$$

Similarly the transition probability with antiparallel spin process [Eq. (7)] is $W_{22}(\uparrow\downarrow)=2\pi|a|^2+2\pi|b|^2$ where

$$
\begin{aligned}
a = \frac{1}{4}\Big\{ & \langle -2'\uparrow 3'\uparrow|\mathrm{T}|-4'\uparrow 1'\uparrow\rangle u_{3'}v_{-2'}v_{-4'}u_{1'}-\langle -2'\uparrow 3'\uparrow|\mathrm{T}|1'\uparrow -4'\uparrow\rangle u_{3'}v_{-2'}u_{1'}v_{-4'} \\
& -\langle 3'\uparrow -2'\uparrow|\mathrm{T}|-4'\uparrow 1'\uparrow\rangle v_{-2'}u_{3'}v_{-4'}u_{1'}+v_{-2'}u_{3'}u_{1'}v_{-4'}\langle 3'\uparrow -2'\uparrow|\mathrm{T}|1'\uparrow -4'\uparrow\rangle \\
& +\langle 4'\downarrow 3'\uparrow|\mathrm{T}|1'\uparrow 2'\downarrow\rangle u_{3'}u_{4'}u_{1'}u_{2'}+\langle -1'\downarrow 3'\uparrow|\mathrm{T}|-4'\uparrow 2'\downarrow\rangle u_{3'}v_{-1'}v_{-4'}u_{-2'} \\
& +\langle 4'\downarrow -2'\uparrow|\mathrm{T}|1'\downarrow -3'\uparrow\rangle v_{-2'}u_{4'}u_{1'}v_{-3'}+\langle -1'\downarrow -2'\uparrow|\mathrm{T}|-4'\downarrow -3'\uparrow\rangle v_{-2'}v_{-1'}v_{-4'}v_{-3'}
\end{aligned}
$$

$$
\begin{aligned}
&-\langle 3'\uparrow 4'\downarrow|\mathrm{T}|1'\uparrow 2'\downarrow\rangle u_4 u_3 u_1 u_{2'} -\langle -2'\uparrow 4'\downarrow|\mathrm{T}|1'\uparrow -3'\downarrow\rangle u_4 v_{-2} u_1 v_{-3'} \\
&-\langle 3'\uparrow -1'\downarrow|\mathrm{T}|-4'\uparrow 2'\downarrow\rangle v_{-1} u_3 v_{-4} u_{2'} -\langle -2'\uparrow -1'\downarrow|\mathrm{T}|-4'\uparrow -3'\downarrow\rangle v_{-1} v_{-2} v_{-4} v_{-3'} \\
&+\langle -1'\downarrow 4'\downarrow|\mathrm{T}|-3'\downarrow 2'\downarrow\rangle u_4 v_{-1} v_{-3} u_{2'} -\langle -1'\downarrow -4'\downarrow|\mathrm{T}|2'\downarrow -3'\downarrow\rangle u_4 v_{-1} u_2 v_{-3'} \\
&-\langle 4'\downarrow -1'\downarrow|\mathrm{T}|-3'\downarrow 2'\downarrow\rangle v_{-1} u_4 v_{-3} u_{2'} +\langle 4'\downarrow -1'\downarrow|\mathrm{T}|2'\downarrow -3'\downarrow\rangle v_{-1} u_4 u_2 v_{-3'} \\
&+\langle 3'\uparrow 4'\downarrow|\mathrm{T}|2'\downarrow 1'\downarrow\rangle u_4 u_3 u_2 u_{1'} +\langle -2'\uparrow -4'\downarrow|\mathrm{T}|-3'\downarrow 1'\uparrow\rangle u_4 v_{-2} v_{-3} u_{1'} \qquad \text{(A2)}\\
&+\langle 3'\uparrow -1'\downarrow|\mathrm{T}|2'\downarrow -4'\uparrow\rangle v_{-1} u_3 u_2 v_{-4'} +\langle -2'\downarrow -1'\downarrow|\mathrm{T}|-3'\downarrow -4'\uparrow\rangle v_1 v_{-2} v_{-3} v_{-4'} \\
&-\langle 4'\downarrow 3'\uparrow|\mathrm{T}|2'\downarrow 1'\uparrow\rangle u_3 u_4 u_2 u_{1'} -\langle -1'\downarrow +3'\uparrow|\mathrm{T}|2'\downarrow -4'\uparrow\rangle u_3 v_{-1} u_2 v_{-4'} \\
&-\langle 4'\downarrow -2'\uparrow|\mathrm{T}|-3'\downarrow 1'\uparrow\rangle v_{-2} u_4 v_{-3} u_{1'} -\langle -1'\downarrow -2'\uparrow|\mathrm{T}|-3'\downarrow -4'\uparrow\rangle v_{-2} v_{-1} v_{-3} v_{-4'}.
\end{aligned}
$$

and

$$
\begin{aligned}
b &= \langle 3'\downarrow 4'\uparrow|H|1'\uparrow 2'\downarrow\rangle = \frac{1}{4}\{\langle -2'\uparrow 4'\uparrow|\mathrm{T}|-3'\uparrow 1'\uparrow\rangle u_4 v_{-2'} v_{-3} u_{1'} \\
&+\langle -2'\uparrow 4'\uparrow|\mathrm{T}|1'\uparrow -3'\uparrow\rangle u_4 v_{-2} u_1 v_{-3'} +\langle 4'\uparrow -2'\uparrow|\mathrm{T}|-3'\uparrow 1'\uparrow\rangle v_{-2} u_{4'} v_{-3} u_{1'} \\
&-\langle 4'\uparrow -2'\uparrow|\mathrm{T}|1'\uparrow -3'\uparrow\rangle v_{-2} u_{4'} u_1 u_{-3'} -\langle 3'\downarrow 4'\uparrow|\mathrm{T}|1'\uparrow 2'\downarrow\rangle u_4 u_3 u_2 u_{1'} \\
&-\langle -1'\downarrow 4'\uparrow|\mathrm{T}|-3'\uparrow 2'\downarrow\rangle u_4 v_{-1'} v_{-3} u_{2'} -\langle 3'\downarrow -2'\uparrow|\mathrm{T}|1'\uparrow -4'\downarrow\rangle v_{-2} u_3 u_1 v_{-4'} \\
&-\langle -1'\downarrow 2'\uparrow|\mathrm{T}|-3'\uparrow -4'\downarrow\rangle v_{-2} v_{-1'} v_{-3} v_{-4'} +\langle 4'\downarrow 3'\downarrow|\mathrm{T}|1'\uparrow 2'\downarrow\rangle u_3 u_4 u_1 u_{2'} \\
&+\langle -2'\uparrow 3'\downarrow|\mathrm{T}|1'\uparrow -4'\downarrow\rangle u_3 v_{-2} u_1 v_{-4'} +\langle 4'\uparrow -1'\downarrow|\mathrm{T}|-3'\uparrow 2'\downarrow\rangle v_{-1} u_4 v_{-3} u_{2'} \\
&+\langle -2'\uparrow -1'\downarrow|\mathrm{T}|-3'\uparrow -4'\downarrow\rangle v_{-1} v_{-2} v_{-3} v_{-4'} -\langle -1'\downarrow 3'\downarrow|\mathrm{T}|-4'\downarrow 2'\downarrow\rangle u_3 v_{-1} v_{-4} u_{2'} \\
&+\langle -1'\downarrow 3'\downarrow|\mathrm{T}|2'\downarrow -4'\downarrow\rangle u_3 v_{-1} u_2 v_{-4'} +\langle 3'\downarrow -1'\downarrow|\mathrm{T}|-4'\downarrow 2'\downarrow\rangle v_{-1} u_3 u_2 v_{-4'} \\
&-\langle 3'\downarrow -1'\downarrow|\mathrm{T}|2'\downarrow -4'\downarrow\rangle v_{-1} u_3 u_2 v_{-4'} -\langle 4'\uparrow 3'\downarrow\ |\mathrm{T}|2'\downarrow 1'\uparrow\rangle u_3 u_4 u_2 u_{1'} \\
&-\langle -2'\uparrow 3'\downarrow|T|-4'\downarrow 1'\uparrow\rangle u_3 v_{-2} v_{-4} u_{1'} -\langle 4'\uparrow -1'\downarrow|T|2'\downarrow -3'\uparrow\rangle v_{-1} u_4 u_2 v_{-3'} \\
&-\langle -2'\uparrow -1'\downarrow|T|-4'\downarrow -3'\uparrow\rangle v_{-1'} v_{-2} v_{-4} v_{-3'} +\langle 3'\downarrow 4'\uparrow|T|2'\downarrow 1'\uparrow\rangle u_4 u_3 u_2 u_{1'} \\
&+\langle -1'\downarrow 4'\uparrow|T|-2'\downarrow -3'\uparrow\rangle u_4 v_{-1} u_2 v_{-3'} +\langle 3'\downarrow -2'\uparrow|T|-4'\downarrow 1'\uparrow\rangle v_{-2} u_3 v_{-4} u_{1'} \qquad \text{(A3)}\\
&+\langle -1'\downarrow -2'\uparrow|T|-4'\downarrow -3'\uparrow\rangle v_{-2} v_{-1'} v_{-4} v_{-3'}\}.
\end{aligned}
$$

The transition probabilities for coalesced processes defined in Eqs. (8) and (9), respectively, become as

$$
\begin{aligned}
W_{31}(\uparrow\uparrow) &= 2\pi\Big\{\Big(|u_4|^2|v_3|^2|u_1|^2|u_2|^2+|v_1|^2|v_2|^2|v_4|^2|u_3|^2-2u_1v_1u_2v_2u_3v_3\Big)\big|\langle 4\uparrow 3\uparrow|\mathrm{T}|1\uparrow 2\uparrow\rangle\big|^2 \\
&+\Big(-|u_4|^2|u_2|^2u_1v_1u_3v_3+|u_1|^2|v_3|^2u_2v_2u_4v_4+|u_3|^2|v_1|^2u_2v_2u_4v_4-|v_4|^2|v_2|^2u_1v_1u_3v_3\Big)
\end{aligned}
$$

$$\times\left(\langle 4\uparrow 3\uparrow|\mathrm{T}|1\uparrow 2\uparrow\rangle^{*}\langle -2\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 1\uparrow\rangle+\langle -2\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 1\uparrow\rangle^{*}\langle 4\uparrow 3\uparrow|\mathrm{T}|1\uparrow 2\uparrow\rangle\right)$$
$$+\left(|v_3|^2|u_2|^2u_1v_1u_4v_4+|u_4|^2|u_1|^2u_2v_2u_3v_3+|v_4|^2|v_1|^2u_3v_3u_2v_2+|v_2|^2|u_3|^2u_1v_1u_4v_4\right)$$
$$\times\left(\langle 4\uparrow 3\uparrow|\mathrm{T}|1\uparrow 2\uparrow\rangle^{*}\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 2\uparrow\rangle+\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 2\uparrow\rangle^{*}\langle 4\uparrow 3\uparrow|\mathrm{T}|1\uparrow 2\uparrow\rangle\right)$$
$$+\left(|u_4|^2|v_2|^2|u_1|^2|u_3|^2+|v_3|^2|v_1|^2|v_4|^2|u_2|^2-2u_1v_1u_2v_2u_3v_3u_4v_4\right)\left|\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 2\uparrow\rangle\right|^2 \tag{A4}$$
$$+\left(|v_1|^2|u_2|^2u_3v_3u_4v_4-|u_3|^2|u_4|^2u_1v_1u_2v_2-|v_3|^2|v_4|^2u_1v_1u_2v_2+|u_1|^2|v_2|^2u_3v_3u_4v_4\right)$$
$$\times\left(\langle -2\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 1\uparrow\rangle^{*}\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 2\uparrow\rangle+\langle -2\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 1\uparrow\rangle^{*}\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 2\uparrow\rangle\right)$$
$$+\left(|v_2|^2|v_3|^2|v_4|^2|u_1|^2+|u_3|^2|v_1|^2|u_4|^2|u_2|^2+2u_1v_1u_2v_2u_3v_3u_4v_4\right)\left|\langle -2\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 1\uparrow\rangle\right|^2\Big\},$$

and

$$W_{31}(\uparrow\downarrow)=2\pi\Big\{\left(|v_2|^2|u_4|^2|u_1|^2|u_3|^2+|v_3|^2|v_1|^2|v_4|^2|u_2|^2+2u_1v_1u_2v_2u_3v_3u_4v_4\right)\left|\langle -2\uparrow 4\uparrow|\mathrm{T}|-3\uparrow 1\uparrow\rangle\right|^2$$
$$+\left(|u_4|^2|u_3|^2u_1v_1u_2v_2-|v_2|^2|u_1|^2u_3v_3u_4v_4-|v_1|^2|u_2|^2u_3v_3u_4v_4+|v_3|^2|v_4|^2u_2v_2u_2v_2\right)$$
$$\times\left(\langle 4\uparrow -1\downarrow|\mathrm{T}|2\downarrow -3\uparrow\rangle\langle -2\uparrow 4\uparrow|\mathrm{T}|-3\uparrow 1\uparrow\rangle^{*}+\langle 4\uparrow -1\downarrow|\mathrm{T}|2\downarrow -3\uparrow\rangle^{*}\langle -2\uparrow 4\uparrow|\mathrm{T}|-3\uparrow 1\uparrow\rangle\right)$$
$$+\left(-|u_3|^2|v_2|^2u_1v_1u_4v_4+|u_4|^2|u_1|^2u_2v_2u_3v_3+|v_1|^2|v_4|^2u_2v_2u_3v_3-|v_3|^2|u_2|^2u_1v_1u_4v_4\right)$$
$$\times\left(\langle -2\uparrow 4\uparrow|\mathrm{T}|-3\uparrow 1\uparrow\rangle^{*}\langle -2\uparrow -1\downarrow|\mathrm{T}|-4\downarrow -3\uparrow\rangle+\langle -2\uparrow -1\downarrow|\mathrm{T}|-4\downarrow -3\uparrow\rangle^{*}\langle -2\uparrow 4\uparrow|\mathrm{T}|-3\uparrow 1\uparrow\rangle\right) \tag{A5}$$
$$+\left(|v_1|^2|u_4|^2|u_2|^2|u_3|^2+|v_2|^2|v_3|^2|v_4|^2|u_1|^2-2u_1v_1u_2v_2u_3v_3u_4v_4\right)\left|\langle 4\uparrow -1\downarrow|\mathrm{T}|2\downarrow -3\uparrow\rangle\right|^2$$
$$+\left(-|v_1|^2|u_3|^2u_2v_2u_4v_4+|u_4|^2|u_2|^2u_1v_1u_3v_3+|v_2|^2|v_4|^2u_1v_1u_3v_3-|v_3|^2|u_1|^2u_2v_2u_4v_4\right)$$
$$\times\left(\langle 4\uparrow -1\downarrow|\mathrm{T}|2\downarrow -3\uparrow\rangle^{*}\langle -2\uparrow -1\downarrow|\mathrm{T}|-3\uparrow -4\downarrow\rangle+\langle -2\uparrow -1\downarrow|\mathrm{T}|-3\uparrow -4\downarrow\rangle^{*}\langle 4\uparrow -1\downarrow|\mathrm{T}|2\downarrow -3\uparrow\rangle\right),$$

Finally, we get the transition probabilities for decay processes [Eqs. (10) and (11)] as

$$W_{13}(\uparrow\downarrow)=2\pi\Big\{\left(|u_3|^2|u_2|^2|u_1|^2|v_4|^2+|u_4|^2|v_1|^2|v_2|^2|v_3|^2-2u_1v_1u_2v_2u_3v_3u_4v_4\right)\left|\langle -2\uparrow 3\uparrow|\mathrm{T}|-4\uparrow 1\uparrow\rangle\right|^2$$
$$+\left(-|u_3|^2|u_1|^2u_4v_4u_2v_2+|u_2|^2|v_4|^2u_1v_1u_3v_3+|u_4|^2|v_2|^2u_1v_1u_4v_4-|v_3|^2|v_1|^2u_2v_2u_4v_4\right)$$
$$\times\left(\langle -2\uparrow 3\uparrow|\mathrm{T}|-4\uparrow 1\uparrow\rangle\langle 4\downarrow 3\uparrow|\mathrm{T}|1\uparrow 2\downarrow\rangle^{*}+\langle -2\uparrow 3\uparrow|\mathrm{T}|-4\uparrow 1\uparrow\rangle^{*}\langle 4\downarrow 3\uparrow|\mathrm{T}|1\uparrow 2\downarrow\rangle\right)$$
$$+\left(-|u_3|^2|v_4|^2u_1v_1u_2v_2+|u_1|^2|u_2|^2u_3v_3u_4v_4+|v_1|^2|v_2|^2u_3v_3u_4v_4-|v_3|^2|u_4|^2u_2v_2u_1v_1\right)$$
$$\times\left(\langle -2\uparrow 3\uparrow|\mathrm{T}|-4\uparrow 1\uparrow\rangle\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\uparrow 2\downarrow\rangle^{*}+\langle -2\uparrow 3\uparrow|\mathrm{T}|-4\uparrow 1\uparrow\rangle^{*}\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\uparrow 2\downarrow\rangle\right)$$

$$
\begin{aligned}
&+\left(|u_3|^2|u_4|^2|u_1|^2|v_2|^2+|u_2|^2|v_1|^2|v_3|^2|v_4|^2-2u_1v_1u_2v_2u_3v_3u_4v_4\right)\left|\langle 4\downarrow 3\uparrow|\mathrm{T}|1\uparrow 2\downarrow\rangle\right|^2\\
&+\left(|u_3|^2|v_2|^2u_1v_1u_4v_4-|u_4|^2|u_1|^2u_2v_2u_3v_3-|v_1|^2|v_4|^2u_2v_2u_3v_3+|v_3|^2|u_2|^2u_1v_1u_4v_4\right)\\
&\times\left(\langle 4\downarrow 3\uparrow|\mathrm{T}|1\uparrow 2\downarrow\rangle^*\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\uparrow 2\downarrow\rangle+\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\uparrow 2\downarrow\rangle^*\langle 4\downarrow\ 3\uparrow|\mathrm{T}|1\uparrow 2\downarrow\rangle\right)\\
&+\left(|u_3|^2|v_1|^2|v_4|^2|v_2|^2+|v_3|^2|u_1|^2|u_2|^2|u_4|^2-2u_1v_1u_2v_2u_3v_3u_4v_4\right)\left|\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\uparrow 2\downarrow\rangle\right|^2\Big\},
\end{aligned}
\tag{A6}
$$

and

$$
\begin{aligned}
W_{13}(\uparrow\uparrow)=2\pi\Big\{&\left(|u_3|^2|u_4|^2|v_2|^2|u_1|^2+|u_2|^2|v_1|^2|v_4|^2|v_3|^2-2u_1v_1u_2v_2u_3v_3u_4v_4\right)\langle 4\uparrow 3\uparrow|\mathrm{T}|1\uparrow 2\uparrow\rangle^2\\
&+\left(-|u_3|^2|u_1|^2u_2v_2u_4v_4+|v_2|^2|u_4|^2u_1v_1u_3v_3+|u_2|^2|v_4|^2u_1v_1u_3v_3-|v_3|^2|v_1|^2u_2v_2u_4v_4\right)\\
&\times\left(\langle 4\uparrow 3\uparrow|\mathrm{T}|1\uparrow 2\uparrow\rangle^*\langle -2\downarrow 3\uparrow\ |\mathrm{T}|-4\downarrow\ 1\uparrow\rangle+\langle -2\downarrow 3\uparrow\ |\mathrm{T}|-4\downarrow 1\uparrow\rangle^*\langle 4\uparrow 3\uparrow|\mathrm{T}|2\uparrow 1\uparrow\rangle\right)\\
&+\left(-|u_3|^2|v_2|^2u_1v_1u_4v_4+|u_4|^2|u_1|^2u_2v_2u_3v_3+|v_1|^2|v_4|^2u_2v_2u_3v_3-|v_3|^2|u_2|^2u_1v_1u_4v_4\right)\\
&\times\left(\langle 4\uparrow 3\uparrow|\mathrm{T}|1\uparrow 2\uparrow\rangle^*\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 2\uparrow\rangle+\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 2\uparrow\rangle^*\langle 4\uparrow 3\uparrow|\mathrm{T}|1\uparrow 2\uparrow\rangle\right)\\
&+\left(|u_3|^2|u_2|^2|v_4|^2|u_1|^2+|u_4|^2|v_1|^2|v_2|^2|v_3|^2-2u_1v_1u_2v_2u_3v_3u_4v_4\right)\left|\langle -2\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 1\uparrow\rangle\right|^2\\
&+\left(|u_3|^2|v_4|^2u_1v_1u_2v_2-|u_1|^2|u_2|^2u_3v_3u_4v_4-|v_1|^2|v_2|^2u_3v_3u_4v_4+|v_3|^2|u_4|^2u_1v_1u_2v_2\right)\\
&\times\left(\langle -2\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 1\uparrow\rangle^*\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 2\uparrow\rangle+\langle -1\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 2\uparrow\rangle^*\langle -2\downarrow 3\uparrow|\mathrm{T}|-4\downarrow 1\uparrow\rangle\right).
\end{aligned}
\tag{A7}
$$