

A combined Wavelet- Artificial Neural Network model and its application to the prediction of groundwater level fluctuations

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Abstract

Accurate groundwater level modeling and forecasting contribute to civil projects, land use, city planning and water resources management. Combined Wavelet-Artificial Neural Network (WANN) model has been widely used in recent years to forecast hydrological and hydrogeological phenomena. This study investigates the sensitivity of the pre-processing to the wavelet type and decomposition level in WANN model for groundwater level forecasting. To this end, the monthly groundwater level time series were collected from October 1997 to October 2007 in 26 piezometers of Qorveh aquifer, Iran. Using discrete wavelet transform method and different mother wavelets (Haar, db2, db3 and db4), these time series were decomposed into sub-signals in various resolution levels. Then, these sub-signals entered to the ANN model to reconstruct the original forecasted time series for 6 months ahead. The Root Mean Square Errors (RMSE) and coefficient of determination (R^2) statistics were used for evaluating the accuracy of the model. The results showed merits of db2 and db4 wavelets in comparison with Haar and db3 because of similarity between the signal of groundwater level and the functions of mother wavelets. For a better and precise analysis, the forecasted results of the model were compared with the observed data not only in the validation stage but also in the test stage.

Keywords: Groundwater Level, Discrete Wavelet Transform, Artificial Neural Network, Decomposition Level, Mother Wavelet.

Introduction

The world's most areas are categorized as warm and arid area because of geographical features and climate conditions. In these areas, due to the low degree of precipitation, the only way to gain potable water and agricultural water is restricted to groundwater resources which are done differently using aquifer derivation. Therefore any changes in these aquifers can influence the inhabitants' lives; or in cases that these changes are drastic, the people's lives might be endangered. For the effective management of groundwater, it is important to predict groundwater level (GWL) fluctuations.

Many methods and models for dealing with groundwater level prediction have been reported based on physical considerations or on other theories (Yang *et al.*, 2009; Shiri & Kisi, 2011; Yoon *et al.*, 2011; Mohammadi, 2008). Typically, physically based numerical models are used for characterizing a groundwater flow system and predicting the GWL fluctuations. These models establish a governing equation simplifying the physics of flow in the subsurface and solve it with proper initial and boundary conditions using numerical methods (Yoon *et al.*, 2011). Simulating of groundwater level by means of numerical models requires various hydrological

and geological parameters. In these models, recognition of boundary conditions, collecting the input data, calibration and verification is difficult, time consuming and expensive. In addition, combination of these models with optimization models for finding optimum groundwater management scenario needs hundreds of run.

Different researchers used stochastic theory-based models as well as numerical models simultaneously. To model a hydrological time series, several models have been developed based on stochastic theory, such as the Time Series (TS) model, the integrated time series (ITS) model (Yang *et al.*, 2009), autoregressive moving average (ARMA) model (Zhou *et al.*, 2008; Kisi, 2010), the seasonal autoregressive moving average (SARMA) model (Gemitzi & Stefanopoulos, 2011; Zhou *et al.*, 2008), the deseasonalized model, ARMAX (Bidwell, 2005), and the periodic autoregressive (PAR) model, threshold autoregressive model (TAR) (Wang *et al.*, 2009), and Artificial Neural Network (ANN), among others (see Ahn, 2000; Daliakopoulos *et al.*, 2005; Wong *et al.*, 2007).

Since hydrogeology, hydrology and water resource engineering sciences mostly have non-linear nature as well as having complicated design along with the inefficiency of physical modeling regarding these sciences, Artificial Neural Network has gained

warm reception as a new perspective in this field. In general the advantages of ANNs over other statistical and conceptual models are:

The application of ANNs does not require a prior knowledge of the process because ANNs have black-box properties.

ANNs have the inherent property of nonlinearity since neurons activate a nonlinear filter called an activation function.

ANNs can have multiple input having different characteristics, which can make ANNs able to represent the time-space variability.

ANNs have the adaptability to represent change of problem environments (Rahnama & Noury, 2008; Nourani *et al.*, 2009).

Artificial Neural Networks have been widely studied in water resources management and planning issues like simulation of quantitative and qualitative variables, rainfall-runoff models, groundwater level fluctuation forecasting and flow and rainfall forecasting. Artificial Neural Networks were utilized by Aziz & Wong (1992) to estimate aquifer parameters of groundwater. Lallahem *et al.* (2005) used ANN to assess water table in fractured media. Arguing that ANN models were more accurate than numerical models (in this case MODFLOW) for groundwater level forecasting, Mohammadi (2008) showed that major shortcoming of numerical models are the large number of input parameters as well as being time consuming.

In spite of suitable flexibility of ANN in modeling hydrologic time series, sometimes there is a shortage when signal fluctuations are highly non-stationary and physical hydrologic process operates under a large range of scales varying from 1 day to several decades. In such a situation, ANNs may not be able to cope with non-stationary data if preprocessing of the input and/or output data is not performed (Nourani *et al.*, 2009).

Different methods have been proposed to resolve the above-mentioned problem among which we can refer to wavelet analysis. Wavelet has been defined as a small wave whose energy is restricted into a short period of time and is an efficient method for signals that are non-stationary, and have short-lived transient components, features at different scales, or singularities (Hsu & Li, 2010). A non-stationary signal can be decomposed into a certain number of stationary signals by wavelet transform. Then ANN

is combined with wavelet transform to improve the prediction accuracy (Zhou *et al.*, 2008).

During recent years, wavelet transforms have become a useful method for analyzing such as variations, periodicities, trends in hydrological time series. Labat (2005) reviewed the most recent wavelet applications in the field of earth sciences and illustrated new wavelet analysis methods in the field of hydrology. Partal and Cigizoglu (2008) used neuro-wavelet technique for forecasting river daily suspended sediment load. Kisi and Cimen (2011) used a wavelet-support vector machine conjunction model for monthly streamflow forecasting.

Using WANN to predict groundwater level fluctuations is a new and fledgling method so that Wang & Ding (2003) and Adamowski & Chan (2011) have used this combined model in this field of study. Wang & Ding (2003) found that WANN models were more accurate than TAR and ARIMA models, and further showed that WANN models prolonged the forecasting time period and hydrology and water resource time series. Also Adamowski & Chan (2011) have shown that the combined model is much more precise than the classical models like ANN and ARIMA.

To the best knowledge of the authors, no study has been carried out to predict groundwater table fluctuations using mother wavelet types and optimum levels of decomposition. In this paper, the sensitivity of the pre-processing to the wavelet type and decomposition level is examined. To this end, the monthly groundwater level time series of Qorveh aquifer was decomposed into sub-signals in various resolution levels; then these sub-signals entered to the ANN model to reconstruct the original forecasted time series.

Methods

Wavelet transforms:

Wavelet transform (WT), a modern tool of applied mathematics, is a signal processing technique that has shown higher performance compared to Fourier transform and Short Time Fourier Transform in analyzing non-stationary signals. These advantages are due to its good localization properties in both, the time- and frequency domain (Cartas *et al.*, 2009). The wavelet transform is executed through discrete and continuous wavelet transform. The continuous wavelet transform changes the signal $x(t)$ into CWT_x^ψ wavelet coefficients:

$$CWT_x^\psi(\tau, s) = \Psi_x^\psi(\tau, s) = \frac{1}{\sqrt{|s|}} \int_{-\infty}^{\infty} x(t) \psi^* \left(\frac{t-\tau}{s} \right) dt \quad (1)$$

Wherein t and s are translation and dilation parameters, respectively (Rao & Bopardikar, 1998). The translation parameter determines the window movement degree and the dilation parameter, having a reverse connection with the frequency, compresses or stretches signal as a mathematical operator. In Formula (1), ψ is mother wavelet and $\psi_{s,\tau}$ is defined as following:

$$\psi_{s,\tau}(t) = |s|^{-\frac{1}{2}} \psi\left(\frac{t-\tau}{s}\right) \quad (2)$$

The mother wavelet should satisfy the following conditions:

1. $\{\psi_{s,\tau}(t)\}$ be a basis for $L_2(\mathbb{R})$
2. $\int_{-\infty}^{\infty} \psi(t) dt = 0$
3. $\int_{-\infty}^{\infty} |\psi(t)| dt < \infty$
4. $\int_{-\infty}^{\infty} \frac{|\psi(\omega)|^2}{|\omega|} d\omega < \infty$

The wavelet transform is in fact the similarity between frequency content of the signal and the basic functions (wavelets). Different scales should be taken into consideration in the continuous wavelet transform and using numerical method, resolved the equation integration (1) for each scale. Calculating the wavelet coefficient is time consuming in all scales and produced huge amount of data. In other words, we can say that the continuous wavelet transform consists of redundant and inefficient sections which are all its weak points.

The discrete wavelet transform has eradicated the continuous wavelet transform pitfalls. Meanwhile, it is an efficient alternative for the discrete data. In DWT, the wavelet transform is just executed for a sub-set of scales and positions. If scales and positions are selected based on powers of two, so-called dyadic scales and positions, the signal analysis is done quickly and precisely. Executing the above-mentioned transform, the raw data are divided into approximation (A) and details (D) (Fig. 1). The approximation consists of high scale and low frequency components of the signal. While the details consist of low scale and high frequency components of the signal which are obtained from low-pass and high-pass filters respectively.

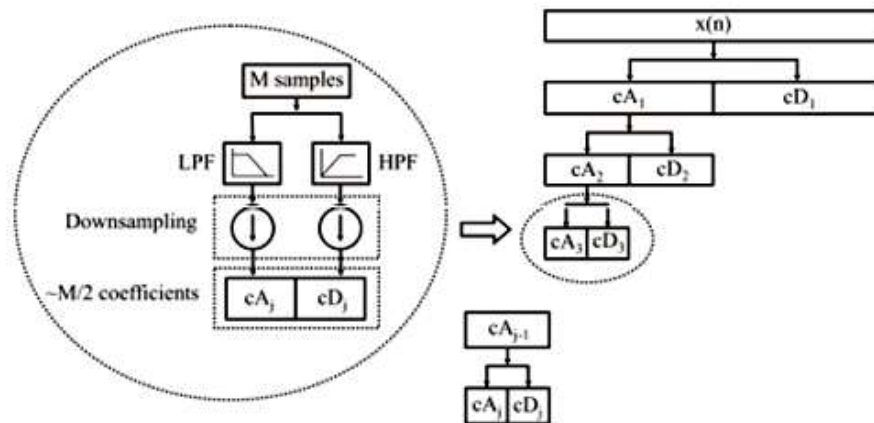


Figure 1: Primary wave ($x(n)$) decomposition to secondary wave of Approximation (cA) and details (cD)

Artificial Neural Network (ANN):

Artificial neural networks (ANN) are massively parallel interconnected networks of simple elements and their hierarchical organizations which are intended to interact with the objects of the real world in the same way as biological nervous systems do (Kohonen, 1998). As illustrated in Fig. 2, feedforward neural networks (FFNNs) are the

most commonly applied in hydrology due to the simple framework that belongs to static networks (Banerjee *et al.*, 2011). These neural networks consist of an input layer, hidden layer and output layer, with each layer including a number of nodes or neurons.

In this network, the input data are fed to input neurons, which in turn pass them on to the hidden

layer neurons after multiplying by a given weight. A hidden layer neuron adds up the weighted input received from each input neuron, associates it with a bias, and then passes the result on through a nonlinear activation function. The output neurons do the same operation as that of a hidden neuron (Lallahem *et al.*, 2005). Therefore a neuron

output in a layer depends on the signal received from the previous layer, its defined weight and activation function type. The linear activation function is most commonly applied to the output layer, whereas the bipolar sigmoid (tansig) function is often used in the hidden layer (Triana *et al.*, 2010) (Fig. 3).

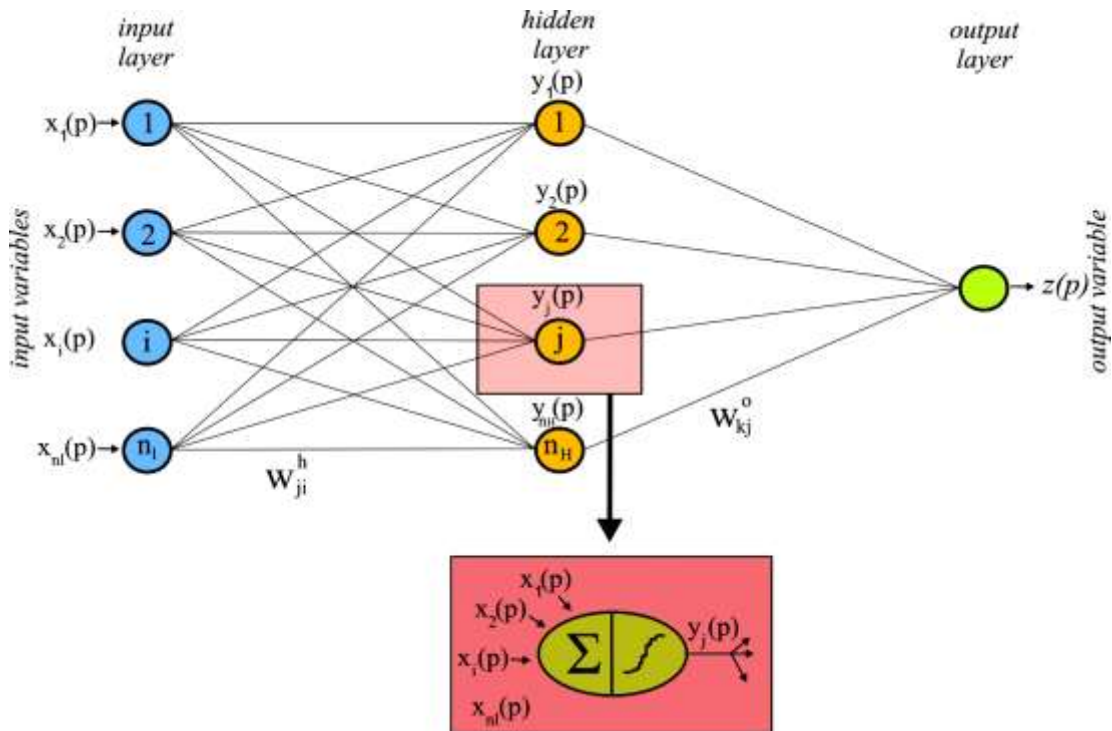


Figure 2: ANN architecture with one hidden layer

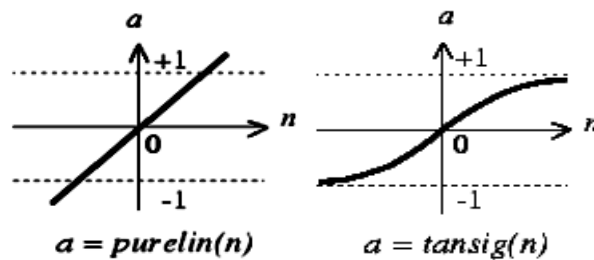


Figure 3. The linear activation function and the bipolar sigmoid (tansig) activation function

Before its application to any problem, the network is first trained, whereby the target output at each output neuron is compared with the network output, and the difference or error is minimized by adjusting the weights and biases through some training algorithm (Lallahem *et al.*, 2005). There are many algorithms for training network among which the Levenberg–Marquardt algorithm is often characterized as more stable and efficient. Also, Coulibaly *et al.*, (2000) point out that it is faster and less easily trapped in local minima than other

optimization algorithms.

The goal of an ANN model is to generalize a relationship of the form:

$$Y^m = f(X^n) \tag{3}$$

Where X^n is an n-dimensional input vector consisting of variables $x_1, \dots, x_i, \dots, x_n$; while Y^m is an m-dimensional output vector consisting of the resulting variables of interest $y_1, \dots, y_i, \dots, y_m$ (Adamowski & Chan, 2011).

Criteria for model performance

Different statistical criteria are used to assess models performance. Such researches proposed that a perfect evaluation of model performance should include at least one 'goodness-of-fit' or relative error measure (e.g. coefficient of determination (R^2)) and at least one absolute error measure (e.g. Root Mean Square Error (RMSE) or Mean Absolute Error (MAE)) (Rajaei, 2011). In this study, two different statistical criteria are used in order to assess model performance and its ability to make precise predictions.

1. The Root Mean Square Error (RMSE) calculated by:

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (y_i - \hat{y}_i)^2}{N}} \text{ or } RMSE = \sqrt{\frac{SSE}{N}} \quad (4)$$

Where y_i is observed data, \hat{y}_i the calculated data and N is the number of observations. RMSE indicates the discrepancy between the observed and calculated values. The lowest the RMSE, the more accurate the prediction is.

2. The coefficient of determination (R^2) given by:

$$R^2 = 1 - \frac{SSE}{SST} \quad (5)$$

$$SST = \sum_{i=1}^N (y_i - \bar{y})^2 \quad (6)$$

$$SSE = \sum_{i=1}^N (y_i - \hat{y}_i)^2 \quad (7)$$

Where SSR is Sum of Square regression, SSE is Sum of Square Error, SST is Sum of Square total and \bar{y} is the mean value of observed data.

The best fit between observed and calculated data, which is unlikely to occur, would have $RMSE=0$ and $R^2=1$.

Groundwater level fluctuations:

Groundwater levels change for many reasons. Some changes are due to natural phenomena, and others are caused by human activities. Water level changes can be divided into several categories. Fluctuations are generally due to one of the following three major factors:

- 1- Changes in the volume of water stored in the aquifer
- 2- Changes in atmospheric pressure
- 3- Changes caused by aquifer deformation

Fluctuations due to aquifer storage changes:

Groundwater is a part of a dynamic flow system that moves into and through aquifers from areas of high to low water level elevation. Groundwater level fluctuations due to aquifer storage changes involve both the addition of water to and extraction of it from the aquifer (recharge and discharge), through natural phenomena and human involvement. Precipitation, runoff, return water from agriculture and industry, and recharge from river, spring, qanat and adjacent aquifers are the most important factors that increase groundwater level. The main factors that decrease groundwater level include: wells pumping for agricultural, industrial and potable purposes, evaporation and evapotranspiration especially in warm regions and/or areas where water table is near the earth surface, drainage, and discharge to river, spring, qanat and adjacent aquifers. The most significant water level changes, due to recharge, generally occur during springtime of the year, when precipitation is generally greatest and evaporation and plant usage rates are low.

Fluctuations due to atmospheric pressure changes:

changes in atmospheric pressure can cause groundwater levels to fluctuate. Moreover, atmospheric pressure is caused by the Earth gravitational attraction of air in the atmosphere. At sea level, the weight of the atmosphere exerts a pressure of about 1013.25hPa (hectoPascals) on the Earth surface (Singh & Aung, 2005). This is equivalent to the pressure exerted by a column of mercury that is 76 cm high.

Changes in barometric pressure causes water levels in some wells, penetrating confined or semi confined aquifers, to change. That is, an increase in air pressure will cause the water level in the well to fall, and a decrease in air pressure will cause the water level in the well to rise.

Fluctuations due to aquifer deformation:

water level changes due to aquifer deformation are commonly due to either earth tides, or earthquakes. Other external stresses caused by heavy trucks and trains can also cause groundwater level fluctuations in some aquifers. However, these fluctuations are generally small and negligible.

Study area and data*Study area*

Qorveh plain which is located in the west of Iran is

selected for this study (between $35^{\circ} 02' 22''$ and $35^{\circ} 30' 54''$ North latitude and $47^{\circ} 38' 52''$ and $48^{\circ} 06' 03''$ East longitude) (Fig. 4). The recharge is a major source of groundwater, as well as rainfall, infiltration from two major rivers in the area, and return flow from irrigation. The average annual rainfall vary between 346 and 480 mm/year

depending on the location. The annual mean temperature is around 12.6°C and varies between -27°C in December and 38°C in July. According to Emberger Climate classification, The Qorveh watershed is under the influence of a cold and semi arid climate.

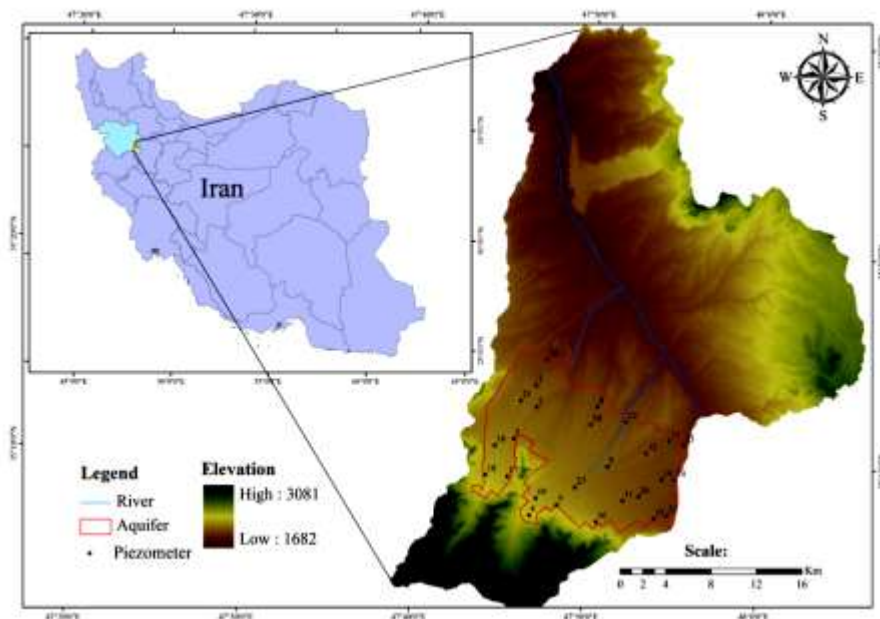


Figure 4: Study area and piezometer position

The Qorveh watershed area is 1063.5 Km^2 and the its aquifer area is 245 Km^2 . The aquifer from south leads to Zagros mountains and from north leads to Mio-Pliocene formations. The oldest rocks in the area include metamorphic rocks of Triassic and Triassic-Jurassic complex in the southern part of the plain and Mio-Pliocene formations in the northern part of it. Triassic complex consists of Amphibolite, Orthogneiss, Metagabbro, Metadiorite, and Marble. They have a lot of cracks and joints and recharge the southern part of the aquifer. Triassic-Jurassic complex consists of Marble, Quartzite and, in some parts, crystalline Limestone and Mica schist. This complex recharges the western part of the aquifer through Oriye valley. Mio-Pliocene and Quaternary formations as almost horizontal layers have covered the northern half of the plain. The formations are mainly composed of clay limestones and early quaternary volcanic rocks. In these formations, Pliocene tuffs are widely spread over the area. These tuffs have high porosity and their water is exploited by semi deep wells. Due to interlayers of marl,

transmissivity of clay limestones is low and, accordingly, they play the role of a bedrock in the northern part of the aquifer. Maximum and minimum value of the aquifer thickness are around wells No. 4 and No. 5 (about 120 m) and the western part of the aquifer (about 50 m), respectively. Transmissivity is $400 \text{ m}^2/\text{day}$ in the center of plain, $600 \text{ m}^2/\text{day}$ in the southeast of it, $50 \text{ m}^2/\text{day}$ in the north of it (the plain output), and about 300 to $400 \text{ m}^2/\text{day}$ along Oriye and Vihaj valleys.

Due to the high density of the pumping wells and the low recharge rate in the eastern part of the aquifer, as well as the recharges in the western part of the aquifer by its limestone and Oriye and Vihaj stream, groundwater level fluctuations on either sides are not the same.

Water is the main factor for environmental construction and agricultural production in this area. Many wells in the region are developed illegally, and pumping is unregulated, resulting in over exploitation of the aquifer and the consequential decrease in groundwater levels over

time. This has led to a sharp decline of groundwater table and the increase of problems associated with exhaustion of the water supply and hence a need for predictions of the future trends in the groundwater table.

Data

Groundwater level fluctuations depend on different factors such as precipitation, pumping, recharge, evapotranspiration, etc.; consequently all these factors exist implicitly in groundwater time series. Therefore it can be concluded that when wavelet decomposition happens wherein time series signal is decomposed into basic signals, the effect of all the factors are taken into consideration. For this reason, the current study just uses groundwater time series as input for WANN.

The current study used the groundwater level data, all obtained of twenty eight piezometer through ten years (from October 1997 to October 2007) in Qorveh plain. Assessing the data, piezometers No. six and seven were deleted from the modeling system due to having faults in the data. The predictions were all made using data obtained from twenty six piezometers.

Model development, result and discussion

The following study aimed at investigating the effects of the used wavelet type as well as decomposition level on the model efficiency. To achieve this purpose, Groundwater level time series should be processed before entering WANN. The scale of the input data should be changed before use based on the tansig activation function in the network hidden layer and the special form of the function along with the wavelet features. If we look carefully at the form of these functions, it is found that their slope is only considerable based on their proportion in the interval [-1, 1]. The changes are

trivial and not worth considering out of this interval. All of the previously used data including input and output are thus transferred to this interval to prevent the saturation of network. Network training is done with these data and the forecasted values will be turned back to the real amount before the operations are finalized. There exist a few methods to normalize the data among which the following formula was conducted (Lallahem *et al.*, 2005; Zhang *et al.*, 2003):

$$x_{normalize} = \frac{(x - x_{min})}{(x_{max} - x_{min})} \times 2 - 1$$

After normalization, the original data was decomposed into sub-series using discrete wavelet transform. In this transform, determining the mother wavelet type and optimum decomposition level is of high importance. One of the fundamental issues in selecting the mother wavelet is the nature of the phenomenon and the type of the time series. The functions of the mother wavelet have different types. The basic and well-known wavelet is called Haar. The biggest problem of Haar wavelet is lacking order (disorganization). The wavelets were either irregular and with a finite support (like the Haar wavelet) or regular and with an infinite support. After the work of Daubechies, a method was created to produce regular wavelets with a finite support which play a crucial role in the most applications. Considering the widespread and relatively successful use of Haar and Daubechies wavelets in hydrological and hydrogeological applications (Rajaei, 2011; Nourani *et al.*, 2009; Labat *et al.*, 2000), the wavelets of Haar, db2, db3 and db4 were used in this study to decompose the signal of groundwater level. Fig.5 shows the graph of these wavelets' function.



Figure5: Haar and Daubechies wavelets function

After determining the type of the wavelet, the most important step is selecting the appropriate level of

decomposition. Since the analysis process is iterative, in theory it can be continued indefinitely.

In reality, the decomposition can proceed only until the individual details consist of a single sample or pixel. It seems not rational to use the maximum level of decomposition in breaking up a signal. In practice, a suitable number of levels can be selected based on the nature of the signal, or according to a suitable criterion such as entropy. First, the experimental formula of INT (log n) was used to determine the level of optimum decomposition where n is the length of time series and INT stands for integer number and log n is common logarithm

(Wang & Ding, 2003). In contrast to what we had expected, we didn't obtain acceptable results after inserting the decomposed data into the neural network and predicting the groundwater level fluctuations. As a result, the data were decomposed up to the 3rd and 4th level in order to analyze the efficiency of the model in predicting the groundwater level fluctuations. Fig.6 shows a sample of decomposing the signal of groundwater with db2 wavelet up to the 3rd level.

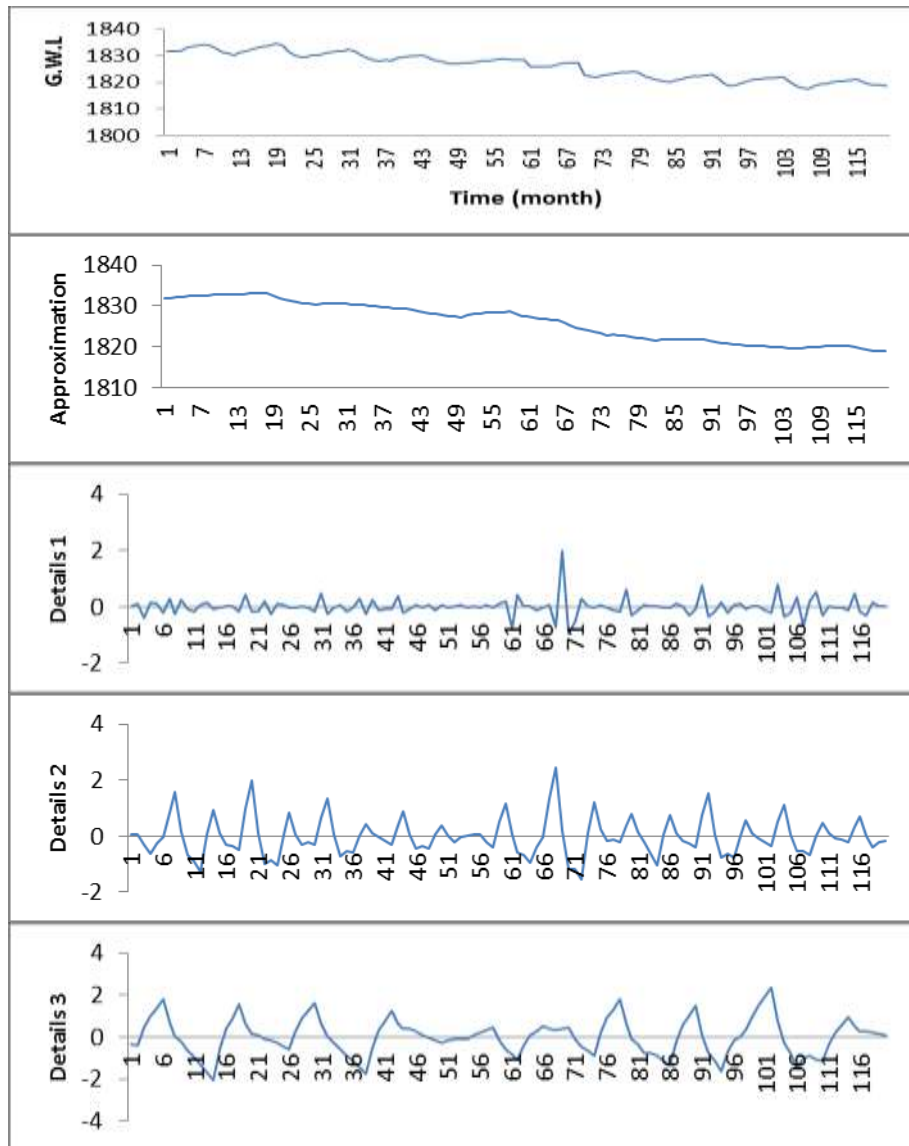


Figure6. Groundwater level time series decomposed by db2 wavelet up to level 3, using DWT

The next step is creating the artificial neural network. In this step, the multi-layer perceptron (MLP) feed forward ANN model with tansig activation function in hidden layer and purelin activation function in output layer was used to

model the monthly groundwater level of the aquifer. This network accompanied by Levenberg–Marquardt training algorithm is widely used in hydrogeological modeling (Daliakopoulos *et al.*, 2005; Sreekanth *et al.*, 2009). The inputs of ANN

model are $X=[D1(t), D2(t), \dots, Dn(t), An(t)]$, The neurons of input layer are $i+1$ where i is wavelet decomposition degree. The output is $Y = [GWL(t+T)]$ where T is prediction time (in this case is 6 month), the neuron of output layer is 1. In addition, the neurons of hidden layer were determined by trial and error and the best option was selected by comparing the error of different options (Fig.7). Another factor which was taken into account to choose the neurons of hidden layer was the simplicity of the network. In other words, the

option with the least number of neurons was selected out of the two with similar output (Fig.7). This was implemented to prevent the complexity of the network. For each one of the inputs, the time series was divided in 3 different subsets. 70% of data for training the ANN (October 1997 to September 2004), 15% for model validation (October 2004 to March 2006) and 15% for model testing (April 2006 to October 2007). The main framework of the model is schematically presented in Fig.8.

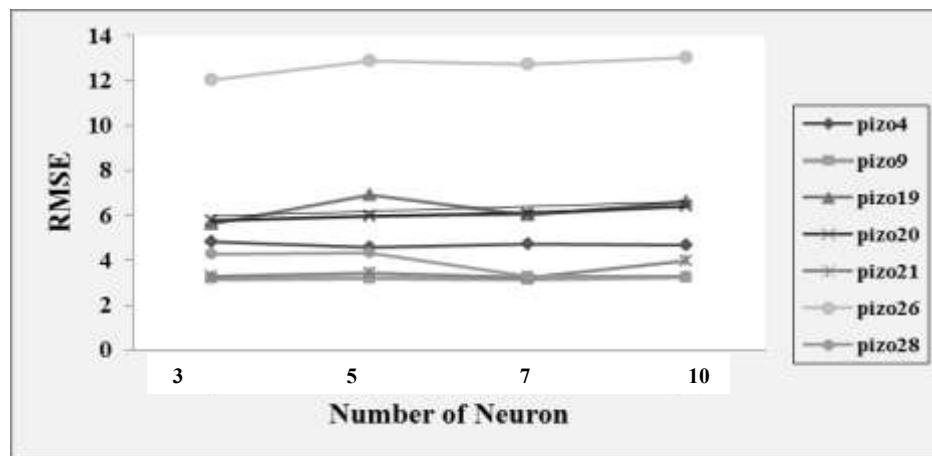


Figure7: RMSE versus number of neuron in selected piezometers of study area

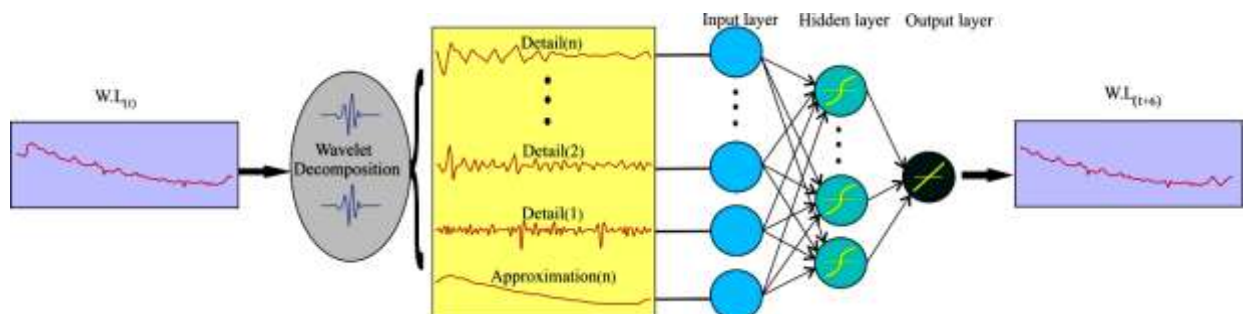


Figure8: General schematic of the WAN

The created network was trained using the training data set and after obtaining the appropriate results, the trained model was validated by the verification data set. The results obtained from this model are presented in Table 1a and 1b for validation stage based on different piezometers, the type of the wavelet and decomposition level. As Fig. 9 shows, the predicted data obtained from the best model output in the validation stage were compared with the observed data.

In general, after analyzing the results obtained from the model of WAN in all of the existing piezometers of the study area, it was found that preprocessing the data with the wavelets of db2 and

db4 have provided better results in comparison with the wavelets of Haar and db3. The reason could be the similarity between the signal of groundwater level and the functions of mother wavelet (frequency similarity). In addition, decomposing the data up to higher levels has indicated more satisfactory results in most cases. This is due to the fact that preprocessing the data with wavelet analysis causes the effect of different time levels to be considered in predicting the fluctuations of groundwater level. In the abovementioned case, we should pay attention to the level of the optimum decomposition of the data which was explained before.

Test stage:

In this step, we used groundwater level data as input dating back to April 2006 to October 2007. Using the data, we predicted the water level from October 2006 to April 2008. For a better and precise analysis, the results of model error WANN

were compared in the test stage with the validation stage. The results of the comparison have been shown in Fig.10. Moreover, the predicted data in the test stage, as the validation stage, were compared with the genuine data.

Table 1a. The results of WANN based on mother wavelet types and decomposition levels

Pizo	Wavelet											
	db2						db3					
	2	3	4	2	3	4	2	3	4	2	3	4
	RMSE%	R ² %	RMSE	R ²	RMSE	R ²	RMSE	R ²	RMSE	R ²	RMSE	R ²
p1	2.83	81.03	3.01	85.54	2.96	80.7	2.89	80.23	2.82	83.79	2.86	85.4
p2	9.9	53.61	2.72	88.15	3.33	95.05	11.94	60.01	5.83	83.71	5.62	82.99
p3	7.17	55.4	6.78	72.09	6.77	69.24	6.68	76.05	5.35	83.9	5.6	83.96
p4	6.6	73.09	4.83	87.81	4.27	83.62	8.18	70.48	4.38	84.7	4.87	87.68
p5	6.39	73.2	3.42	92.35	2.69	96.02	7.68	69.3	4.49	87.02	4.45	84.78
p8	14.21	51.53	5.26	91.12	4.96	92.49	13.98	52.21	5.64	84.72	7.01	76.7
p9	5.51	70.98	3.16	88.79	3.36	85.07	6.04	72.16	3.93	84.06	3.49	87.87
p10	12.27	62.53	10.71	58.21	9.54	55.6	12.54	58.8	9.78	64.2	9.96	67.76
p11	8.24	64.63	2.61	96.27	3	94.98	8.27	64.11	4.1	87.94	3.25	91.98
p12	13.88	50.73	5.96	87.44	1.59	98.78	15.85	58.64	4.75	88.88	2.91	96.05
p13	6.79	80.14	3.49	94.71	2.08	98.11	12.72	54.67	5.29	85.91	3.07	95.35
p14	13.89	62.68	4.37	93.1	2.92	96.58	16.94	50.6	5.99	87.76	3.48	95.52
p15	7.65	71.18	4.84	89.61	4.84	87.2	10.45	53.71	5.45	84.9	4.21	91.82
p16	2.03	90.06	2.63	87.93	2.59	90.75	2.32	88.65	2.47	91.98	2.23	91.96
p17	4.8	75.52	3.72	87.81	2.8	89.63	5.41	77.38	3.41	82.85	4.45	85.07
p18	13.93	62.72	10.06	79.46	6.77	84.75	10.95	61.72	5.36	89.59	6.84	87.13
p19	21.35	52.42	5.62	92.96	6.56	90.12	19.97	52.45	11.79	71.7	5.76	92.23
p20	7.15	71.38	5.76	72.14	5.91	74.02	7.01	75.91	5.82	67.79	5.9	71.25
p21	6.66	76.44	3.25	93.69	3.54	89.77	5.58	77.02	3.21	91.15	3.23	90.92
p22	2.07	96.06	1.89	96.1	1.55	97.79	2.16	95.27	1.23	98.61	1.27	98.34
p23	3.74	86.26	2.39	94.89	3.12	91.79	3.1	88.03	3.17	89.13	3.13	89.28
p24	8.62	75.56	3.8	90.18	3.12	94.25	8.95	54.63	4.29	87.6	3.5	93.42
p25	5.14	80.97	1.85	96.62	1.67	96.76	9.57	60.81	2.95	89.62	1.48	97.1
p26	22.3	51.09	6.89	81.87	8.75	78.67	15.68	40.66	10.96	71.51	7.08	81.48
p27	4.35	81.97	4.14	92.12	4.43	85.51	3.93	80.68	2.47	93.72	2.86	93.12
p28	7.66	73.79	3.25	94.79	4.47	89.17	6.9	66.81	6.77	70.79	4.5	87.23

Table 1b: The results of WANN based on mother wavelet types and decomposition levels

pizo	Wavelet									
	db4						Haar			
	2	3	4	2	3	4	3	4	3	4
	RMSE	R ²	RMSE	R ²	RMSE	R ²	RMSE	R ²	RMSE	R ²
p1	2.99	83.62	2.79	82.33	2.91	82.28	2.99	85.81	3.11	83.71
p2	12.07	62.04	5.62	89.05	4.87	91.11	6.63	81.62	6.86	79.6
p3	8	59.58	8.44	71.63	7.38	73.46	6.94	76.01	7.73	71.25
p4	7.25	72.03	5.99	83.02	5.62	82.96	5.68	77.44	5.45	85.9
p5	8.03	57.27	2.69	96.23	2.36	96.83	5.32	76.49	3.39	92.66
p8	15.22	53.01	5.94	86.44	4.44	93.8	11.81	54.26	7.81	75.6
p9	5.24	70.01	3.98	83	2.89	91.8	4.77	78.07	3.35	82.6
p10	11.08	63.59	11.17	51.8	8.51	78.53	9.66	65.58	7.59	71.07
p11	7.38	69.9	2.93	94.18	2.36	96.45	4.56	89.26	4.47	87.9
p12	11.09	63.93	4.31	92.99	3.33	95.18	3.3	95.05	2.65	97.15
p13	12.56	55.37	4.67	90.14	4.38	90.96	5.26	87.55	4.77	91.77
p14	11.73	61.4	4.29	93.15	3.93	94.55	5.39	90.13	5.57	89.96
p15	10.78	55.81	5.09	86.39	4.79	88.43	6.38	83.31	5.54	81.87
p16	2.82	91.6	2.99	89.75	2.76	88.56	3.44	80.68	1.36	96
p17	4.83	79	3.7	85.2	2.84	88.81	3.83	85.16	3.14	86.65
p18	10.88	72.72	7.25	85.14	2.23	92.66	13.29	66.08	10.77	78.07
p19	19.68	49.23	7.47	88.24	6.31	92.23	13.62	66.42	8.03	83.27
p20	6.98	69.88	5.6	63.03	6.46	69.93	6.66	72.52	5.93	66.6
p21	5.99	75.95	3.78	88.4	2.98	93.51	4.47	82.74	4.57	87.07
p22	1.98	96.46	1.53	97.61	1.88	96.77	2.1	96.07	2.06	95.43
p23	3.74	84.04	2.89	89.96	3.25	90.9	3.87	85.01	3.62	86.11
p24	9.85	60.7	3.62	91.11	4.78	85.65	4.88	89.14	3.77	90.06
p25	4.94	77.59	1.39	97.98	1.48	97.41	3.17	86.89	2.34	93.52
p26	15.07	54.68	11.78	72.82	7.79	85.2	13.62	66.49	13.05	64.13
p27	3.99	70.12	2.89	91.09	4.22	83.25	12.07	74.76	3.58	87.88
p28	6.64	55.75	4.59	86.21	4.34	85.54	6.96	67.46	6.08	68.56

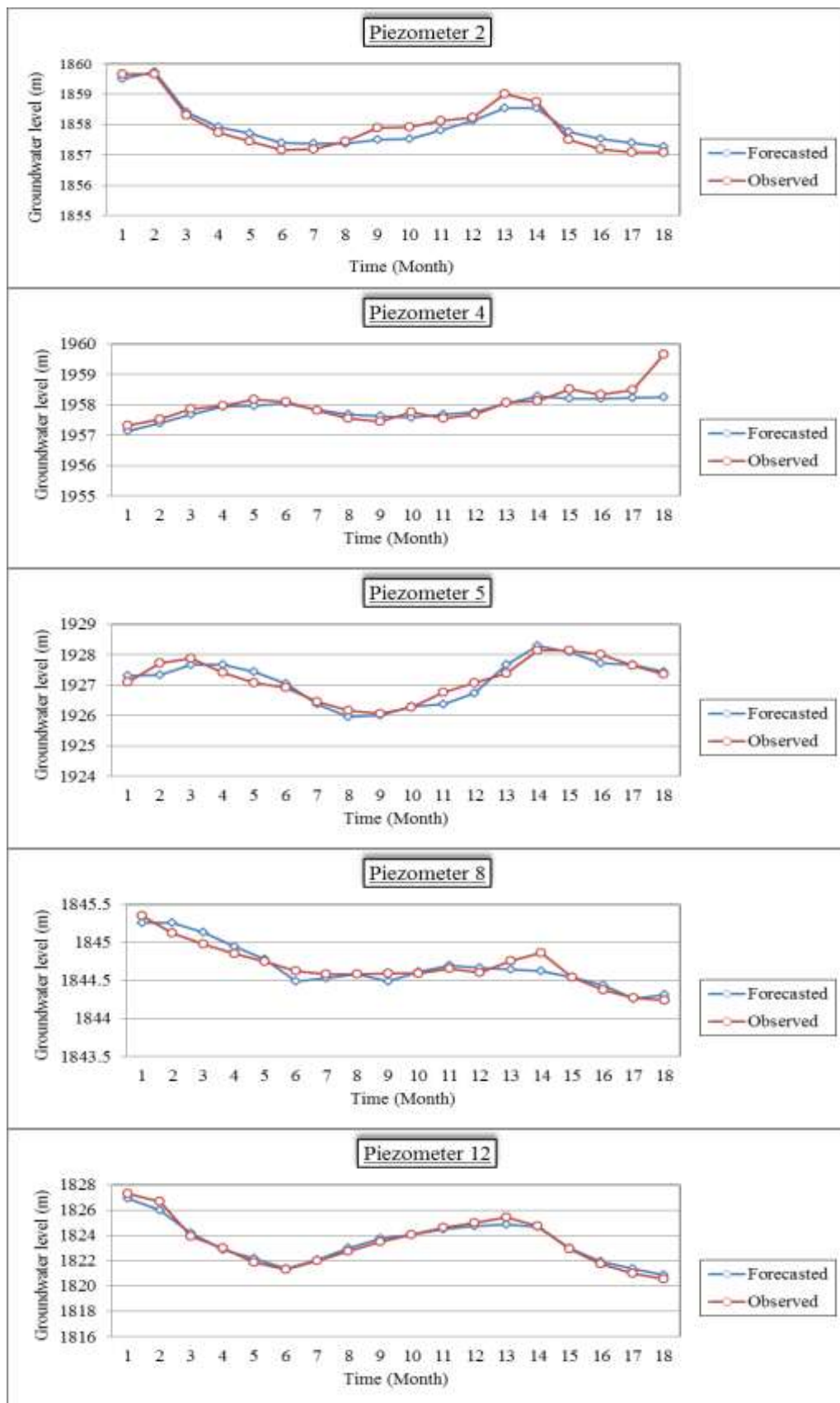


Figure9: Comparison of forecasted versus observed groundwater level at selected piezometers using the best WANN model for Six-month ahead forecasting in validation stage.

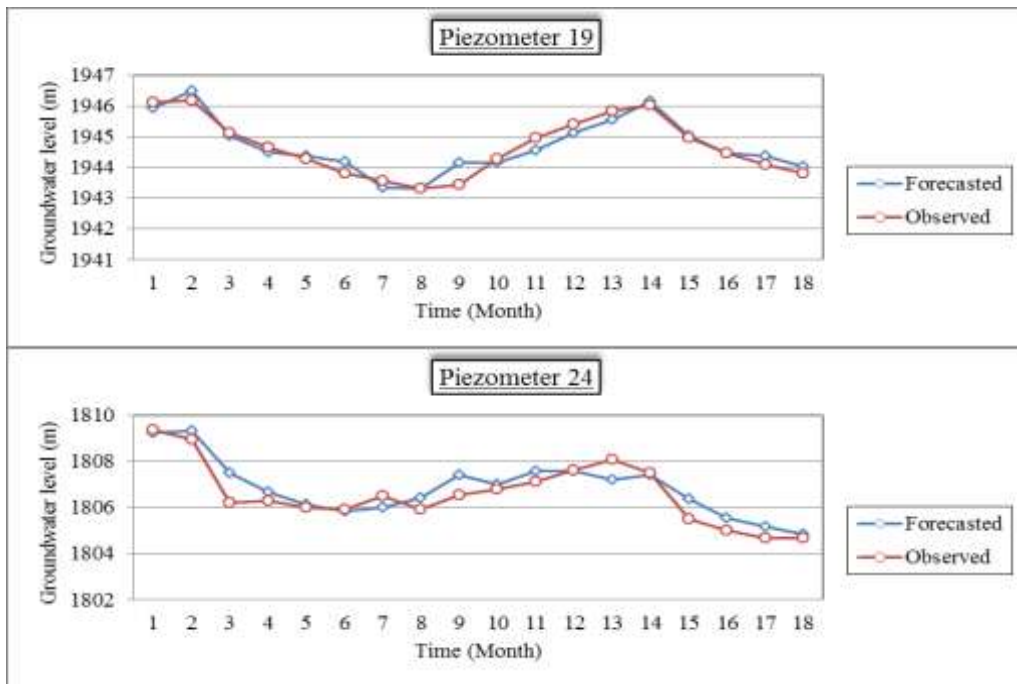


Figure9. (Continued)

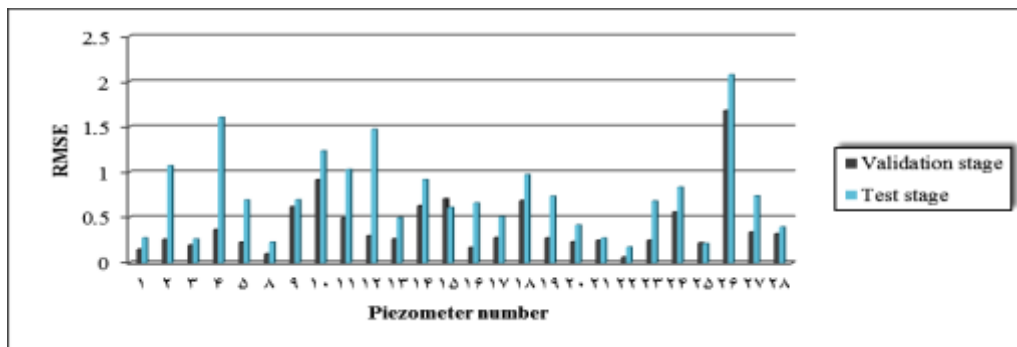


Figure10: Comparison of WANN model error results in test stage and validation stage

The results obtained from WANN model for each piezometer indicated that if they are able to be trained with the least error and have a high correlation in the training stage, they can produce appropriate results. These models predicted the water level in all piezometers satisfactorily and can

be thus considered as a suitable alternative for numerical models in predicting the fluctuations of groundwater level Fig.11 shows the observed values of water level fluctuations in some of the piezometers and its comparison with the values computed by WANN at test stage.

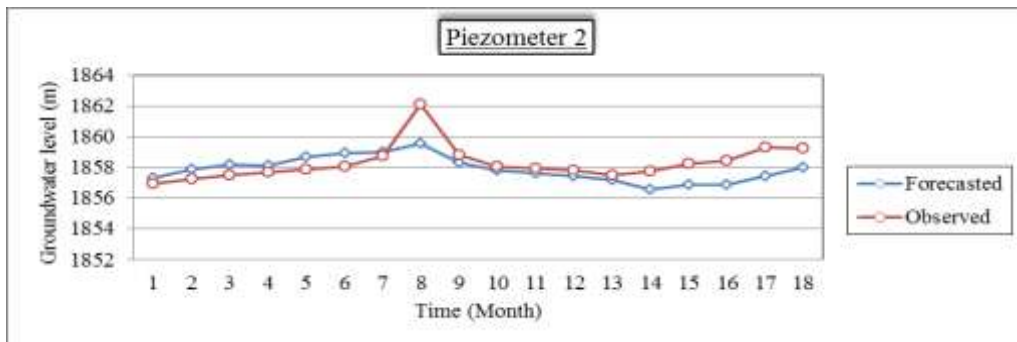


Figure 11. Comparison of forecasted versus observed groundwater level at selected piezometers using the best WANN model for Six-month ahead forecasting in test stage.

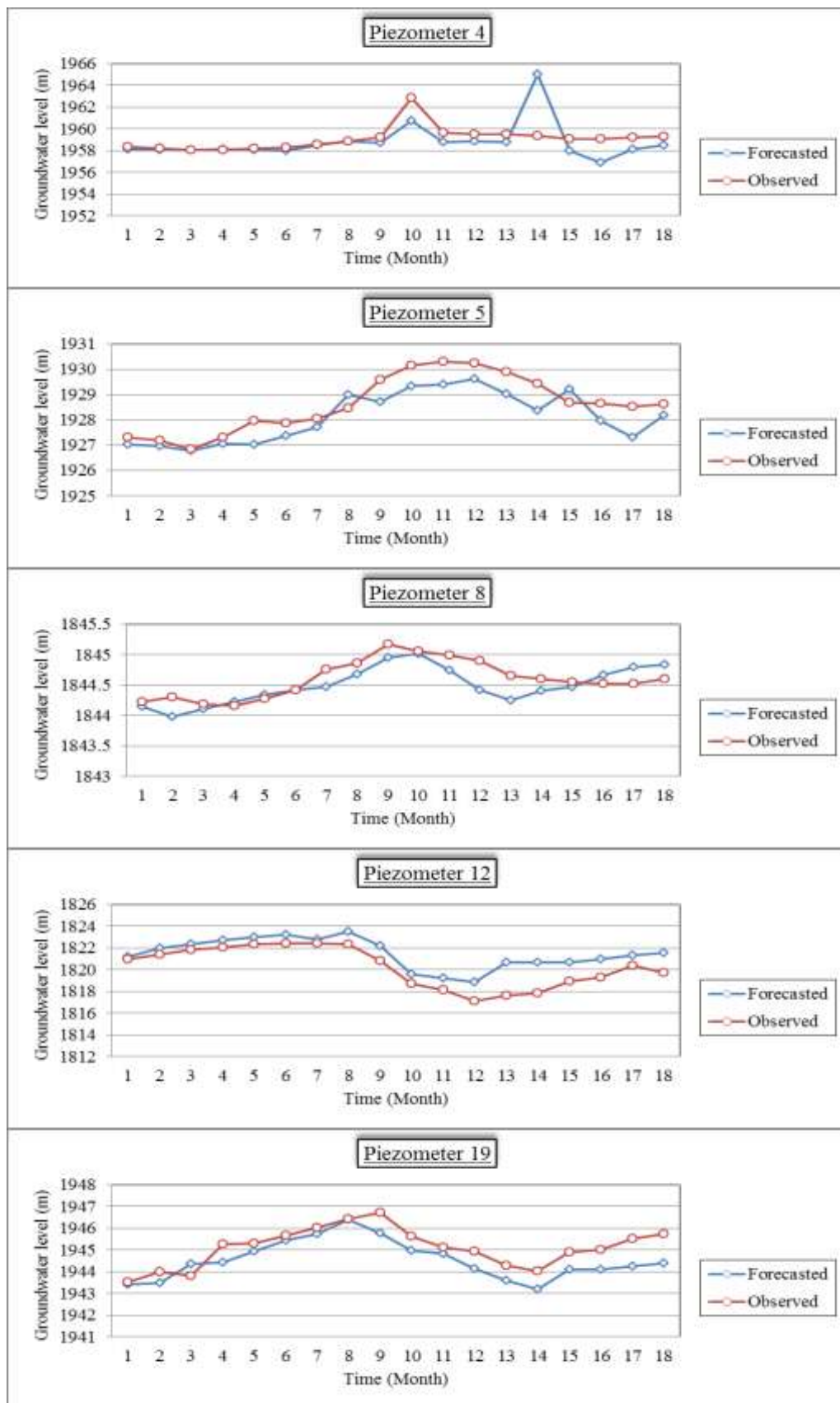


Figure 11. (Continued)

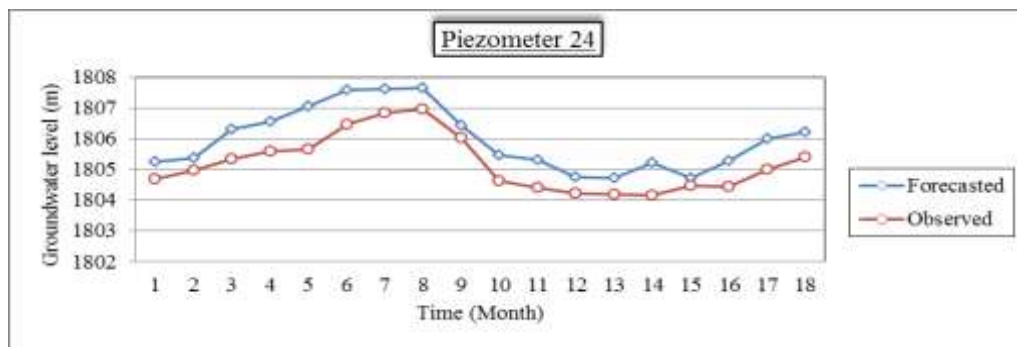


Figure 11. (Continued)

Conclusion

In this study, the discrete wavelet transform, which can capture the multi-scale features of signals, was used to decompose the Qorveh groundwater level time series. The sub-signals were then used as input to the ANN model to predict the groundwater level six months ahead. The discrete wavelet transform allowed most of the 'noisy' data to be removed and it facilitated the extraction of quasi-periodic and periodic signals in the original data time series.

Although the previous studies had shown that this combined model was much precise than classic models such as ANN and ARIMA (Wang & Ding, 2003; Adamowski & Chan, 2011), the current study shows that specifying the mother wavelet and optimum decomposed level both lead to much more precise results. The effect of mother wavelet type

on the model performance was investigated using four different kinds of mother wavelet. The results obtained from the model show low merits of Haar and db3 wavelets in comparison with db2 and db4 because of dissimilarity between the signal of groundwater level and the functions of mother wavelet.

Moreover, the obtained results from the current study were not in line with studies carried out by Nourani *et al.*, 2009 and Wang & Ding, 2003 who argued that to determine the optimum decomposed level entails using INT (log n) experimental formula. They had claimed that an optimum level can be chosen on the basis of the signal length. But the current study, against our expectations, didn't lead to appropriate results in using the formula applied in the current model.

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