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Introducing Busy Customer Portfolio Using Hidden Markov Model

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Abstract

Due to the effective role of Markov models in customer relationship management (CRM), there is a lack of comprehensive literature review which contains all related literatures. In this paper the focus is on academic databases to find all the articles that had been published in 2011 and earlier. One hundred articles were identified and reviewed to find direct relevance for applying Markov models in CRM. Forty four articles were selected and categorized on two major subclasses: articles which ha cha models (MCM) in CRM a a hidden Ma models (HMM) in CRM. Findings of this pa HMM in CRM is a since it conta articles. To complete investigation a two-step framework has been suggested for using HMM in busy customer portfolio management. It is for the first time that two important concepts (busy customer and HMM) are used to achieve a common goal. Also the model parameters have been estimated in order to analyze a real firm's data.

Keywords:

Customer relationship management, Markov chain models, Hidden Markov models.

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Introduction

Customer relationship management (CRM) comprises a set of process and enabling systems supporting a business strategy to build long term, profitable relationship with specific customers (Ling & Yen, 2001). From another point of view CRM is a customer-focused business strategy that dramatically integrates sales, marketing and customer care service in order to create and add value for the company and its customers (Chelmeta, 2006).

CRM had developed as an approach based on maintaining positive relationships with customers, increasing customer loyalty and expanding customer lifetime value (Blattberg & Deighton, 1996), (Brassington & Pettit, 2000),(Ahn, Kim, & Han, 2003) .

Undoubtedly, we can find many definitions for CRM which emphasize on CRM's critical role in achieving benefits for both firm and customer.

Articles in consideration show that different kinds of tools and techniques have been applied in CRM. One of the most practical ones are the Markov models.

Markov models are stochastic models that assume the Markov property; these introduce a general class of mathematical models which are appropriate for modeling customer relationships (Pfeifer & Carraway, 2000).

Chan et al. (2010) believe Markov models are suitable for modeling customer segmentation, acquisition, retention and migration situation under influences of marketing strategy.

Markov models, when applying to CRM are probabilistic models accounting for uncertainties and evaluating future relationships.

Pfeifer and Carraway (2000) prefer Markov chain model through their research due to its flexibility and well developed theoretical supports.

This paper presents an accurate literature review of the usage of Markov models in CRM and classifying these models to two subclasses:

1. Markov chain models (MCM) that includes all Markov models except hidden ones.
2. Hidden Markov models (HMM).

Research Methodology

Utilizing a particular method that covers all articles is the aim of this paper and requires a comprehensive search in the following online journal databases to provide an encyclopedic bibliography of the articles.

1. ScienceDirect
2. ProQuest
3. SpringerLink
4. Emerald Full text

The literature search was based on these descriptors: customer relationship management, Markov chain models and hidden Markov models which produced approximately one hundred articles. Figure 1 depicts the covered range in this research.

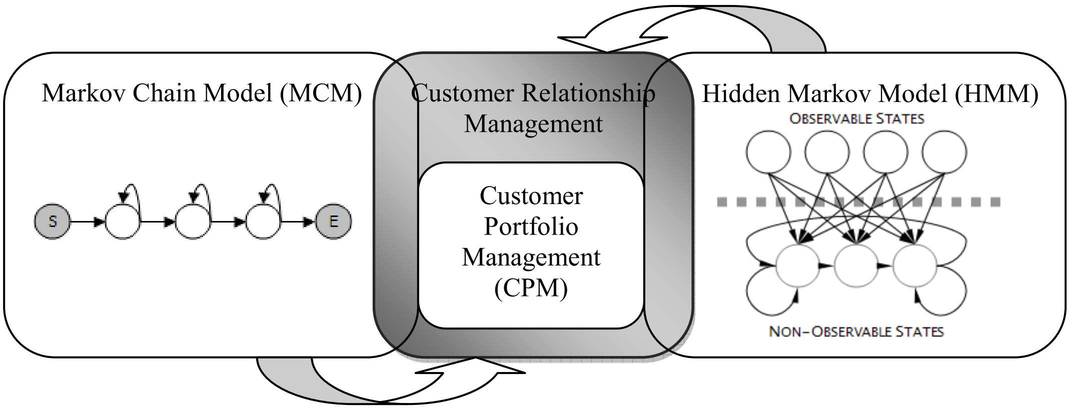


Figure 1. Subjects which were covered in this paper

The purpose of this figure is to depict the procedure of selecting articles and classifying them to two subclasses which is the topic of next sections.

Each article was reviewed thoroughly to eliminate those which were not precisely related to the desired topic. All the articles which had been published in 2011 and earlier, and clearly described how Markov models could be applied and assisted in CRM were picked out. Intended CRM dimensions are: customer retention, customer loyalty, customer portfolio management (CPM) and customer lifetime value (CLV).

Figure 2 shows a flowchart for determining article selection method.

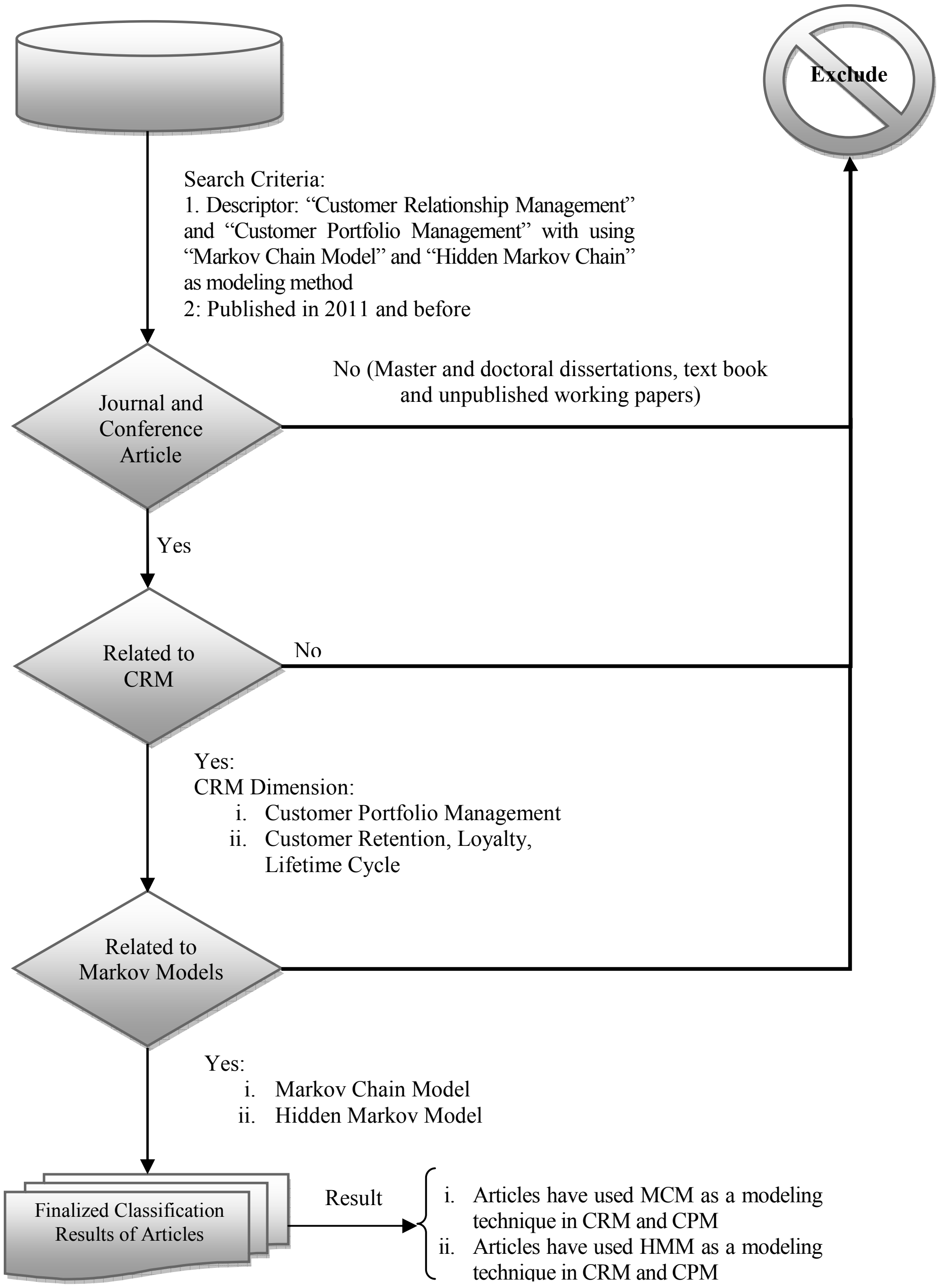


Figure 2. Selection criteria and evaluation framework

Application of Markov Chain Models in CRM

From forty four articles which were actually related to topic, thirty two articles (72.2%) had used Markov chain models in CRM.

Harary and Lipstein (1962) was one of the first researchers attempting to apply the probabilistic method of Markov chain to systematic study of brand switching and brand loyalty. After that this application developed slowly.

Researchers tried to use it in other dimensions of CRM such as analyzing and predicting customer special behavior (Feinberg, Kahn, & McAlister, 1994).

Studying past transaction patterns to develop a dynamic CRM model is also another MCM usage (Li, Xu, & Li, 2005).

Wu and Shieh (2006) applied MCM in quality function deployment to analyze customer requirements. Their proposed approach provides a decision maker to investigate and then satisfy both present and future customers' needs. Therefore, probabilistic nature of Markov chain has a crucial impression on decision making (Ma, Li, & Chen, 2008).

These kinds of mathematical models are used for customer segmentation and measuring CLV (Ha, Bae, & Park, 2002). Thus researchers for achieving a valuable customer's segmentation try to exert Markov property (Chan, Ip, & Cho, 2010), (Ip, Chen, Lau, Choy, & Chan, 2008), (Chan & Ip, 2008), (Haenlein, Kaplan, & Beeser, 2007) (Chen & Kai, 2006), (Xiao- Wei & Fu, 2004). Pie and Zhao (2006) applied MCM to cluster internet customers through their relationships with the firm.

Analyzing customer behavior to figure out their migration patterns can be vital for predicting their future value (Mark & Csaba, 2007).

Jenamani et al. (2003) proposed a data mining model that considers e-customer activities through a discrete-time semi-Markov process. This highlights that using Markov models is not limited to simple and first order Markov models (Ching, Ng, & Fung, 2008)

Markov models have been used in bundling products during the product planning stage, because there are a number of possible combinations that can be offered to customers, so even a small mistake can effect on customer satisfaction (Ferreira & Wu, 2011).

Jonker et al. (2004) use this modeling technique in a macro marketing decision level, applying the Markov decision process to forecast markets share and determine customer's decision pattern in the diverse circumstances that are common (Sokele, Moutinho, & Hudek, 2009)

Customer relations is increasingly influencing the performance of inventory management in a supply chain, thus customer retention and migration directly affect the number of customer orders and demands as well as the inventory level. Lam and Ip (2011) suggested a customer satisfaction model, where the probabilistic concepts of Markov chains of uncertainties in customer relationships are adopted.

Another research area is selecting the best sale strategy (Chen, Ip, & Yingjie2005) (Chen, Ip, & Yingjie, 2007). They could present the best sale strategy based on Pfeifer & Carraway's (2000) proposed model.

Some of the researchers tried to compare or improve existence models with using creative solution in estimating model's elements such as transition matrix (Chen, Li, & Liu, 2010), (Chen, Liu, & Yingjie, 2008), (Liu & Yingjie, 2008), (Ip, B. C., 2005).

Finally, determining relationships between customers and firm and attempting to develop it in a profitable way is another CRM dimension that can be modeled by MCMs (Xiaoyun, 2009). Also one article was found which had accurately mentioned CPM (Homburg, Steiner, & Totzek, 2009). They proposed a model for managing customer relations and behaviors besides measuring CLV for each portfolio, so this is a customer portfolio evaluation method too.

According to the subjects raised, all of the considered articles have utilizd Markov models except HMMs.

Figure 3 shows the distribution of considered articles by the year of publication.

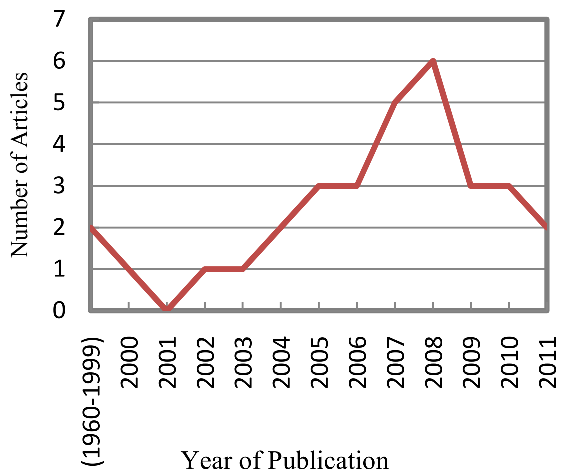


Figure 3. Distribution of articles by the year of publication (application of MCM in CRM)

This diagram shows a growth till 2008 and a fall off after that.

Application of Hidden Markov Models in CRM

Twelve out of forty four (27.2%) articles categorized in this class contain HMM applications in CRM dimensions.

Netzer et al. (2008) constructed and estimated a non-homogenous HMM to model the transitions among latent relationship states. They used HMM to overcome the problem of unobserved state, as well as to describe a set of latent states and transitions between them, also they translated these latent states into observed customers' behaviors.

Hassan and Nath (2005) applied this modeling technique to forecast stock prices for interrelated markets.

Hidden Markov models often encountered researchers with estimating the models' parameters. There are several estimation methods and algorithms such as expectation-maximization (EM). Wang et al. (2000) used this algorithm to estimate HMM's parameters and utilized Viterbi algorithm to find some path which were describing customers migration pattern in an online retailer.

Nowadays, predicting the intention of the internet users is vital in e-business. Wu et al. (2005) constructed a specific HMM for the web browsing which could prospect whether the users have the intention to purchase in real time. It can be effective to be ready for customers' needs in buying process and predicting customer churn situations (Burez & Van Den Poel, 2007),(Hadden, Tiwari, Ror, & Ruta, 2007) .

For another HMM application in CRM, putting forward a customer loyalty analysis process based on customer purchase, customer price perception, service perception and quality perception have been researched (Bouchaffra & Tan, 2004), (Poulsen, 1990). Also, Shen & Zhao (2006) according to Bayesian rules obtained the conditional probability and calculated the equation that was referred as likelihood function and then designed a classifier based on HMM for discovering which customer is loyal and which is not loyal.

Predicting demands of mobile customers to provide business information they need is another HMM usage.

Xia and Tingjie (2009) mined the latent information requirement concepts using HMM.

Ching et al. (2004) suggested an operative estimation method instead of EM algorithm. Authors claimed their offered method is better in both quality of estimation and computational complexity, they also applied it to classify the customers of a computer service company.

Another HMM usage in CRM which have been researched is credit card risk analysis. Oguz and Gurgen (2008) investigated the performance of HMM for credit card risk analysis in term of classification.

Through evaluated articles in this section only two of them used HMM as a portfolio optimization technique (Liu H. , 2010), (Elliot & Hinz, 2002). Their basic idea is to describe essential movement of the stock price using HMM and to calculate the optimal portfolio using HMM's recursive algorithms.

Figure 4 portraits the distribution of investigated articles by the year of publication.

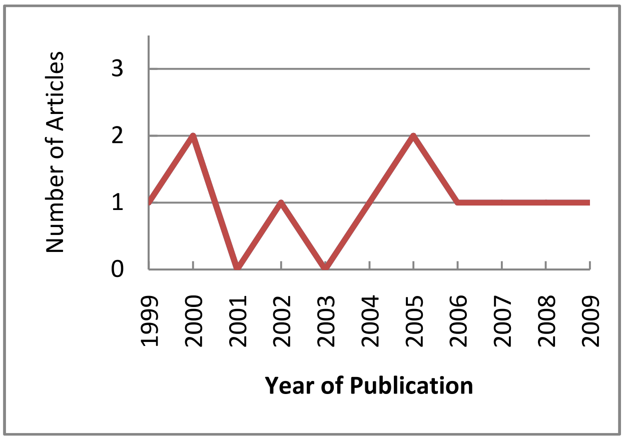


Figure 4. Distribution of articles by the year of publication (application of HMM in CRM)

This diagram shows that since 2006 the growth rate of article publication is consistent. Despite of HMM potency, it can be a significant deficiency.

Suggested Framework for Further Researches

According to articles investigated, several researches have been done around using Markov models in CRM concepts. Nevertheless, none of them had a special focus on application of HMM in busy customers' behaviors and their relationships portfolios. These customers are who, as Chaffey (2003) calls them "busy customers" and characterizes them by phrases such as:

- Cash-rich, time-poor
- Time is money
- Value for time
- Personal disposal time

So, they can make value with rich cash flows but can be attracted by competitors with low time-consuming services. Creating a distinguished portfolio for busy customers is suggested for managing their relationships and predicting their migration pattern. These customers can be determined by two important criteria: cash flows they have expended and time they have consumed in the market. It is possible to monitor their interactions with the firm by strong stochastic assumptions and approaches. In Figure 5, we introduce a comprehensive framework consisting of two major steps.

First Step:

1. Hidden Markov models due to the following features are a suitable method(Hassan & Nath, 2005) :

- Strong statistical function
- Handling new data robustly
- Computational efficiency in development and evaluation
- Predicting similar patterns efficiently

2. In the estimation procedure of the model which is very significant, we act as below:

- For estimating observation symbol probability distribution we prefer to apply exponential distribution (Mark, 2007):

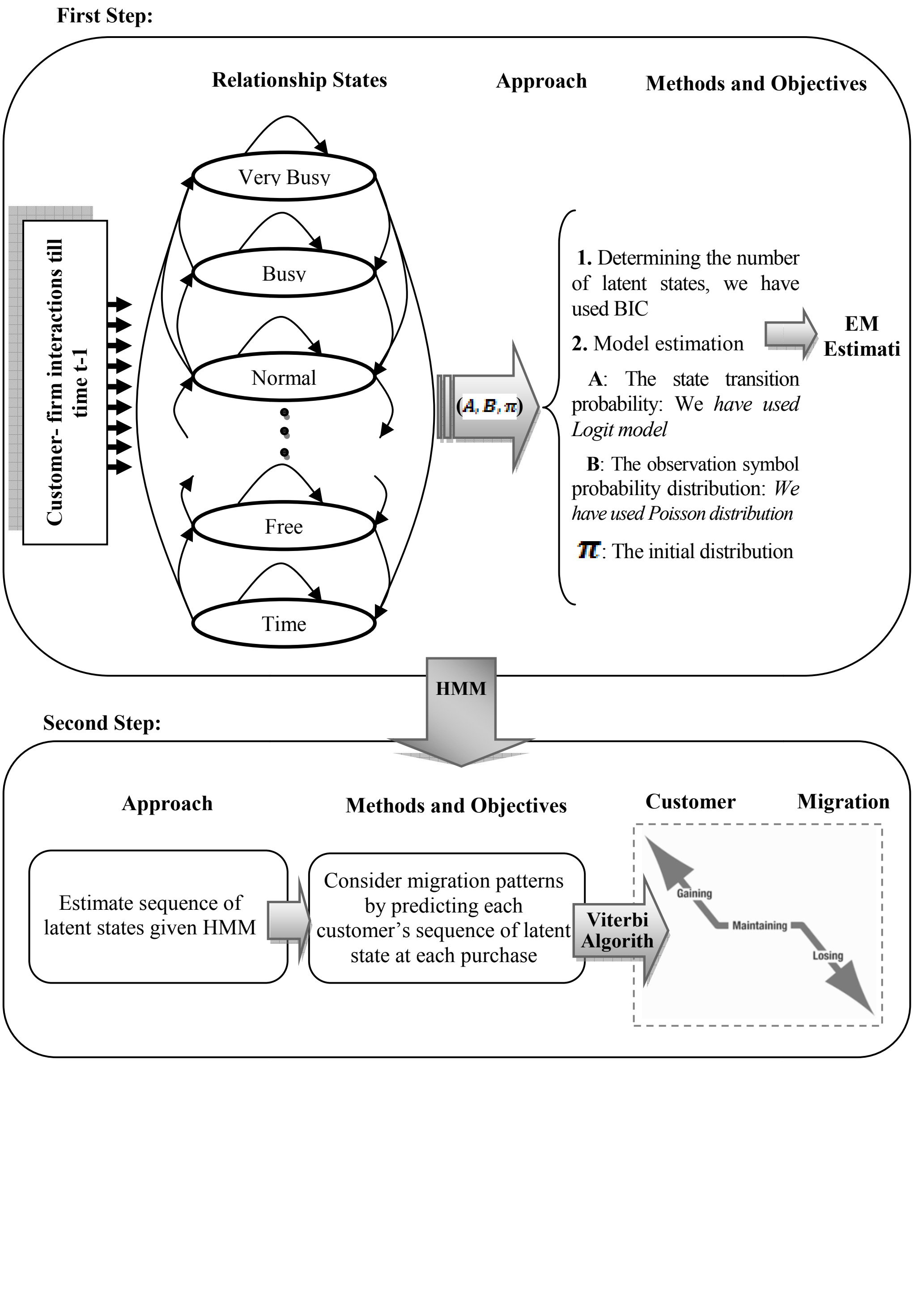
  $$b_{ij} = P\{X_i = x_i | C_j = j\} = -\lambda_j exp\,(-\lambda_j x_i) \quad (1)$$

  Where $X_{\mathrm{i}}, \mathrm{i} = 0,1, \dots, \mathrm{m}$, denotes the observed sequence of customer purchase time in time t and C_{j}, $\mathrm{i} = 0,1, \dots, n$ denotes the hidden Markov chain at time t with n states. Therefore, the HMM has a exponential distribution where the initial rate, λ , is the mean purchase time across customer's base. Because we are interested in shopping behavior of customers over time, the exponential distribution is well suited. Also the aim of considering the purchase time is specifying busy customer's features.
- The transition probabilities represent the probability of a customer migrating from one relationship state to another at next purchase occasion based on current period's buying behavior. It can follow a logit model such as below:

  $$a_{ij} = P\{C_{it+1} = k | C_{it} = j\} = \frac{exp(z\beta_j)}{\left[1+\sum_{h=1}^{J} exp(z\beta_h)\right]} \quad (2)$$

  Where $z_{ij} = (1, z_{ij1}, \dots, z_{ij}\mathrm{t})$ is a vector of time-varying buying behavior covariates for customer i and β is the vector denoting the effect of the covariates on transition to another latent state.
  The purpose of using logit model in estimating HMM's parameters is proven by the application in modeling transition probabilities and also it counts in calculating the probability of belonging to several states.
- Initial distribution also can be computed based on firm's nature.

3. To determine the number of latent states, the log-likelihood (LL) can be used. This criterion is a conservative statistic due to its penalizing in proportion to the number of parameters estimated. The LL has been widely used for model identification in time series. Also it can be applied quite widely to any set of maximum likelihood-based models.



4. To ensure the model converged to the local maximum likelihood is the closest way to the global maximum, so it is essential to have suitable initial estimates of the model parameters. The pervious sections have provided these essential and prepared circumstances for final estimation by the expectation-maximization (EM) algorithm. This is an iterative method for performing likelihood estimation which has a proven usage in estimation of HMM's parameters (Zucchini & McDonald, 2009).

Second Step:

In this step, in order to get marketing results from the estimated model, we suggest to perform it by estimating the sequence of latent states trough considering migration patterns. It is practical by executing Viterbi algorithm in predicting each customer's sequence of latent state at each purchase. Viterbi algorithm is a common method in estimating best state sequence.

Model Estimation

Data

The data for this research are from an Iranian online retailer. The retailer sells both apparel and household goods only through internet. To calibrate the model in busy customer portfolio management regarding to suggested framework, data were collected over a two year period. The data include daily transaction records for each customer. Due to data restriction only the numbers of catalogues are included in the analysis. After data preparation, we started working on approximately four hundred records. Table 1 shows the summery of descriptive characteristics.

Table 1. Customer Characteristics

Characteristics	Min	Average	Max
Number of Purchase	1	2.825758	8
Purchase Amount	35000	1019602	25020000
Time Spent	15	62.36111	109
The Number of Catalogues	0	5.113636	10

First Step

Determining the number of latent states

The first parameter of the HMM is the number of states. Because there is no theoretical guidance to influence the number of states for this model, log-likelihood is assessed to determine the number of states. According to two-state likelihood model, three-state one best fits the data. Also the models with more than three steps were not investigated due to the aim of the research. The focus is on distinguished portfolio for busy customer whereas investigating more-than-three-step portfolios can lead into sub-portfolios which cannot show the goal of the research obviously. Moreover, investigating more-than-three-state frameworks can be done in further researches

Table 2. Log-likelihood for model selection

Number of States	Log-likelihood
2	897.2
3	**2196.818**

Due to this result we can label these three states as "free customers" for state one, "normal customers" for state two and "busy customers" for statethree .

Starting values of transition probability matrix

The probabilities were estimated by counting the number of customer transition from state *i* to state *j* and dividing it by the number of transitions from state *i* to any state (Rabiner, 1989). See Table 3 for these starting values.

Table 3. Starting values for transition probability matrix

	State 1	State 2	State 3
State 1	0.89	0	0.11
State 2	0.11	0	0.89
State 3	0.45	0.2	0.35

Estimating observation symbol probability distribution

Another important parameter which should be estimated is λ. It is the mean purchase time for each state. Using the segmentation result from *k*-mean algorithm, the starting values for lambda are 63.3 for state one, 67.07 for state two and 60.8 for state three. The mean purchase time suggests that state three is the most valuable to the firm, in relation to state one and two.

Transition intensity matrix

Using *R* language and environment, the HMM parameters were estimated. We have used Baum Welch algorithm, which is similar to expectation-maximization.

One of the most important results is transition intensity matrix. It shows the intensities of transitions between different states and has following characteristics:

1. $q_{ij} = 0$, if there is no connection between i and j, otherwise it would never be zero
2. $q_{ii} = -\sum_{j=1}^{n} q_{ij}$

Before analyzing the transition intensity matrix for this case we should see the results of transition matrix in Table 4.

Table 4. Transition probability matrix

	State 1	State 2	State 3
State 1	0.9388371	0.005073453	0.5608945
State 2	0.4470133	0.144398977	0.40858776
State 3	0.5819617	0.058784945	0.359225337

In this case the transition intensity matrix was estimated and the result can be seen in Table 5.

Table 5. Transition Intensity Matrix

	State 1	State 2	State 3
State 1	-0.09891	0	0.09891

State 2	0.2336	-2.348	2.114
State 3	1.005	0.3071	-1.312

It shows that the rate of transition to state one is higher than other states, thus it demonstrates that there is a positive inclination toward becoming free customer. On the other hand, the transition intensity from state two to state three is high and it shows that a considerable number of normal customers are tending to become busy in next period.

Therefore, it shows the need of further planning for normal customers to transpose them to the most valuable firm's customers.

The effect of marketing activities on membership to the latent states

The type of HMM we have investigated here is non-homogeneous. In this HMM the transitions between the states are a function of time-varying covariates such as the number of catalogues or the time which have been spent by customers.

Table 6 shows the effect of catalogue numbers received by customers in transition intensity through states.

Table 6. The impact of catalogue numbers on transition

	State 1	State 2	State 3
State 1	0	0	-1.045
State 2	-0.869	0	0.3395
State 3	0.6523	0.7678	0

This table shows that the highest effect is on customers who are moving from state three to state two. It means that the numbers of catalogues decrease the number of valuable customers and it is because of the low quality of catalogues that not only retained the busy customers in their states but also let them to migrate to a less valuable state.

Table 7 illustrates the effect of purchase time on transition. As what we expected, the highest impact of purchase time is on the transition between state two to three. The firm can achieve beneficial advantages by offering time-dependant services to normal customers who can slowly change to busy ones.

Table 7. The impact of purchase time on transition

	State 1	State 2	State 3
State 1	0	0	0.3961
State 2	0.4224	0	0.6374
State 3	0.207	-0.3554	0

Second Step

In this step we estimated the sequence of latent states for each customer by using Viterbi algorithm.

Seventy four out of one hundred and ninety eight customers are tending to migrate (equal to 37%), so it shows that most of the customers like to remain on their current position.

By a precise investigation on estimated sequence we see that most of migrations to state two are from state one, so the probability of becoming busy in free customers are high.

Another important result of Viterbi algorithm is the increase in the number of busy customers at the end of second year. So, the firm can plan more effectively by these kinds of visions. By analyzing the migration pattern among customers we could define three kinds of pattern: increasing (1-2, 1-3 and 2-3), decreasing (2-1, 3-1 and 3-2) and stable (1-1, 2-2 and 3-3). Table 8 shows the frequency of customers in each pattern.

Table 8. Frequency of migration patterns

Migration pattern	Frequency of pattern
1-1	237
2-2	1
3-3	2
1-2	15
1-3	17
2-3	0
2-1	9
3-1	17
3-2	4

Conclusion

According to this research, application of Markov models in CRM has attracted the attention of researchers. Forty four articles which are precisely related to the desired subject and published in 2011 and earlier prove the above claim.

The aim of this paper was investigating the application of HMM in busy customer portfolio management. Thus, paying attention to the result of literature review can be helpful: only 27.2% of investigated articles had used HMM as their main modeling technique in CRM, it is clearly lesser than the application of other kinds of Markov models. We believe HMM, notwithstanding its complicated estimation method can be used more forcefully in CRM especially with more attention to CPM dimension. Also we proposed a two-step framework for estimating a hidden Markov model for busy customer portfolio management, it consists of:

1. The application of exponential distribution in calculating the observation symbols probabilities.
2. Usage of logit model for estimating transition probability matrix.
3. Estimating the HMM parameters by EM algorithm.
4. State sequence estimation by Viterbi algorithm.

We implemented both two steps on the real data and tried to analyze their result. This research showed that how HMM can be useful for modeling new customer's behavior which was not considered before.

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