The Impact of Bullwhip Effect in a Highly Volatile Market

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Abstract

The bullwhip effect plays an important role in supply chain management especially in a highly volatile market where prices change due to many unexpected reasons brought about by different phenomenon such as global warming. Traditionally, one may expect a reduction on demand when there is a significant move on market price. However, the recent changes on global economy imply that the demand for a particular product may significantly increase as the price goes up in short time and it will come down in long run. There are many evidences to confirm this theory and as an example we could study the behaviour of price and demand for rice in September, 2008 in Iran's economy. We present a mathematical model where demand is not only affected by price but also is influenced by the speed of price changes. Our model behaves identical the traditional demand model, where demand is only a function of price and price elasticity, when price rise is sluggish. However, in the event that there is a big shock in market price, the model has completely different attitude. The proposed model examines the bullwhip effect using the Lyapunov exponent.

Keywords: Bullwhip effect, Lyapunov exponent, Supply chain, Price Fluctuation

Introduction

The Bullwhip effect is a performance index for the instability in a supply chain [1]. In a supply chain when the variation of the orders received by the supplier is greater than the variation of demand observed by the customer, we can face the bullwhip effect [2]. End users form the demand for the last level in the supply chain, but the demand for upstream levels is formed by the levels in the immediate downstream supply chain. The demand seasonality and forecast error can increase as we proceed up the supply chain. The bullwhip effect is a demand distortion and can create inefficiencies for upstream levels of supply chain [3]. Demand amplification is not a new phenomenon, and its existence has been recorded in the start of the 20th century, especially in economy. The bullwhip effect can be very costly in terms of capacity and stock-out [4].

The most important reasons of the bullwhip effect can be listed as: Demand forecast updating, order batching, price fluctuation and rationing and shortage gaming. Demand forecast updating is the demand amplification caused by the safety stock and long lead time. Order batching causes surges in demand at a particular time period. Price discount or promotion can cause price fluctuation that modifies the buying pattern of customers and creates undesirable variations in demand. Irregular behaviours occurred in both buyers and supplier's part causes the bullwhip effect. Also the information and material delays might be the causes of the bullwhip effect [5,6]. It is shown that by implementing Vendor Managed Inventory (VMI) system, both rationing and shortage gaming effect can be completely eliminated [7,8]. The use of VMI system also can reduce the impact of price variations or the promotion effect on the bullwhip effect. Taylor [9,10] introduced the supply variability as a possible cause of the bullwhip effect. Supply variability includes machine reliability and quality problems [11]. One of the most important reasons for bullwhip is the wrong information flow across the chain. Metters [3] studied the information distortion from the end to the beginning of the supply chain. Zhenxin [12] studied the

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effects of the information sharing among the participants in the supply chain. To reduce the negative effect of the bullwhip effect, Lee [13] categorize the topics as follows,

1) Reducing the information distortion from the end to the beginning of the supply chain.
2) Information sharing among the participants for the supply chain.
3) Introducing distributed controls among the supply chain, and reducing the uncertainty.

Geary [4] introduced 10 principals about bullwhip reduction: control system principle, time compression principle, information transparency principle, echelon elimination principle, synchronization principle, multiplier principle, demand forecast principle, order batching principle, price fluctuation principle and gaming principle. Lu [14] studied a nonlinear model for the bullwhip effect for order up to policy based on demand signal policy and analyse the complexity of the bullwhip effect in a supply chain. Miragliotta [15] have a complete review on bullwhip effect literature. Based on this review, some conclusions about the opinions of two schools of thoughts are extracted. The system thinking school views the bullwhip effect as an irrational reaction to a complex system and suggests teaching and training. On the other hand, the operations managers school, views the bullwhip effect as rational reaction to isolated and well perceived factors. Ozelkan [16] analysed the impact of procurement price variability in the upstream for a supply chain on the downstream retail prices. They used a game theory framework to model a serial supply chain and analysed the price variability which occurred in the upstream of the supply chain and showed that this variability could be amplified under some certain scenarios. Because of the reverse direction of price variability, compared to the direction of bullwhip effect in order variability, they named it reverse bullwhip effect in pricing (RBP). In RBP it is assumed that by augmenting the price, the demand will be diminished [16]. Ma [17] studied the behaviour of a supply chain system with a retailer and customer. In their model, a discount rate is offered by the retailer when the demand increases based on a basic level. Their analysis shows a chaotic behaviour and the bullwhip effect in the supply chain. Pan and Sinha [18] consider financial markets as complex systems in non-equilibrium steady state that one of whose most important properties is the distribution of price fluctuations. They show that the price fluctuations in the Indian stock market have a distribution that is identical to that observed for developed markets (*e.g.*, NYSE). They represent the self-organization of price fluctuation distribution in stock market as an important component of complex systems. There are many methods to quantify the bullwhip effect [19]. Warburton [20] uses control theory and solves the fundamental differential delay equations for a retailer's inventory reacting to a surge in demand. Bradley [21] develops model to describe the performance of supply chains based on their elasticity's of supply and demand. The model predicts the supply chain's ability to respond to supply interruptions, cost increases, and demand shifts, and can quantify the degree to which it is prone to the bullwhip effect. Hsieh et al [1] apply the bootstrap technique to analyze the bullwhip effect in supply chain. In this paper the method introduced by Makui and Madadi [22] using Lyapunov exponent, have been used. Alexander Mikhailovich Lyapunov [23] introduced a method to measure the rate of convergence between two orbits, one been perturbed. The quantity obtained, named Lyapunov exponent gives important information on the system's behaviour. When Lyapunov exponent is less than zero, this means that the system is insensitive to initial conditions, a value greater than zero means that the system tend to go away from the stable attractor and has a sensitive dependence on initial conditions.

Problem statement

Raisudin and Bernard [24] have a study on rice price fluctuation in Bangladesh. They examine the sources and extend of rice price variability in Bangladesh and provide a framework for the implementation of an effective and simple mechanism to limit the variability in future. In their report the main causes of variation are shown to be the interactions between demand, supply and import. In the September 2008, the rice market in Iran faced a new phenomenon. As suggested in the literature, by augmenting the price of a good, its demand will be diminished. But in a short interval of time, the price of different types of rice rise suddenly and simultaneously the demand of rice rises with a considerable speed. This phenomenon is not consistent with the traditional price and demand elasticity theorems and it becomes our main incentive for this research. Our studies show that the demand for essential and critical goods is not only dependent on the price, but also it has a considerable correlation to the elevation speed of price. It is obvious that the reason can be found in psychological behaviours of customers. In this situation, a new mathematical function is developed to simulate the relation between demands, price and the speed of price elevation. When the price elevation speed is low or zero, the proposed function's behaviour is like the classical iso-elastic function and when the price rises, the demand will be diminished. But when the speed is high enough, the behaviour of the proposed function becomes different from classical one. When the price rises, the demand also rises.

Consider demand (*D*) for a product which is an exponential function of price (*p*), velocity (*v*) of price with the following function,

$$D = (ab^p P^v) * D_0, \qquad (1)$$

Where *a* and *b* are assumed to be nonnegative arbitrary numbers. The following is an example graph which shows the behaviour of the function,

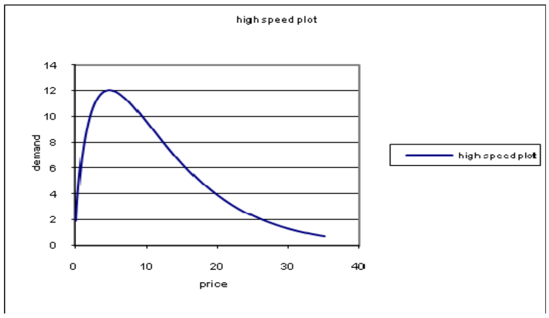

Figure 1: The change on price in highly volatile market price

With a= 8.4341, b= 0.874 and v= 0.6398. As we can observe, for $0.1 < p < 5$, the demand is increasing, but for $p > 5$, the demand begin to diminishes. Now assume v= 0.05 meaning a low speed then the function has different behaviour which is shown in the following graph,

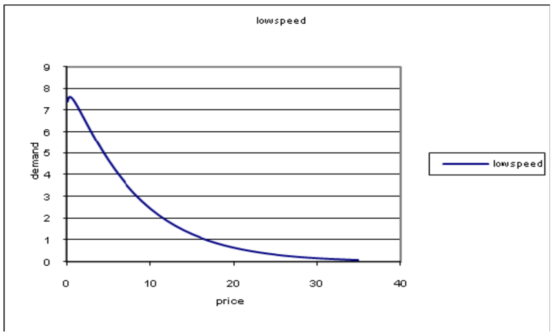

Figure 2: The price change in sluggish price change

Now consider a simple example where the initial price is $P_0=1$ and for this price the basic demand is $D_0=100$. let a=10 and b=0.1 and v=3. Therefore we have,

P=1 → D=100
P=1.1 → D=105
P=1.2 → D=109
P=1.5 → D=106
P=1.6 → D=102
P=2 → D=80
P=2.5 → D=49.4

As we can observe, equation (1) is sensitive to the speed of the price variation. This phenomenon causes a secondary event that is the bullwhip effect. The important note is that the reverse pricing effect firstly takes place and then, the bullwhip effect occurs in the supply chain. In the next section we will analyse these effects mathematically.

Mathematical Analysis

Assume a supply chain with *N* agents. Let q^k be the order received by agent *k* from the agent (*k-1*) and L_k is the ordering lead time for agent *k*. We assume that the forecasting method is a moving average method with *p=1*. Taylor expresses the following relationship (Taylor, 2000).

$$\frac{\text{var}(q_k)-\text{var}(q_{k-1})}{\text{var}(q^1)-\text{var}(D)}=\frac{(L_k)^2+L_k}{(L_1)^2+L_1} \quad (2)$$

The reverse pricing generates the bullwhip in price and its direction is from the end of the chain to the customer. Thus,

$$D_k=(a.b^p.p^v).D_{k-1} \quad (3)$$

The speed of the price variation in the time interval Δt is as follows,

$$v=\frac{p_k-p_{k-1}}{\Delta t} \quad (4)$$

In the supply chain, D_k is the order quantity of agent *k*.

$$q_k=(a.b^{p_k}.p_k^{\frac{p_k-p_{k-1}}{\Delta t}})q_{k-1} \quad (5)$$

Therefore we have,

$$q_k-q_{k-1}=\text{var}(q_k)= q_{k-1}(a.b^{p_k}.p_k^{v_k}-1) \quad (6)$$

By inference, equation (6) can be written as:

$$q_{k-1}-q_{k-2}=q_{k-2}(a.b^{p_{k-1}}.p_{k-1}^{v_{k-1}}-1) \quad (7)$$

$$q_{k-2}-q_{k-3}=q_{k-3}(a.b^{p_{k-2}}.p_{k-2}^{v_{k-2}}-1) \quad (8)$$

…

$$q_2-q_1=q_1(a.b^{p2}.p_2^{v2}-1) \quad (9)$$

$$q_1-D_0=D_0(a.b^{p1}.p_1^{v1}-1) \quad (10)$$

D_0 is the demand implemented by the customer. So we have:

$$q_1=a.b^{p_1}.p_1^{v_1}.D_0 \quad (11)$$

$$q_2=a^2.b^{(p_1+p_2)}.p_2^{(v_1+v_2)}.D_0 \quad (12)$$

$$q_3=a^3.b^{(p_1+p_2+p_3)}.p_3^{(v_3+v_2+v_3)}.D_0 \quad (13)$$

Therefore we have,

$$q_k=a^k.b^{\sum_{j=1}^{k}p_j}.p_k^{\sum_{j=1}^{k}v_j}.D_0, \quad (14)$$

and

$$q_k-q_{k-1}=[a^k.b^{\sum_{j=1}^{k}p_j}.p_k^{\sum_{j=1}^{k}v_j} -a^{k-1}.b^{\sum_{j=1}^{k-1}p_j}.p_{k-1}^{\sum_{j=1}^{k-1}v_j}].D_0 \quad (15)$$

and

$$q_1-D_0=a.b^{p_1}.p_1^{v_1}.D_0 \quad (16)$$

Which yields,

$$\frac{q_k-q_{k-1}}{q_1-D_0}= \frac{a^{k-1}.b^{\sum_{j=1}^{k-1}p_j}.p_{k-1}^{\sum_{j=1}^{k-1}v_j}(a.b^{p_k}.p_k^{v_k}-1)}{a.b^{p_1}.p_1^{v_1}} \quad (17)$$

$$=\frac{1}{p_1^{v_1}}[a^{(k-2)}.b^{\sum_{j=2}^{k-1}p_j}.p_{k-1}^{\sum_{j=1}^{k-1}v_j}.(a.b^{p_k}.p_k^{v_k}-1)] \quad (18)$$

Equation (18) can be interpreted as:

$$\frac{\text{var}(q_k)}{\text{var}(q_1)}=e^{\lambda'.k} \quad (19)$$

Where λ' the Lyapunov exponent is created by the bullwhip effect and can be set to the Lyapunov exponent created by the relation (1).

$$\lambda'=\frac{1}{kp_1^{v_1}}\ln[a^{(k-2)}.b^{\sum_{j=2}^{k-1}p_j}.p_{k-1}^{\sum_{j=1}^{k-1}v_j}(ab^{p_k}.p_k^{v_k}-1)] \quad (20)$$

The condition for having the bullwhip effect is that $\lambda'>0$, so:

$$a^{(k-2)}.b^{\sum_{j=2}^{k-1}p_j}.p_{k-1}^{\sum_{j=1}^{k-1}v_j}(a.b^{p_k}.p_k^{v_k}-1)>1 \quad (21)$$

Numerical Example

Consider a supply chain with four stages as follows:

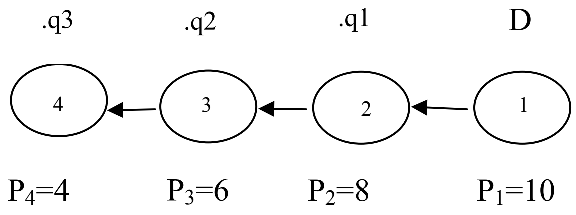

Figure 3: A supply chain with four stages

Where p_1,p_2,p_3 and p_4 are the prices in stages 1 to 4 for the supply chain. The relation between q_k and q_{k-1} is supposed to be as follows:

$$q_k = [8.4341 * (0.874)^{p_k} * p_k^{\,2}] * q_{k-1} \qquad (22)$$

and the demand (D) is obtained as follows:

$$D = 8.4341 * (0.874)^{p} * p^2 \qquad (23)$$

Where p is assumed to be a random uniform variable. The results of computations are represented in appendix. For instance the results of q_4 are plotted versus q_4 (n th value of q_4 versus the (n-1) th value of q_4). In this case the value of λ' will be 0.043 which is greater than zero that is the sign of being bullwhip.

Conclusion

By augmenting the price of a good, we expect that its demand will be diminished. But in a short interval of time, the price of some goods rise suddenly and simultaneously their demand rises with a considerable speed. It is obvious that the reason must be in psychological behaviours of customers. This fact is not consistent with the traditional price and demand elasticity theorems. We have presented a new mathematical function where demand for a price is influenced by price and price velocity. When the price elevation speed is low or zero, the proposed function's behaviour is like the classical iso-elastic function and when the price rises, the demand will be diminished. But when the speed is high enough, the behaviour of the proposed function becomes different from classical one. When the price rises, the demand also rises. The proposed model could explain the turmoil on market price and demand fluctuations for some products such as rice in some countries like Iran where demand and price had similar trend in short run and opposite trend for long run. The bullwhip effect has also been studied for the market where our proposed model could explain the behaviour of the changes on price and demand. It is shown that the variation of price causes the variation in demand and in fact we are faced a positive feedback between the price and demand. While these two factors amplify each other, the bullwhip occurs in both price and demand**.**

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Appendix:

The computations of a numerical example of a supply chain with four stages:

p	D	q1	q2	q3	q4
11.142	233.4991877	51219.87	6024605	257978491	1783365671
8.622341	196.3273239	43065.93	5065519	216909648	1499463074
9.636627	213.9224416	46925.56	5519498	236349382	1633846963
2.813828	45.71481819	10027.9	1179506	50507413	349149983
12.57638	245.2311586	53793.37	6327307	270940404	1872969384
11.02782	232.282606	50953	5993216	256634366	1774073947
6.763404	155.1627941	34036.17	4003417	171429562	1185066223
7.359987	169.5577889	37193.82	4374828	187333681	1295008959
7.335901	168.9971371	37070.84	4360362	186714253	1290726943
11.66803	238.5544295	52328.77	6155038	263563708	1821975418
7.792673	179.3202737	39335.3	4626714	198119634	1369570590
15.71958	250.9024448	55037.41	6473634	277206249	1916284212
10.95969	231.5360998	50789.25	5973955	255809599	1768372456
7.471405	172.1279942	37757.62	4441143	190173339	1314639074
13.51904	249.5865294	54748.75	6439681	275752377	1906233819
6.232482	141.5245743	31044.52	3651532	156361555	1080903407
11.73868	239.1655804	52462.83	6170806	264238930	1826643124
8.048343	184.8058131	40538.6	4768248	204180259	1411466764
7.955233	182.8329124	40105.82	4717345	202000526	1396398603
11.60518	237.9973763	52206.58	6140665	262948255	1817720887
12.7531	246.2406481	54014.81	6353353	272055725	1880679427
7.746074	178.2975087	39110.95	4600325	196989645	1361759154
10.82773	230.0462932	50462.45	5935516	254163606	1756993958
10.43144	225.220196	49403.81	5810996	248831556	1720134317
7.902805	181.7094646	39859.39	4688358	200759300	1387818195
11.08986	232.9489874	51099.18	6010409	257370609	1779163480
9.544892	212.4778843	46608.68	5482226	234753381	1622814059
9.802608	216.4619237	47482.61	5585020	239155095	1653242427
8.726091	198.2903913	43496.54	5116169	219078517	1514456132
10.88199	230.6658866	50598.36	5951502	254848155	1761726144
9.296512	208.4198743	45718.52	5377524	230269943	1591820737
15.01659	251.6996179	55212.27	6494202	278086995	1922372674
10.81629	229.9144025	50433.52	5932113	254017888	1755986634
6.390795	145.6666395	31953.11	3758403	160937861	1112538707
11.16201	233.707971	51265.67	6029992	258209162	1784960268
15.1627	251.6211998	55195.07	6492179	278000356	1921773751
10.17486	221.8107553	48655.92	5723027	245064681	1694094487
7.167254	165.0220204	36198.87	4257799	182322398	1260366723
8.172325	187.3882778	41105.08	4834879	207033462	1431190511
5.646499	125.701835	27573.68	3243283	138880011	960056176
7.003156	161.0726394	35332.54	4155899	177958976	1230203062
10.5009	226.1044323	49597.77	5833810	249808493	1726887732
9.484903	211.5174245	46398	5457445	233692230	1615478484
12.90196	247.0204915	54185.87	6373474	272917324	1886635534
8.850844	200.6018559	44003.58	5175808	221632309	1532110098
16.40888	249.1522027	54653.48	6428475	275272517	1902916620
15.91503	250.4992213	54948.96	6463230	276760753	1913204565
9.909448	218.0460722	47830.11	5625893	240905320	1665341467
10.05359	220.1207752	48285.21	5679424	243197528	1681187150
7.667392	176.554702	38728.65	4555358	195064128	1348448351
15.51814	251.2374046	55110.88	6482276	277576325	1918842490
2.578523	39.62476631	8692.001	1022374	43778899	302636804
11.85649	240.1496621	52678.7	6196197	265326179	1834159114
9.405057	210.2196532	46113.32	5423961	232258405	1605566668
5.423568	119.5066909	26214.72	3083440	132035388	912740349
p	D	q1	q2	q3	q4
10.66133	228.0844093	50032.09	5884896	251996044	1742009938
9.402247	210.1735718	46103.21	5422772	232207492	1605214718
14.64598	251.6826235	55208.55	6493763	278068219	1922242879
6.534286	149.3667807	32764.77	3853872	165025913	1140798782
9.672203	214.474797	47046.72	5533749	236959644	1638065614
16.41341	249.1377512	54650.31	6428102	275256551	1902806246
12.2445	243.0846742	53322.52	6271924	268568889	1856575466
15.21389	251.5826359	55186.61	6491184	277957749	1921479216
7.919183	182.0613819	39936.58	4697438	201148111	1390505987
13.0917	247.9225656	54383.75	6396749	273913969	1893525184

7.31644	168.5428394	36971.19	4348641	186212328	1287257214
10.99502	231.9251782	50874.6	5983994	256239467	1771344068
11.64425	238.3450966	52282.85	6149637	263332429	1820376625
9.306416	208.5857945	45754.92	5381805	230453257	1593087963
9.951917	218.6647999	47965.83	5641857	241588913	1670067041
7.52534	173.3583004	38027.49	4472886	191532626	1324035619
6.125048	138.6795551	30420.44	3578126	153218273	1059174382
5.366925	117.9196085	25866.59	3042491	130281921	900618901
10.52185	226.368037	49655.59	5840612	250099733	1728901030
9.022657	203.6973408	44682.6	5255676	225052314	1555752071
14.45808	251.5519064	55179.87	6490391	277923798	1921244518
1.183722	10.07636957	2210.33	259984.4	11132744	76958946.9
5.919457	133.162483	29210.23	3435778	147122809	1017037375
12.82487	246.6246369	54099.04	6363260	272479970	1883612168
9.75474	215.7393541	47324.11	5566377	238356773	1647723754
14.27985	251.3495187	55135.48	6485169	277700193	1919698769
5.000053	107.5330616	23588.22	2774503	118806481	821290953
12.79959	246.4910563	54069.73	6359814	272332385	1882591936
11.25735	234.6847549	51479.93	6055195	259288349	1792420519
8.273797	189.4636704	41560.33	4888427	209326432	1447041461
10.81653	229.9171596	50434.12	5932184	254020934	1756007692
12.76571	246.309207	54029.84	6355122	272131471	1881203051
13.54612	249.6752209	54768.21	6441970	275850367	1906911207
8.304578	190.0863849	41696.93	4904494	210014429	1451797484
11.95421	240.9327679	52850.48	6216402	266191383	1840140137
10.73892	229.0107886	50235.3	5908798	253019542	1749085222
6.920762	159.0600629	34891.06	4103972	175735408	1214831874
7.997872	183.7399496	40304.79	4740747	203002654	1403326160
12.41389	244.2214968	53571.89	6301256	269824892	1865258024
12.21727	242.8938123	53280.65	6267000	268358017	1855117745
7.907491	181.8102487	39881.5	4690958	200870650	1388587940
10.06346	220.2601719	48315.79	5683020	243351539	1682251802
9.069972	204.5318668	44865.66	5277208	225974329	1562125819
6.371791	145.172681	31844.76	3745658	160392117	1108766066