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Depth and shape factor determination of gravity anomalies by linearized least-squares

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Abstract

A new method derived by Salem et al. (2004) for interpretation of gravity and magnetic anomalies is used to determine the depth and shape factor of the gravity anomalies.

The depth and shape factor of some rectangular prisms as synthetic models are estimated using the method. The gravity effects of the models are contaminated with some random noise and then the parameters of the models are extracted through the erroneous data.

A field example is interpreted and the depth and shape factor of the main anomaly has also been estimated through the method.

Key words: Depth, Shape factor, Linearized least-squares

تعیین عمق و فاکتور شکل بی‌هنجاری‌های گرانی با روش حداقل مربعات خطی

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چکیده

روش جدیدی که سالم و همکاران (۲۰۰۴) در تفسیر داده‌های گرانی و مغناطیسی برای برآورد عمق و فاکتور شکل بی‌هنجاری‌های گرانی به کار برده است.

عمق و فاکتور شکل مدل‌های مکعبی درحکم بی‌هنجاری‌های مصنوعی با این روش برآورد شده است.

سپس نوفه‌های تصادفی به اثر گرانی این مدل‌ها اضافه شده و پارامترهای مجهول شامل عمق و فاکتور شکل با روش پیش‌گفته برآورد شده است.

در مجموعه‌ای از داده صحرایی نیز این پارامترهای مجهول برای یک بی‌هنجاری اصلی برآورد شده‌اند.

بعضی عمق‌های به‌دست آمده با این روش از راه حفاری‌های محلی نیز تأیید شده‌اند.

واژه‌های کلیدی: عمق، فاکتور شکل، روش حداقل مربعات خطی

1 INTRODUCTION

A variety of methods, based on the use of the derivatives of the gravity anomalies has been developed for the determination of source parameters such as the location of the boundaries, depths and the geometry.

For the depth estimation, they include Werner de-convolution (Hartman et al. 1971), Euler de-convolution (Thompson, 1982), least-square minimization (Abdolrahman, 1990).

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Recently, efforts have been made to identify both the shape and the location of the buried sources, using some approaches like the analytic signal (Nabighian, 1972) and the non-linear least-squares minimization (Abdolrahman and El-Araby, 1993).

As the estimation of both the model type and the depth from potential field data is a non-linear problem, solving these problems for more than one parameter is generally difficult. Salem et al. (2004) presented a new method to estimate these model parameters based on having isolated anomalies with symmetric field variation about the center of the source. They developed a linearized least squares method involving the symmetric field and its horizontal derivatives.

2 SOURCES OF SYMMETRIC FIELD IN GRAVITY

In gravity, fields of many simple bodies are symmetric about the location of the source. For example, the general gravity effect *g* caused by simple models (such as a sphere, an infinite horizontal cylinder and a semi-finite vertical cylinder) which is given by Abdolrahman (1990) as,

$$g(x) = \frac{A}{(x^2 + z^2)^q} \qquad (1)$$

where q is called shape factor characterizing the nature of the source (q=1/2 for a vertical cylinder, q=1 for a horizontal cylinder and q=3/2 for a sphere) and A is an amplitude factor related to the radius and density contrast of the source.

3 THE METHOD

Taking the horizontal derivative of equation (1) yield,

$$\frac{\partial g(x)}{\partial x} = \frac{-2qA}{\left(x^2 + z^2\right)^{q+1}} \qquad (2)$$

Then considering equations (1) and (2), the relation of the field and its horizontal derivative will be,

$$\frac{g(x)}{\frac{\partial g(x)}{\partial x}} = \frac{x^2 + z^2}{-2qx} \qquad (3)$$

With a simple rearrangement of equation (3), we obtain (Salem et al.2004),

$$x^2 \frac{\partial g(x)}{\partial x} = -2qxg(x) - z^2 \frac{\partial g(x)}{\partial x} \qquad (4)$$

equation (4) is a linear equation which can be solved numerically for estimation of depth (z) and shape factor (q).

When information about the nature of the source is available (q is known) the depth can be obtained from equation (4) in a least-square sense as,

$$z^2 = \frac{\sum_{i=1}^{N} \left(-2qx_i g(x_i) - x_i^2 \frac{\partial g(x_i)}{\partial x}\right) \frac{\partial g(x_i)}{\partial x}}{\sum_{i=1}^{N} \left(\frac{\partial g(x_i)}{\partial x}\right)^2} \qquad (5)$$

where N is the number of observations around the peak of the anomaly.

4 NUMERICAL PROCEDURE

The gravity effect of the models are computed using Talwani's algorithm (Talwani et al 1959) and then contaminated by random noise generated by following command in MATALA"NORMRND(MU,SIGMA.,[M N]" which return an M by N matrix of random numbers chosen from the normal distribution with parameters MU and SIGMA.

Then the horizontal derivative of gravity effects are computed by three point Lagrangian operator using MATLAB.

Having the gravity effects and the horizontal derivatives the depth and shape factor could be obtained from equation (4) and any conventional method of linear matrix inversion.

5 SYNTHETIC MODELS

Rectangular prisms with different depth are applied as synthetic models.

The gravity effect of the models are computed by Talwani's algorithm. Then random noises are added to the gravity effects. The models and their gravity effects of them with and without noises are demonstrated in figurs 1-4.

Equation (4) and (5) are applied to estimate the depth and shape factor of the models both with and without the presence of random noises and the results are demonstrated in Table (1).

Z1 and Z2 are the depths computed using equations (4) and (5) by applying on noise free gravity effects and Z3 and Z4 are the same depths with the noisy gravity effects.

Considering the depth of the models, the computed depths are quite acceptable and the small difference could be generated from this fact that the original formula (equation (1)) derived for gravity effects of horizontal and vertical cylinders and sphere and not the rectangular prism.

6 REAL EXAMPLE

A field example is also used to test the capability of the method.

The Bouguer gravity anomalies and the Euler solution of the anomalies are shown in figure 5. The gravity effect of this profile (AB) is demonstrated in figure 6.

The method is applied along the profile (AB). This profile is selected in such a way that it crosses the main negative anomaly in an east-west direction.

Estimating the depth of this negative anomaly and two depths equal to 5.7 and 14.3 meters are computed for the left and right side of the profile respectively. These depths show a good agreement with the Euler solution around the anomaly in figure 5.

Table 1. Depth and shape factor of models

Model	q	Z1(m)	Z2(m)	Z3(m)	Z4(m)
1	2.6	15.9	6.7	16	6.8
2	1.8	15.9	6.92	12.6	7.3
3	.2	-	12.07	-	13.29
4	0.46	6.0	12.7	3.0	12.53

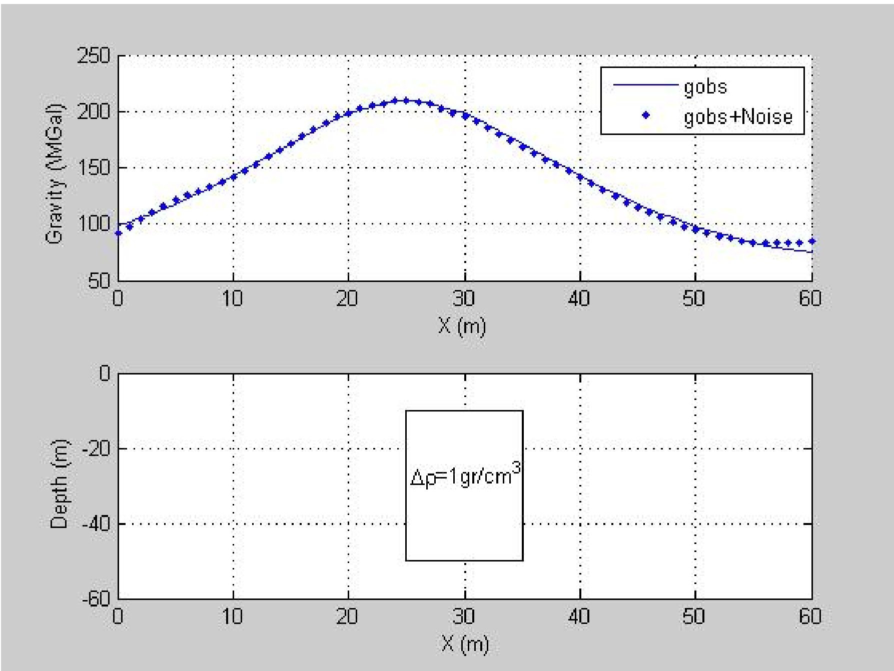

Figure 1. The gravity effects of model in Micro-Gal.

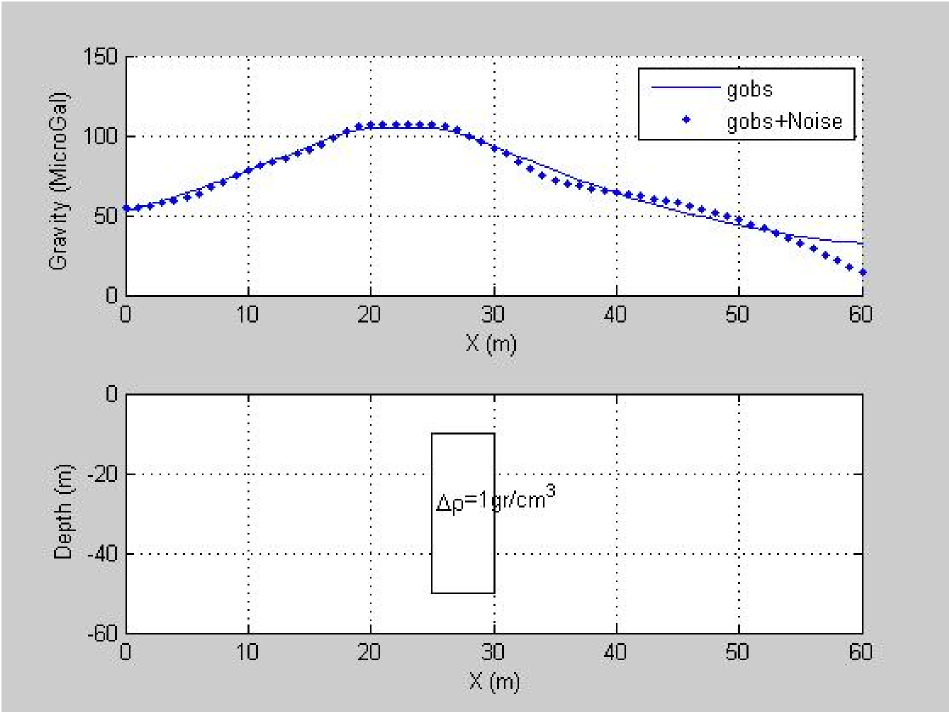

Figure 2. The gravity effects of model in Micro-Gal.

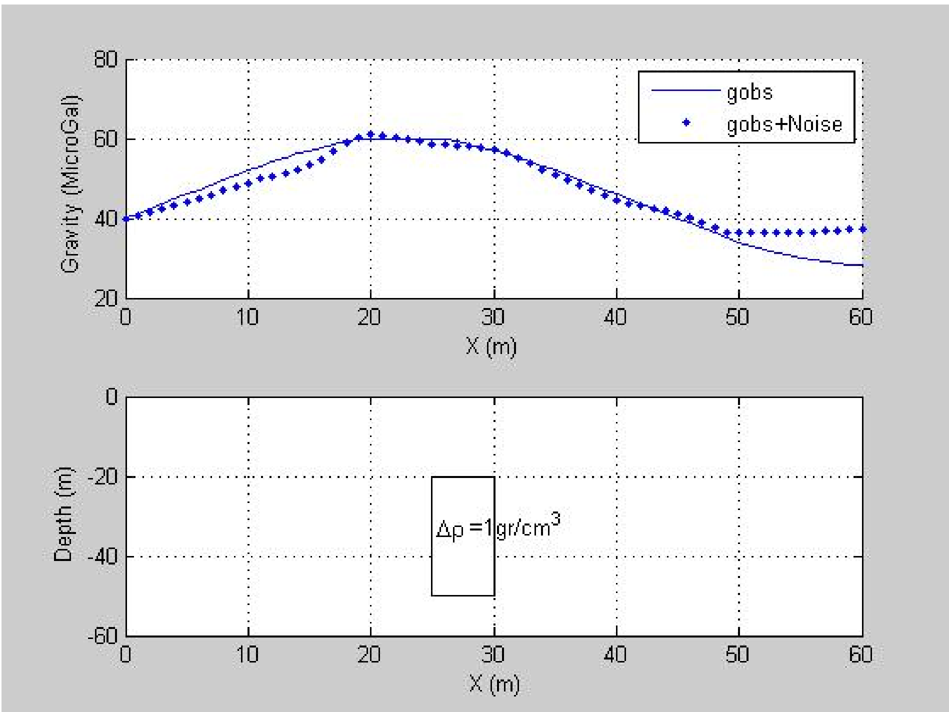

Figure 3. The gravity effects of model in Micro-Gal.

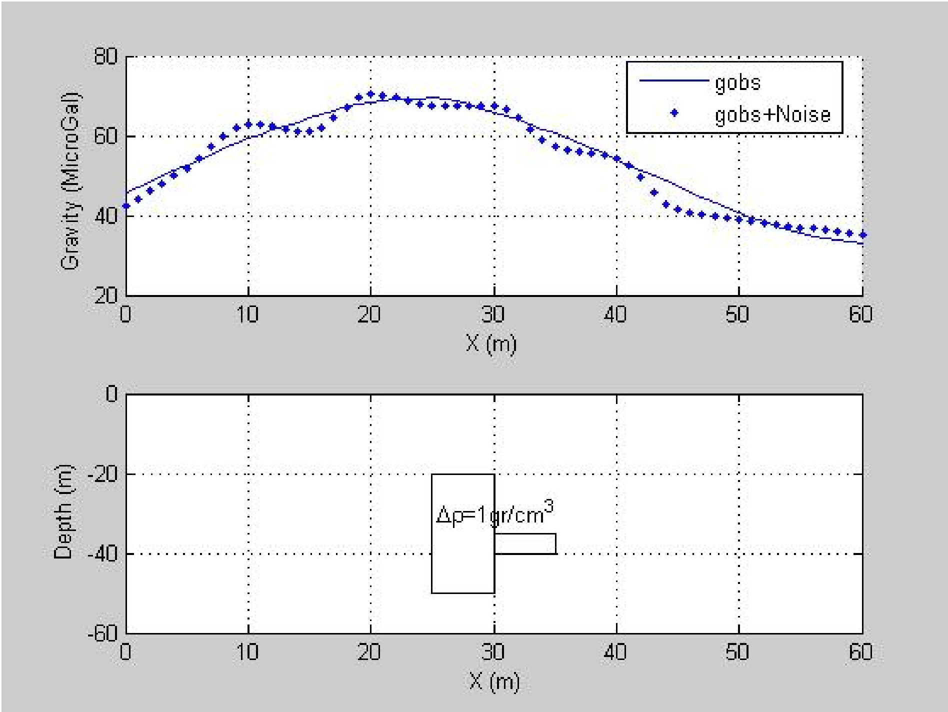

Figure 4. The gravity effects of model in Micro-Gal.

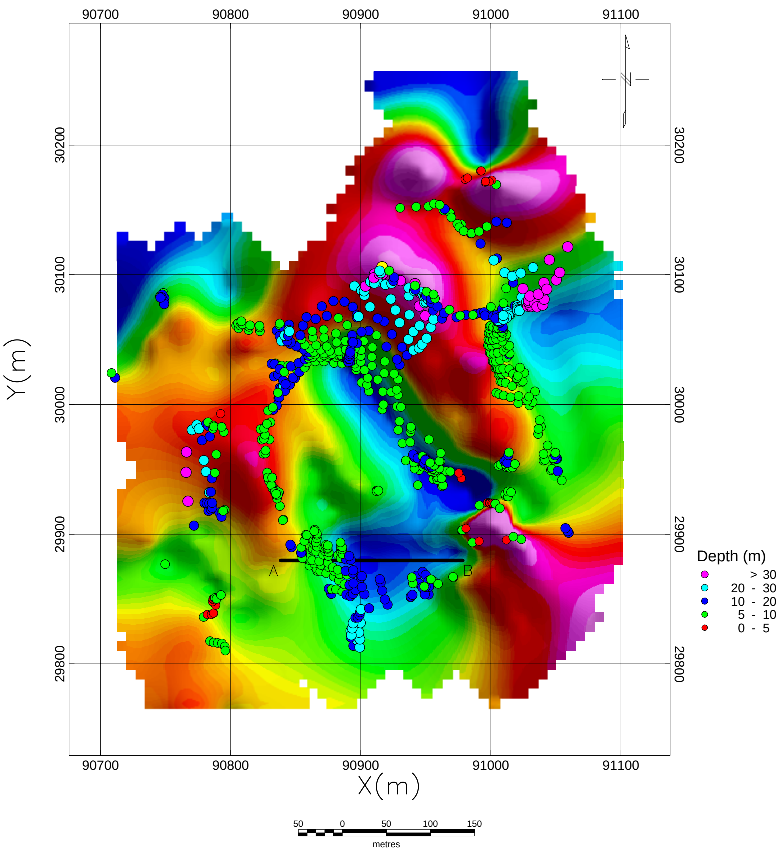

Figure 5. The Bouguer gravity anomalies (mGal) and Euler depths.

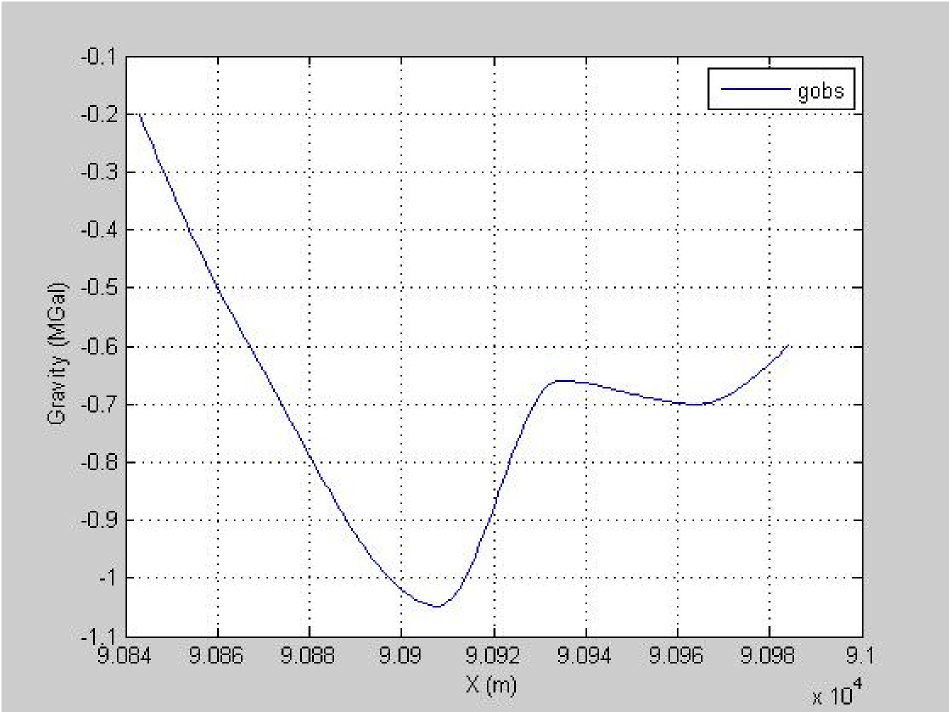

Figure 6. The Bouguer gravity anomalies along profile AB.

7 CONCLUSION

The method which has originally been used for the determination of the depth and shape factor of the magnetic anomalies is quite capable in the case of the gravity anomalies. The method is quite feasible and is applicable with a few numerical computations and works for narrow anomalies such as dikes in the presence of random noises.

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REFERENCES

Abdolrahman, E. M., 1990, A least-squares approach to depth determination from gravity data by O.P. Gupta: Geophysics, **55**, 376-378.

Abdolrahman, E. M., and El-Araby, H. M., 1993, Shape and depth solutions from grsvity using correlation factors between successive least-squares residuals. Geophysics, **59**, 1785-1791.

Hartman, R. R., Tesky, D. J., and Friedberg, J. L., 1971, A system for rapid digital aeromagnetic interpretation. Geophysics, **33**, 891-918.

Nabighian, M. N., 1972, The analytic signal of two-dimensional magnetic bodies with polygonal cross-section: Its properties and use for automated anomaly interpretation, Geophysics, **37**, 507-517.

Salem, A., Ravat, D., Mushuyandebvu, M. F., and Ushijima, K., 2004, Linearized least-squares method for interpretation of potential field data from sources of simple geometry. Geophysics, **69**, 783-788.

Talwani, M., Worzel, J. L., and Landisman, M., 1959, Rapid computations for two-dimensional bodies with application to the Mendocino Submarine fracture zone, J Geophys Res, **64**, 49-59.

Thompson, D. T., 1982, "EULDPH" A new technique for making compute-assisted depth estimates from magnetic data. Geophysics, **47**, 31-37.