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Prediction of Dissolved Oxygen through Mathematical Modeling

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ABSTRACT: Water quality aspect with regard to dissolved oxygen was studied for a 24 kilometer stretch of Malaprabha River in Karnataka State, India. Main objective of the research was to simulate the predicted dissolved oxygen depletion due to the waste load allocation in the river with the ambient observed values of dissolved oxygen, and to ascertain the application of mathematical modeling for predicting the dissolved oxygen in a mixing zone. The entire river stretch selected for the study was divided in to four stations and water samples were collected from each station, and analyzed for different parameters. Station S1 was purely the upstream of the sewage discharge point and hence reflected the water quality of the river without pollution. Station S2 and downstream stations were the part of mixing zone where the discharged sewage at S2 kept spreading. Dissolved oxygen levels at these four stations were calculated through mathematical modeling, considering all variables that affect dissolved oxygen variation. Further, this predicted dissolved oxygen found with the application of mathematical modeling was then simulated with the actual observed values of dissolved oxygen in these four stations. The results were highly encouraging, with the predicted values almost agreeing with the observed values. Thus the application of such mathematical modeling may be useful for a river stretch where water pollution due to the discharge of effluents is expected, and will help the engineers in arriving at a proper waste-load allocation for the river.

Key word: Mixing zone, In-stream, Transects, Sources and sinks,Diffusion factor

INTRODUCTION

Analytical solutions for steady state dissolved oxygen (DO) prediction in mixing zones of rivers are constructed by employing the product law concept. Dissolved Oxygen (DO) is an important water quality parameter affecting the health of a river and hence, great importance is attached to maintain the DO at desirable level. Dissolved oxygen is important to aquatic life, as detrimental effects can occur when DO levels drop below 4 mg/ lt. or 5 mg/ lt. depending on the aquatic species (Chapra, 1997; Thomann and Mueller, 1987).

Oxygen level in Malaprabha River is adversely affected by the discharge of untreated sewage from a town on its bank in to the river. Upstream part of the river water quality before the discharge of sewage is fairly good, and the oxygen level is mainly controlled by the natural forces in this stretch. An effort is made in this paper to predict dissolved oxygen in the mixing zone due to the impact of sewage disposal from the nearby town. Dissolved oxygen of a stream plays an important role in the river eco-system affecting the flora and fauna of the stream (Wang *et al.*, 2003). Generally, a single point measurement of dissolved oxygen by the traditional method is not reliable, and sometimes may be misleading in assessing the state of the river water body. This is due to the fact that it does not reflect the effect of biological activity and also do not reveal extremes

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in dissolved oxygen concentrations. The aim of the study was to arrive at a mathematical solution through modeling which would be able to predict the dissolved oxygen variation with distance. Attention was also focused on the lateral distribution of dissolved oxygen across the river width. The cross section of the river channel within the mixing zone was divided in to two parts, viz., Limited Use Zone or Zone of non-compliance –LUZ and Zone of Passage – ZOP, requiring water quality compliance along designated LUZ boundary, as shown in Fig.2 (Thomann, 1987; Chapra, 1997). The location of the Malaprabha River is as in (Fig.1).

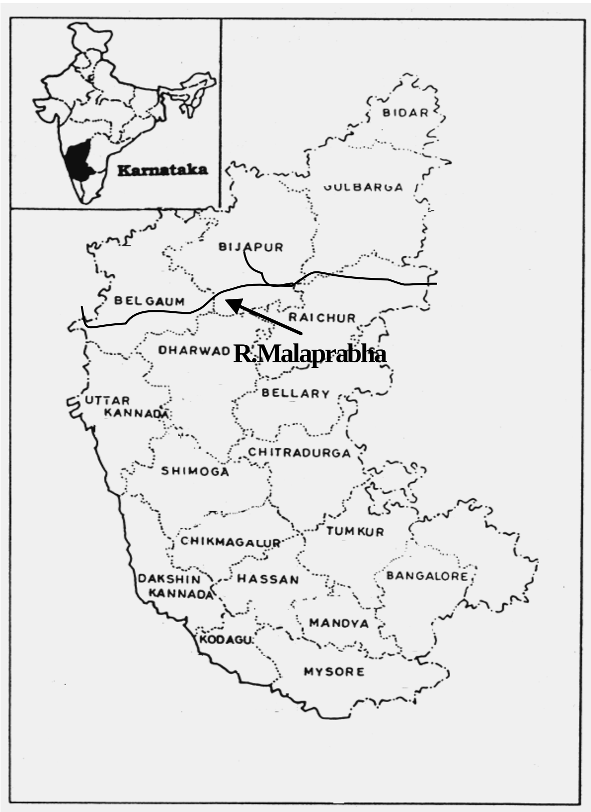

Fig. 1. Location of Malaprabha River

MATERIALS & METHODS

Field tests were conducted from June, 2005 to May, 2006 to monitor the water quality parameters at 4 Stations – S1, S2, S3 and S4 as shown in Fig.2. The study stretch selected was 24 kilometer, starting from its birth place. A part of the stretch which formed the upstream of the sewage disposal point was 5 km, and the downstream stretch from the discharge of sewage was 19 kilometer and sewage from the town is discharged into the stream through a piped outfall located at the left bank (looking d/s), as shown in Fig.2. The channel alignment is fairly straight, but significant variations occur in widths and depths. The upper portion of the stretch is shallow with depths up to 1m, with rocky bottom, whereas the lower portion is deep, up to 2 to 2.5 meter, with muddy bottom. The in-stream and sewage characteristics were analyzed during this period. The method of sample collection, transport to the laboratory, preservation and analyses were all done as per the standard procedures(American Public Health Administration, 1998). The in-stream monitoring was carried out for four to five lateral points at each station. Water temperature and DO were measured on site only during the sampling period. The analyzed result of sewage for important parameters, re-oxygenation and de-oxygenation rate constants, respiration and photosynthesis rates and other hydro-geometric characteristics of the river stretch are presented in Table 1. The decay and re-aeration rates for CBOD and DO, respiration and photosynthesis rates along with sediment oxygen demand – were all found by applying standard methods and procedures (Thomann, 1987; Chapra, 1997). The measured DO profile just at the u/s of sewage disposal and the downstream for S1, S2 and S3 are presented in Fig.3. which exhibit both lateral and longitudinal variations. The DO values along the outfall (left) bank at S1, S2, S3 were found to be lower than those at the opposite bank, displaying increase in the lateral direction, clearly indicating the plume behavior of the waste dispersion. The other stations show relatively uniform distribution of DO laterally. The analytical formulation for 2-D DO model assumes that, steady state conditions prevail; cross sectional distribution of background water quality constituents are uniform, densities of sewage and receiving water are same, far-field concentrations are not affected by near-field mixing processes, longitudinal dispersion effects are negligible. The DO distribution within the mixing zone is affected by pollutant loadings from two principle inputs, i.e. the upstream flow and the sewage effluent discharge. If the initial DO is lower than DO saturation value, there would be initial DO deficit in each input (Morgan *et al.*, 2006). The oxygen sinks considered here, include the two DO deficits, carbonaceous biochemical

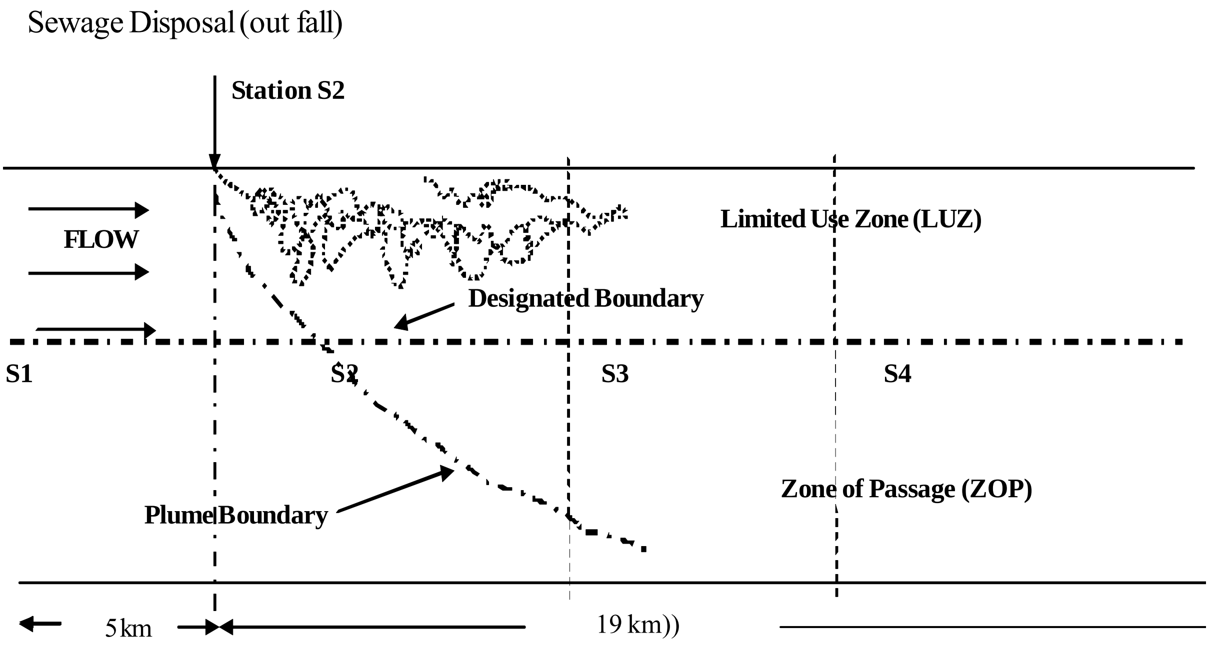

Fig. 2. Mixing Zone of the Study Area

Table 1. Hydro Geometric values for Pre Monsoon Season 2005- 2006

Sl. No.	Parameters	Stations			
		S1	S2	S3	S4
1	Water Temp. C	24.8	25.4	24.7	24.6
2	Saturation DO C_s	8.40	8.12	8.46	8.10
3	Ambient DO C_b	7.40	7.07	7.30	7.40
4	Dist. From origin, x m	5315	11815	19415	24565
5	Flow velocity, u m/s.	0.15	0.21	0.19	0.16
6	Flow velocity, u m/day.	12960	18144	16416	13824
7	Av. stream flow, Q m^3/s	5.28	5.28	5.47	5.47
8	Av. Effluent flow, Qe m^3/s	0	0.19	0	0
9	Background CBOD L_{bo}	0.46	1.18	0.68	0.62
10	Background NBOD N_{bo}	1.55	3.7	2.2	2.0
11	TKN	0.31	0.79	0.46	0.42
12	Nitrate N. NO_2	0.12	0.11	0.08	0.06
13	Re-aeration const K_a	1.06	1.72	2.01	1.81
14	Decay rate, river K_d	0.43	0.46	0.19	0.17
15	Decay rate, efflnt K_d	0.18	0.18	0.18	0.18
16	Decay for Nitrgn Kn	0.48	0.51	0.24	0.22
17	Sediment oxygen demand S	0.48	1.4	3.5	1.5
18	Respiration R	0.2	-1.71	-0.12	0.9
19	Photosynthesis P	0.6	0	1.8	1.6
20	Effluent DO D_{eo}	0	0	0	0
21	Effluent BOD - Le	0	0	0	0
22	Effluent NOD -- Ln	0	0	0	0

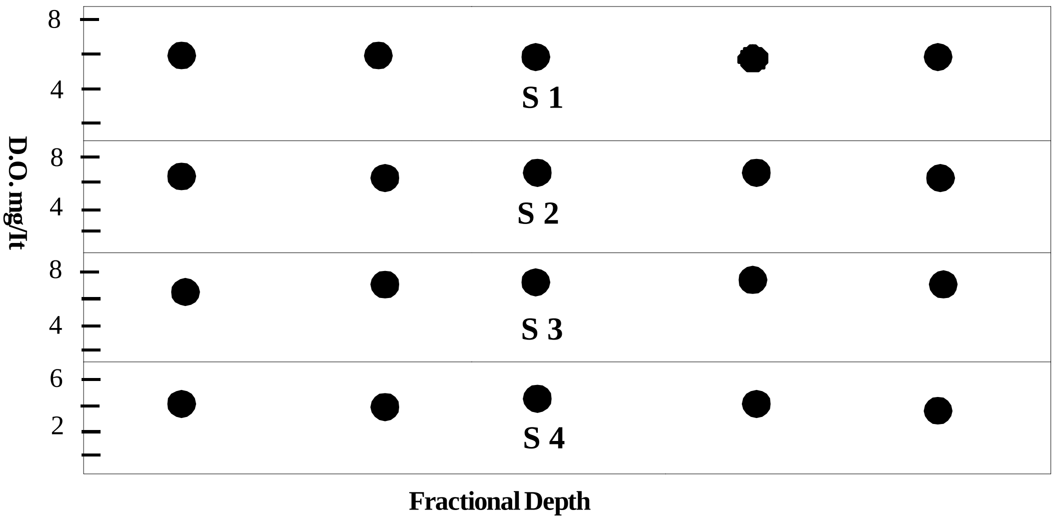

Fig. 3. Variation of Dissolved Oxygen across the river width

oxygen demand (CBOD) and nitrogenous oxygen demand (NOD) that are present in the upstream water and outfall discharge, while the DO source is due to re-aeration.

The formulation for DO distribution in mixing zone consists of three steps;

I) The 1-D relationships for DO deficits due to upstream loadings and distributed internal sources and sinks are considered assuming cross sectional uniformity.

II) 2-D relationships describe the effect of sewage discharge which include:

a) the distributions of CBOD and NOD due to the lateral dispersion and longitudinal decay of the sewage CBOD and NOD;

b) the distribution of DO deficit due to the sewage CBOD and NOD, and re-aeration;

c) The DO deficit distribution in mixing zone due to the sewage DO deficit.

iii) The principle of superposition is invoked to obtain the total DO deficit as the sum of the 1-D and the 2-D deficits (Gowda, 1980).

The DO deficit, D_b due to the combined effects of the decay of background CBOD, NOD and initial DO deficits, and of re-aeration are described by the 1-D transport equation (Chapra, 1997).

$$u.\frac{\partial D_b}{\partial x} = K_d.L_b + K_n.N_b - K_a.D_b \qquad (1)$$

where:

D_b = $(C_s - C_b)$, in mg/ lt.

C_s = DO saturation value, mg/ lt

C_b = DO value, in mg/ lt. at any distance 'x' w.r.t. origin at the outfall

L_b & N_b = CBOD & NOD concentrations at 'x'

K_d & K_n = first order decay rates for CBOD & NOD

K_a = Re-aeration rate

The solution for eqn.(1) is:

$$D_b = D_{bo}.e^{\left(-K_a.\frac{x}{u}\right)} + L_{bo}.F_c + N_{bo}.F_n \qquad (2)$$

Where, L_{bo} & N_{bo} are the initial values of background CBOD & NOD. $D_b = (C_s - C_b)$ in mg/ lt.

$D_{bo} = (C_s - C_{bo})$ = Initial background DO deficit at x =0

C_{bo} = Initial background DO value

The diffusion factor F_c is given by:

$$F_c = \left[\frac{K_d}{(K_a - K_d)}\right]\left[e^{\left(-K_d.\frac{x}{u}\right)} - e^{\left(-K_a.\frac{x}{u}\right)}\right] \qquad (3)$$

And the factor F_n is given by:

$$F_n = \left[\frac{K_n}{(K_a - K_n)}\right].\left[e^{\left(-K_n \cdot \frac{x}{u}\right)} - e^{\left(-K_a \cdot \frac{x}{u}\right)}\right] \quad (4)$$

The concentration of NOD is calculated as

$$NOD = 4.57 \times TKN + 1.14 \times NO_2 \quad (5)$$

TKN = Total Kjeldahl Nitrogen; and NO_2 = Nitrogen Dioxide in the water sample.

For the convenience, the effects of effluent DO and of CBOD & NOD on the in stream DO deficit are considered separately. The readily available 1-D &2-D equqtions[3] are utilized to construct analytical solution for the 2-D deficit distribution in mixing zones through the application of product law concept, as outlined below:

i. Effect of Effluent DO: The in-stream DO deficit due to the effluent DO deficit may be described by the 2-D transport equation:

ii. $$u.\frac{\partial D_e}{\partial x} = u.E_q.\frac{\partial^2 D_e}{\partial q^2} - m_x.K_a.D_e \quad (6)$$

where:

D_e = DO deficit at a point (x,q) in the mixing zone due to the effluent DO deficit D_{eo}

E_q = Transverse diffusion factor

q =Cumulative discharge denoting the lateral co-ordinate

m_x = Scaling factor

The solution to eqn. (4) is given by:

$$D_e = \left(Q_e.\frac{D_{eo}}{Q}\right).e^{\left(-K_a \cdot \frac{x}{u}\right)}.G(\phi, p) \quad (7)$$

Q_e = Effluent flow rate And G (Φ, p) = A factor depending on the type of outfall and location and equal to 1 for steady state condition

ii) Effect of Effluent CBOD: The 2-D distribution of CBOD in the stream due to the effluent CBOD is given by:

$$u.\frac{\partial L}{\partial x} = u.E_q.\frac{\partial^2 L}{\partial q^2} - K_d.L \quad (8)$$

wherein, L = CBOD concentration at a point (x, q)

Solution for the above equation for an effluent CBOD value L_e is

$$L = \left(Q_e.\frac{L_e}{Q}\right).\left[e^{\left(-K_d \cdot \frac{x}{u}\right)}\right].G(\phi, p) \quad (9)$$

The 2-d distribution of DO deficit in the stream channel due to the combined effect of decay and lateral dispersion of CBOD, and re-aeration, is described by the transport equation

$$u.\frac{\partial D_c}{\partial x} = u.E_q.\frac{\partial^2 D_c}{\partial q^2} + K_d.L - K_a.D_c \quad (10)$$

in which, D_c=DO deficit at a point (x, q) due to CBOD.Solution to eqn. (10) is:

$$D_c = \left(Q_e.\frac{L_e}{Q}\right).F_c.G(\phi, p) \quad (11)$$

By following the similar procedure as above for CBOD, we can obtain the expressions the distributions of NOD, N and the corresponding DO deficit, D_n can be obtained, by replacing N, N_e, K_n and F_n in place of L, L_e, K_d and F_c respectively in above equations.

Total DO Deficit:

The distribution of total DO deficit, D_t due to the combined effects of upstream inputs, internal source and sinks, re-aeration and effluent loading is described by the transport equation:

$$u.\frac{\partial D_t}{\partial x} = u.E_q.\frac{\partial^2 D_e}{\partial q^2} + K_d.L_b + K_n.N_b + K_d.L + K_n.N - K_a.D_t + S_p \quad (12)$$

where: $S_p = (S + R - P)$ denoting the algebraic sum of the distributed sinks and source.

$$D_t = (D_b + D_p + D_s)$$

$$D_s = (D_e + D_c + D_n)$$

D_p is the DO deficit due to S_p given by equation

$$D_p = \left(\frac{S_p}{K_a}\right) \cdot \left[1 - e^{\left(K_a \cdot \frac{x}{u}\right)}\right] \qquad (13)$$

The analytical solution for the equation (12) is given by the sum of the DO deficits of the corresponding solutions, which can be written as:

$$D_t = \left\{ \begin{array}{l} \left[D_{bo} \cdot e^{\left(-K_a \cdot \frac{x}{u}\right)} + L_{bo} \cdot F_c + N_{bo} \cdot F_n \right] + \\ \left(\frac{S_p}{K_a}\right) \cdot \left[1 - e^{\left(K_a \cdot \frac{x}{u}\right)} \right] + \left[\begin{array}{l} \left(\frac{Q_e}{Q}\right) \cdot G(\phi, p) \cdot \\ \left(D_{eo} \cdot e^{\left(-K_a \cdot \frac{x}{u}\right)} + L_e \cdot F_n + N_e \cdot F_n \right) \end{array} \right] \end{array} \right\} \qquad (14)$$

In the above equation, the terms within the three sets of the square brackets on the right hand side denote, in order, the DO deficits due to the background, distributed source & sinks and effluent. However, the effect of S_p is not considered in this article for sake of simplicity.The predicted values for the stream under the study, is estimated with the use of the above equations and the values are as shown in Table 2. These predicted and the observed values of Dissolved Oxygen are presented in Table 3. The comparison between the observed and the predicted DO profile for the selected study area of Malaprabha River is simulated as shown in Fig.4.

RESULTS & DISCUSSION

Analyzed results of sewage sample for important parameters, re-oxygenation and de-oxygenation rate constants, respiration and photosynthesis rates and other hydro-geometrical characteristics for the study area are presented in Table 1. Measured dissolved oxygen profile just at the upstream of sewage disposal (S1) and the downstream stations S2 and S3 is presented in Fig.3, which exhibits both lateral and longitudinal variations (Tiseer *et al.*, 2008). Due to gradual spreading of the effluent, lateral concentration

Table 2. Calculated Predicted Dissolved Oxygen Values for Pre Monsoon Season 2005 – 2006

Sl. No.	Parameters	Stations			
		S1	S2	S3	S4
1	Initial DO deficit Dbo	1.00	1.05	1.16	0.70
2	Dispersion coefficient for BOD Fc	0.130	0.151	0.07	0.073
3	Dispersive coefficient for NOD Fn	0.144	0.165	0.09	0.088
4	Nitrogenous BOD Nbo	1.545	3.738	2.17	1.97
5	DO deficit due to u/s loading Db	0.930	1.138	0.35	0.246
6	Effect of effluent DO De	0.000	0.000	0	0
7	Effect of effluent CBOD Dc	0.000	0.000	0	0
8	Effect of effluent NOD Dcn	0.000	0.000	0	0
DO deficit due to sources and sinks					
9	Sources and sinks for DO Sp	0.08	-0.31	1.58	0.8
10	DO deficit due to Sp Dp	0.027	-0.12	0.71	0.424
11	Total DO deficit Dt	0.956	1.016	1.07	0.670
Hence, predicted Ambient DO		**7.44**	**7.10**	**7.39**	**7.43**

Table 3. Predicted and Observed values of Dissolved Oxygen in the study area

Station	S1	S2	S3	S4
Observed DO values, mg/lt.	7.4	7.07	7.3	7.4
Predicted DO values, mg/lt. (as Table 3)	7.44	7.1	7.39	7.43

gradients were established in the mixing zone. This had resulted in spatial distribution of DO as shown pictorially in Fig.3, with minimum DO distribution occurring along the outfall bank, while it was near to saturation along the opposite bank. The distribution factor, however, would depend on the type of outfall, i.e. piped or diffused – and its location in the river. Further, at first two stations, i.e. S1 and S2, the DO concentrations near the opposite bank was close to those at the upstream station S1, but lower DO values at the other two stations – S3 and S4. This clearly indicated the plume spreading effect as in Fig.2. where extent and the distance depending on the velocity and nature of flow of the river water (JCE, 2005; Vellidas et *al*., 2006).

Values of predicted dissolved oxygen deficit are presented in Table 2. Steady states analytical DO model for the mixing zone of Malaprabha River was applied and these DO deficit results were obtained. Actual field DO values were also observed at these stations, and both the values are presented in Table 3. It can be observed from Fig. 4 that the observed and predicted values almost agreed, and the predicted DO values showing slightly higher results. As the DO variations occur due to many factors, such as biological activities in the river, re-aeration and de-oxygenation rates etc., - a 2 to 3 percent variation in DO variation is acceptable (Wang et *al*., 2003). The results indicated that the concentration of DO was less at the effluent disposal point and also along the bank, whereas it was more near the opposite bank. Dispersion coefficient for biochemical oxygen demand varied from 0.151 to 0.07, with 0.151being highest at station S2, indicating the maximum spread of the effluent at the disposal location. Reduced values of dispersion coefficients were observed at other stations, which showed that the effluent dispersion was on lower side due to the effect of dilution (Steele, 1989). Similar observations can be made with other parameters presented in Table 2. Sources

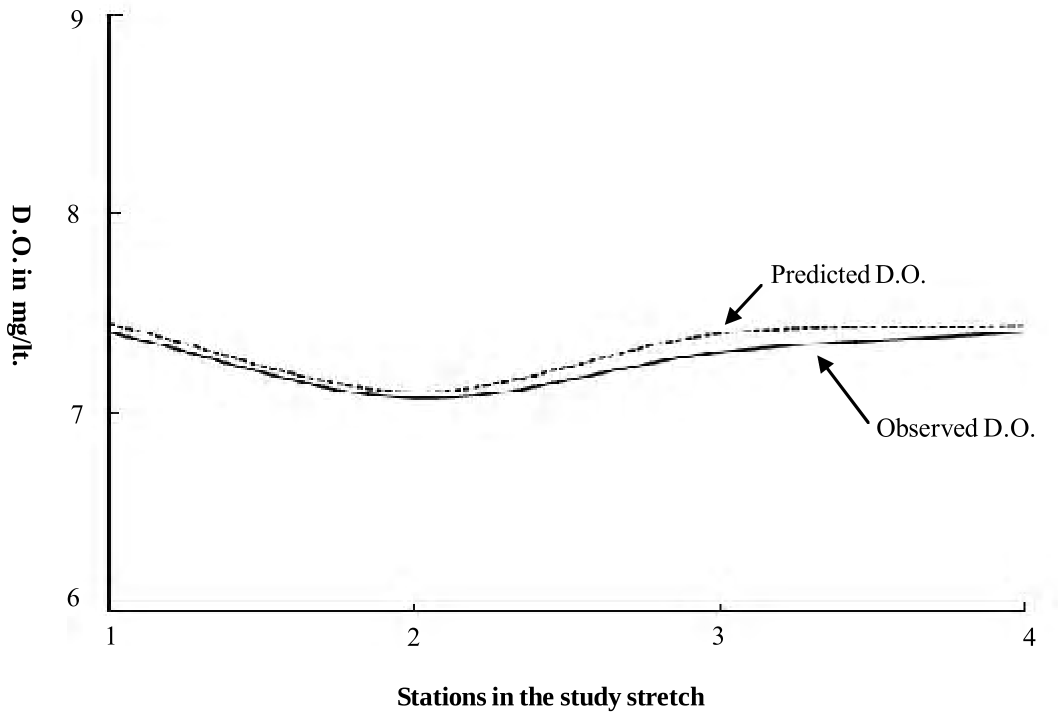

Fig. 4. Simulation of Observed and Predicted D.O. values

and sinks, determined by standard procedures (Chapra, 1997; Thomann, 1987) for the dissolved oxygen indicated less biological activities at station S1 and more at S2. This clearly showed that contamination of river water is very less at upstream of the effluent disposal point and more on the downstream side (Dogan *et al*., 2009).

CONCLUSION

Steady state 2-D analytical solutions are presented for predicting DO distributing in mixing zones of Malaprabha River, by the use of the concept of product law. The effect of background conditions and the effect of sewage quality on DO deficit are considered separately for convenience (Gowda, 1984). For each Station, the deficit due to initial DO level and those due to CBOD and NOD are independently considered. The product law is applied for the analytical solution for the DO deficit prediction.For the Malaprabha River, for which this analytical modeling approach is adopted, the observed and predicted vales of DO deficit are presented in Fig.4. From observations, it can be seen that both the values of the DO deficits agree on the upstream and downstream stretch of the sewage disposal except between the transect 5 and 10, which may be attributed to local conditions (Araujo and Garutti, 2005). Hence, this simulation with respect to DO deficit, confirms the application of this method for predicting the DO deficit values for any changes in the sewage characteristics and stream water, and also for the changes in hydraulic characteristics of both, sewage and river water. The analytical approach can be further extended for other parameters of the river water, such as pH-value, temperature, nitrates, phosphates etc., with modifications in the above equation.

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