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New families of pair difference cordial graphs

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ABSTRACT

In this paper we investigate the pair difference cordial labeling behaviour of diamond ladder graph,lattitude ladder, octopus graph,pagoda graph, planter graph and semi jahangir graph.

Keyword: diamond ladder graph, lattitude ladder, octopus graph, pagoda graph, planter graph, semi jahangir graph.

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1 Introduction

In this paper we consider only finite, undirected and simple graphs. Cordial labeling was introduced by Cahit[2] in the year 1987. Prime labeling behaviour of planter graph, duplication of planter graph, fusion of planter graph,switching of planter graph, joining of two copies of planter graph were studied by A.Edward samuel and S.Kalaivani[4]. Dafik, Riniatul Nur Wahidah hve been examined the Rainbow Antimagic coloring of special graphs like volcano, sandat graph, sunflower,octpus, semijahangir [3]. Prihandini,R M., at.el have been studied the elegant labeling of shackle graphs and diamond ladder graphs [16]. Classical meanness of some graphs such as one-side step graph,double-sided step graph,grid,slanding ladder,diamond ladder,lattitude ladder was studied by Alanazi et. al [1]. In [17] Yeni Susanti et. al studied the edge odd geaceful labeling behaviour of prism, antiprism, cartesian product graphs. The notion of pair diference cordial labeling of a graph was introduced in [7]. The pair difference cordial labeling behaviour of certain graphs like path, cycle, star, wheel,triangular snake,alternate triangular snake, butterfly etc have been investigated in [8-15]. In this paper we investigate the pair difference cordial labeling behaviour of diamond ladder graph,lattitude ladder, octopus graph,pagoda graph, planter graph, semi jahangir graph . Terms not defined here are follow from Gallian[6] and Harary[7].

2 Preliminaries

Definition 2.1. [1]. *The diamond ladder graph is the graph obtained from ladder L_n and it is denoted by DL_n. Let $V(DL_n) = \{x_i : 1 \leq i \leq n\} \cup \{y_i : 1 \leq i \leq n\} \cup \{z_i : 1 \leq i \leq 2n\}$ and $E(DL_n) = \{x_i x_{i+1}, y_i y_{i+1} : 1 \leq i \leq n-1\} \cup \{x_i y_i : 1 \leq i \leq n\} \cup \{x_i z_{2i-1}, y_i z_{2i-1} : 1 \leq i \leq n\} \cup \{x_i z_{2i}, y_i z_{2i} : 1 \leq i \leq n\} \cup \{z_{2i} z_{2i+1} : 1 \leq i \leq n-1\}$. It is easy to verify that the DL_n has $4n$ vertices and $6n-1$ edges.*

Definition 2.2. [4]. *Let $F_n = P_n + K_1$, $n \geq 2$ where P_n be the path $u_1 u_2 u_3 \cdots u_n$ and $V(K_1) = \{u\}$. Let C_n be the cycle $v_1 v_2 v_3 \cdots v_n v_1, n \geq 3$. The planter graph R_n, $n \geq 3$ is obtained from F_n and C_n by identifying the vertices u and v_1. That is $V(R_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(R_n) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_1 u_i : 1 \leq i \leq n\} \cup \{v_1 v_n\}$. The planter graph has $2n$ vertices and $3n-1$ edges.*

Definition 2.3. [3]. *The octopus graph O_n is the graph whose vertex set $V(O_n) = \{u, u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq n-1\}$ and the edge set $E(O_n) = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{uu_i, uv_i : 1 \leq i \leq n\}$. The octopus graph has $2n+1$ vertices and $3n-1$ edges.*

Definition 2.4. [1]. *Let n be an even integer . The lattitude ladder graph LL_n,$n \geq 4$ is the graph with vertex set $V(LL_n) = \{x_i : 1 \leq i \leq n\}$ and edge set $E(LL_n) = \{x_i x_{i+1} :$*

$1 \leq i \leq n-1\} \cup \{x_i x_{n+2-i} : 2 \leq i \leq \frac{n}{2}\}$.

Definition 2.5. [6]. *The pagoda graph PG_n,$n \geq 3$ is the vertex set $V(PG_n) = \{u, u_i, v_i : 1 \leq i \leq n\}$ and edge set $E(PG_n) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq n\} \cup \{u_1 u, v_1 u\}$. The pagoda graph has $2n+1$ vertices and $3n$ edges.*

Definition 2.6. [6]. *D_n^* is the graph with the vertex set $V(D_n^*) = \{x_i : 1 \leq i \leq n\} \cup \{y_i : 1 \leq i \leq n\} \cup \{z_i : 1 \leq i \leq 2n\}$ and the edge set $E(D_n^*) = \{x_i x_{i+1}, y_i y_{i+1} : 1 \leq i \leq n-1\} \cup \{x_i z_{2i-1}, y_i z_{2i-1} : 1 \leq i \leq n\} \cup \{x_i z_{2i}, y_i z_{2i} : 1 \leq i \leq n\} \cup \{z_{2i} z_{2i+1} : 1 \leq i \leq n-1\}$. The D_n^* graph has $4n$ vertices and $7n-3$ edges.*

Definition 2.7. [6]. *The volcano graph V_n is the graph with the vertex set is $V(V_n) = \{x, y, z\} \cup \{x_i : 1 \leq i \leq n\}$ and the edge set is $E(V_n) = \{xy, yz, xz\} \cup \{xx_i : 1 \leq i \leq n\}$. V_n has $n+3$ vertices and $n+3$ edges.*

3 Pair difference cordial labeling

Definition 3.1. Let $G = (V, E)$ be a (p, q) graph.
Define

$$\rho = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd} \end{cases}$$

and $L = \{\pm 1, \pm 2, \pm 3, \cdots, \pm \rho\}$ called the set of labels.
Consider a mapping $f : V \longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to p-1 elements of V and repeating a label for the remaining one vertex when p is odd.The labeling as defined above is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $|f(u) - f(v)|$ such that $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \leq 1$, where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the number of edges labeled with 1 and number of edges not labeled with 1.A graph G for which there exists a pair difference cordial labeling is called a pair difference cordial graph.

4 main results

Theorem 4.1. The pagoda graph PG_n is pair difference cordial for all values of $n \geq 3$.

Proof. Let us consider the vertex set and edge set from definition 2.1.

Assign the label $1, 2, 3, \cdots, n$ to the vertices $u_1, u_2, u_3, \cdots, u_n$ respectively. Assign the labels $-1, -2$ respectively to the vertices v_1, v_2 and assign the labels $-4, -3$ respectively to the vertices v_3, v_4 . Next assign the labels $-5, -6$ to the vertices v_5, v_6 respectively and assign the labels $-8, -7$ respectively to the vertices v_7, v_8. Proceeding like this until reach v_n. Finally assign the label 2 to the vertex u.

Note that the vertices v_{n-1}, v_n to the vertex $-n, -n+1$ when $n \equiv 0 \pmod 4$. The vertices v_{n-1}, v_n to the vertex $-n+2, -n$ when $n \equiv 1 \pmod 4$.
The verticesv_{n-1}, v_n to the vertex $-n+1, -n$ when $n \equiv 2 \pmod 4$. The vertices v_{n-1}, v_n to the vertex $-n+1, -n$ when $n \equiv 3 \pmod 4$.

The Table 1 given below establish that this vertex labeling is a pair difference cordial labeling of PG_n for all values of $n \geq 3$.

Nature of n	Δ_{f_1}	$\Delta_{f_1^c}$
$n \equiv 0 \pmod 4$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 1 \pmod 4$	$\frac{3n+1}{2}$	$\frac{3n-1}{2}$
$n \equiv 2 \pmod 4$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 3 \pmod 4$	$\frac{3n+1}{2}$	$\frac{3n-1}{2}$

Table 1:

Theorem 4.2. O_n is pair diffference codial if and only if $2 \leq n \leq 4$.

Proof. **Case 1.** $2 \leq n \leq 4$.
The pair difference cordial labeling of O_n , $2 \leq n \leq 4$ is shown in the following figure 1
Case 2. $n \geq 5$.

The maximum possible number of edges with the label 1 is
$\Delta f_1 = \underbrace{n-1}_{\text{for the path}P_n} + \underbrace{2}_{\text{for the star}}$. Therefore $\Delta f_1 = n+1$. Since $|E(O_n)| = 3n-1$, ${\Delta f_1}^c = (3n-1) - (n+1) = 2n-2$. Hence $|\Delta f_1 - {\Delta f_1}^c| = 2n-2-n-1 = n-3 > 1$, which is a contradiction.
Hence the octopus graph O_n is not pair difference cordial for all values of $n \geq 5$.

Theorem 4.3. The volcano graph V_n is pair difference cordial if and only if $1 \leq n \leq 4$.

Proof. Let us consider the vertex set and edge set from definition 2.7.
There are two cases arises.
Case 1. $1 \leq n \leq 4$.
The pair difference cordial labeling of V_n , $1 \leq n \leq 4$ is shown in the following figure 2

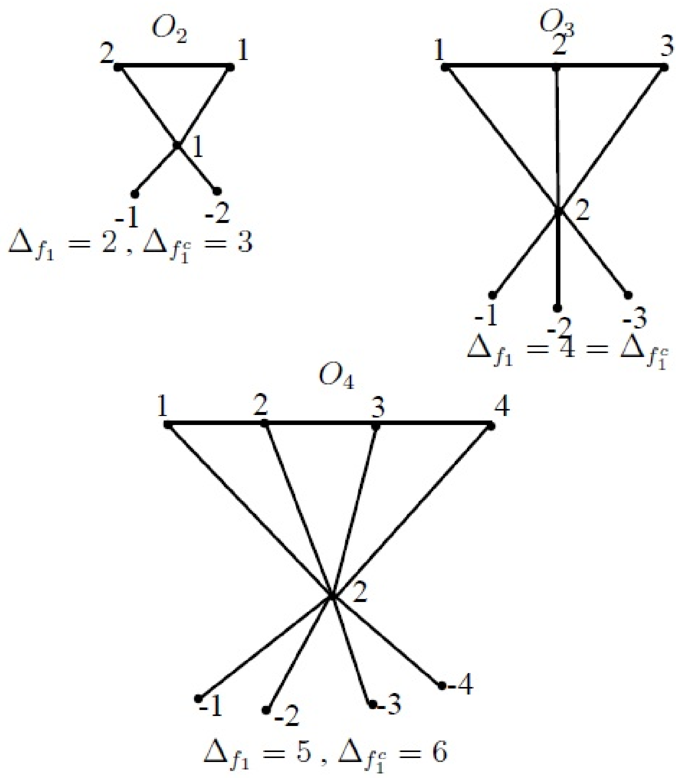

Figure 1: case 1

Case 2. $n \geq 5$.

The maximum possible number of edges with the label 1 is $\Delta f_1 = 3$. Since$|E(V_n)| = n+3$, ${\Delta f_1}^c = (n+3) - 3 = n$. Hence $|\Delta f_1 - {\Delta f_1}^c| = n - 3 > 1$ when $n \geq 5$, which is a contradiction.
Hence the volcano graph V_n is not pair difference cordial for all values of $n \geq 5$.

Theorem 4.4. The Planter graph R_n is pair difference cordial for all values of $n \geq 3$.

Proof.
Let us consider the vertex set and edge set from definition 2.2.

Assign the label $1, 2, 3, \cdots, n$ to the vertices $u_1, u_2, u_3, \cdots, u_n$ respectively. Assign the labels $-1, -2$ respectively to the vertices v_1, v_2 and assign the labels $-3, -5$ respectively to the vertices v_3, v_4 . Next assign the labels $-4, -6$ to the vertices v_5, v_6 respectively and assign the labels $-7, -9$ respectively to the vertices v_7, v_8. Proceeding like this until

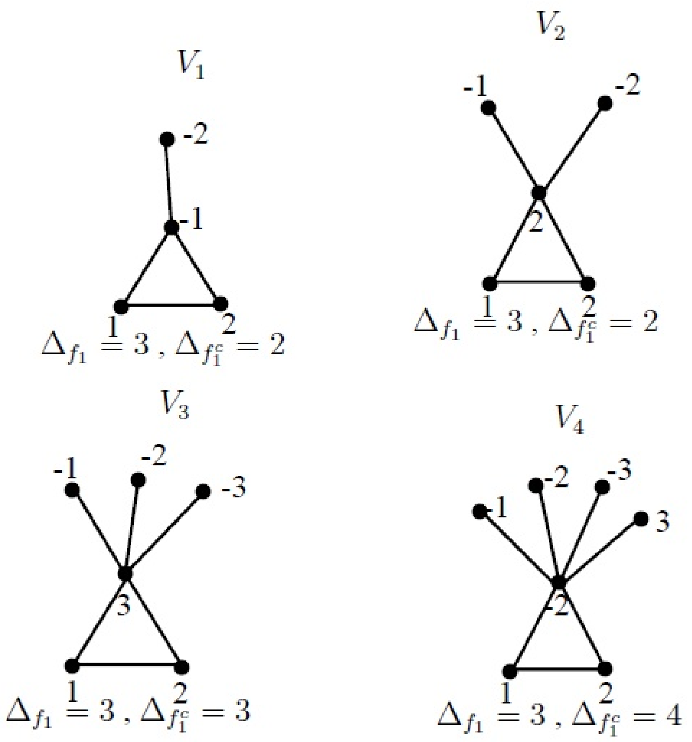

Figure 2: case 2

reach v_n.

Note that the vertices v_{n-1}, v_n to the vertex $-n+1, -n$ when $n \equiv 0 \pmod 4$. The vertices v_{n-1}, v_n to the vertex $-n, -n+1$ when $n \equiv 1 \pmod 4$.
The verticesv_{n-1}, v_n to the vertex $-n+2, -n$ when $n \equiv 2 \pmod 4$. The vertices v_{n-1}, v_n to the vertex $-n+1, -n$ when $n \equiv 3 \pmod 4$.

The Table 2 given below establish that this vertex labeling is a pair difference cordial labeling of R_n for all values of $n \geq 3$.

Nature of n	Δ_{f_1}	$\Delta_{f_1^c}$
$n \equiv 0 \pmod 4$	$\frac{3n}{2}$	$\frac{3n-2}{2}$
$n \equiv 1 \pmod 4$	$\frac{3n-1}{2}$	$\frac{3n-1}{2}$
$n \equiv 2 \pmod 4$	$\frac{3n-2}{2}$	$\frac{3n}{2}$
$n \equiv 3 \pmod 4$	$\frac{3n-1}{2}$	$\frac{3n-1}{2}$

Table 2:

Theorem 4.5. The diamond ladder graph DL_n is pair difference cordial for all values of

$n \geq 2$.

Proof.
Let us consider the vertex set and edge set from definition 2.1.
There are two cases arises.
Case 1. n is even.

Assign the labels $2, 6, 10, \cdots, 2n-2$ to the vertices $x_1, x_3, x_5, \cdots, x_{n-1}$ respectively and assign the labels $-2, -6, -10, \cdots, -(2n-2)$ respectively to the vertices $x_2, x_4, x_6, \cdots, x_n$. Next assign the labels $4, 8, 12, \cdots, 2n$ to the vertices $y_1, y_3, y_5, \cdots,$ y_{n-1} respectively and assign the labels $-4, -8, -12, \cdots, -2n$ respectively to the vertices $y_2, y_4, y_6, \cdots, y_n$.

Assign the labels $1, 5, 9, \cdots, 2n-3$ to the vertices $z_1, z_5, z_9, \cdots, z_{2n-3}$ respectively and assign the labels $-1, -5, -9, \cdots, -(2n-3)$ respectively to the vertices $z_3, z_7, z_{11}, \cdots, z_{2n-1}$. Now assign the labels $3, 7, 11, \cdots, 2n-1$ to the vertices $z_2, z_6, z_{10}, \cdots, z_{2n-2}$ respectively and assign the labels $-3, -7, -11, \cdots, -(2n-1)$ to the vertices $z_4, z_8, z_{12}, \cdots, z_{2n}$ respectively.
Case 2. n is odd.

Assign the labels $2, 6, 10, \cdots, 2n-4$ to the vertices $x_1, x_3, x_5, \cdots, x_{n-2}$ respectively and assign the labels $-2, -6, -10, \cdots, -(2n-4)$ respectively to the vertices $x_2, x_4, x_6, \cdots, x_{n-1}$. Next assign the labels$4, 8, 12, \cdots, 2n-2$ to the vertices $y_1, y_3, y_5, \cdots, y_{n-2}$ respectively and assign the labels $-4, -8, -12, \cdots, -(2n-2)$ respectively to the vertices $y_2, y_4, y_6, \cdots, y_{n-1}$.

Assign the labels $1, 5, 9, \cdots, 2n-5$ to the vertices $z_1, z_5, z_9, \cdots, z_{2n-5}$ respectively and assign the labels $-1, -5, -9, \cdots, -(2n-5)$ respectively to the vertices $z_3, z_7, z_{11}, \cdots, z_{2n-3}$. Now assign the labels $3, 7, 11, \cdots, 2n-3$ to the vertices $z_2, z_6, z_{10}, \cdots, z_{2n-4}$ respectively and assign the labels $-3, -7, -11, \cdots, -(2n-3)$ to the vertices $z_4, z_8, z_{12}, \cdots, z_{2n-2}$ respectively.
Finally assign the labels $2n-1, 2n, -(2n-1), -2n$ to the vertices $x_n, z_{2n-1}, z_{2n}, y_n$ respectively.
The Table 3 given below establish that this vertex labeling is a pair difference cordial labeling of DL_n for all values of $n \geq 2$.

Nature of n	Δ_{f_1}	$\Delta_{f_1^c}$
$n \equiv 0, 2 \pmod 4$	$3n$	$3n-1$
$n \equiv 1, 3 \pmod 4$	$3n-1$	$3n$

Table 3:

Theorem 4.6. The lattitude ladder graph LL_n is pair difference cordial for all even values of $n \geq 4$.

Proof. Let us consider the vertex set and edge set from definition 2.4.
Assign the labels $1, 2, 3, \cdots, \frac{n}{2}$ to the vertices $x_1, x_2, x_3, \cdots, x_{\frac{n}{2}}$ and assign the labels $-1, -2, -3, \cdots, \frac{n-4}{2}$ respectively to the vertices $x_{\frac{n+}{2}}, x_{\frac{n+}{2}}, x_{\frac{n+}{2}}, \cdots x_{n-2}$. Finally assign the labels $\frac{n}{2}, \frac{n-2}{2}$ to the vertices x_{n-1}, x_n.
The Table 4 given below establish that this vertex labeling is a pair difference cordial labeling of LL_n for all even values of $n \geq 4$.

Nature of n	Δ_{f_1}	$\Delta_{f_1^c}$
$n \equiv 0 \pmod 4$	$\frac{3n-8}{2}$	$\frac{3n-6}{2}$
$n \equiv 2 \pmod 4$	$\frac{3n-4}{2}$	$\frac{3n-4}{2}$

Table 4:

Theorem 4.7. The graph D_n^* is pair difference cordial for all values of $n \geq 3$.

Proof.
Let us consider the vertex set and edge set from definition 2.6.
There are two cases arises.
Case 1. n is even.

Assign the labels $2, 6, 10, \cdots, 2n-2$ to the vertices $x_1, x_3, x_5, \cdots, x_{n-1}$ respectively and assign the labels $-2, -6, -10, \cdots, -(2n-2)$ respectively to the vertices $x_2, x_4, x_6, \cdots, x_n$. Next assign the labels $4, 8, 12, \cdots, 2n$ to the vertices $y_1, y_3, y_5, \cdots,$
y_{n-1} respectively and assign the labels $-4, -8, -12, \cdots, -2n$ respectively to the vertices $y_2, y_4, y_6, \cdots, y_n$.

Assign the labels $1, 5, 9, \cdots, 2n-3$ to the vertices $z_1, z_5, z_9, \cdots, z_{2n-3}$ respectively and assign the labels $-1, -5, -9, \cdots, -(2n-3)$ respectively to the vertices $z_3, z_7, z_{11}, \cdots, z_{2n-1}$. Now assign the labels $3, 7, 11, \cdots, 2n-1$ to the vertices $z_2, z_6, z_{10}, \cdots, z_{2n-2}$ respectively and assign the labels $-3, -7, -11, \cdots, -(2n-1)$ to the vertices $z_4, z_8, z_{12}, \cdots, z_{2n}$ respectively.

Case 2. n is odd.

Assign the labels $2, 6, 10, \cdots, 2n-4$ to the vertices $x_1, x_3, x_5, \cdots, x_{n-2}$ respectively and assign the labels $-2, -6, -10, \cdots, -(2n-4)$ respectively to the vertices $x_2, x_4, x_6, \cdots, x_{n-1}$. Next assign the labels $4, 8, 12, \cdots, 2n-2$ to the vertices $y_1, y_3, y_5, \cdots, y_{n-2}$ respectively and assign the labels $-4, -8, -12, \cdots, -(2n-2)$ respectively to the vertices $y_2, y_4, y_6, \cdots, y_{n-1}$.

Assign the labels $1, 5, 9, \cdots, 2n-5$ to the vertices $z_1, z_5, z_9, \cdots, z_{2n-5}$ respectively and assign the labels $-1, -5, -9, \cdots, -(2n-5)$ respectively to the vertices $z_3, z_7, z_{11}, \cdots, z_{2n-3}$. Now assign the labels $3, 7, 11, \cdots, 2n-3$ to the vertices $z_2, z_6, z_{10}, \cdots, z_{2n-4}$ respectively and assign the labels $-3, -7, -11, \cdots, -(2n-3)$ to the vertices $z_4, z_8, z_{12}, \cdots, z_{2n-2}$ respectively.
Finally assign the labels $2n-1, 2n, -(2n-1), -2n$ to the vertices $x_n, z_{2n-1}, z_{2n}, y_n$ respectively.
The Table 5 given below establish that this vertex labeling is a pair difference cordial labeling of D_n^* for all values of $n \geq 3$.

Nature of n	Δ_{f_1}	$\Delta_{f_1^c}$
n is even	$3n-1$	$3n-1$
n is odd	$3n-1$	$3n-1$

Table 5:

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